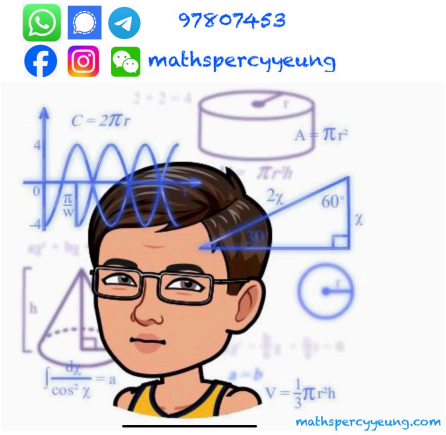
2023-2024 Second Term Examination

F. 3 Mathematics

Paper 1



Time allowed : 1 hour 30 minutes

Full mark : 80

This question-answer book consists of 20 printed pages.

Instructions to candidates:

1. This paper must be answered in English with a blue / black ball pen, unless otherwise specified.

2. Write your name, class and class number in the spaces provided on this cover.

3. This paper consists of TWO sections, A and B.
   Section A carries 40 marks and Section B carries 40 marks.

4. Answer ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

5. All diagrams / graphs / charts as part of the answers must be clearly drawn with an HB pencil.

6. Graph paper and supplementary answer sheets will be supplied on request. Write your name, class and class number on each sheet, and fasten them INSIDE this book.

7. Unless otherwise specified, all working must be clearly shown.

8. The diagrams in this paper are not necessarily drawn to scale.

9. Unless otherwise specified, numerical answers must be exact or correct to 3 significant figures.

10. Calculator pad printed with the "HKEA Approved" / "HKEAA Approved" label is allowed.
    Remove the calculator cover / jacket.

Section A (40 marks)

1. Simplify $\frac{m^{-12}n}{(m^3n^6)^{-2}}$ and express your answer with positive indices. (3 marks)

2. Factorize

   (a) $6a^2 - 13ab - 28b^2$,

   (b) $8a - 28b + 6a^2 - 13ab - 28b^2$.

(3 marks)

Answers written in the margins will not be marked.

3. (a) Solve the inequality $\frac{10x+7}{4}-1>4x$.

(b) Write down the greatest integer satisfying the inequality in (a).

(3 marks)

4. Sandy wants to deposit \$40 000 in a bank for 6 years. Bank *A* offers a compound interest at 4% p.a., compounded half-yearly. Bank *B* offers a simple interest of *r*% p.a.. If the interest earned from Bank *A* is twice that earned from Bank *B*, find the value of *r* correct to 4 decimal places.

(3 marks)

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5. (a) Simplify $\dfrac{\sin^3\theta + \sin\theta\sin^2(90^\circ - \theta)}{\cos\theta}$.

(b) Hence, solve $\dfrac{\sin^3\theta + \sin\theta\sin^2(90^\circ - \theta)}{\cos\theta} = \dfrac{1}{\tan(\theta + 10^\circ)}$.

(5 marks)

Answers written in the margins will not be marked.

6. The stem-and-leaf diagram below shows the distribution of the scores obtained by a group of students in a test.

Stem (tens)	Leaf (units)
6	*a* *a* *a* 4 5 5 6
7	0 *a* *a* 8 8 9 9
8	1 1 2 3 4 5

The median of the distribution is 75.

(a) Find the mean and the mode of the distribution.

(b) If a student is randomly selected from the group, find the probability that the score obtained by the student is higher than the mode of the distribution.

(c) If the score of each student is increased by 220%, write down the median of the new distribution.

(5 marks)

Answers written in the margins will not be marked.

7. In Figure 1, the true bearing of R from P is 118°. It is given that $PR = 2$ km and $\angle PQR = 84°$. P and R are equidistant from Q.

   (a) Find the true bearing of Q from P.

   (b) Find the shortest distance from R to PQ.

   (c) S is a point such that $PQRS$ is a rhombus. Find the compass bearing of S from P.
   (S is not shown in the figure.)

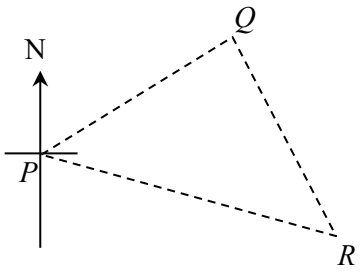

Figure 1

(6 marks)

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8. In Figure 2, E is the point of intersection of AC and BD . It is given that C is the orthocentre of ΔACB , D is the orthocentre of ΔADB and $AD = BC$.

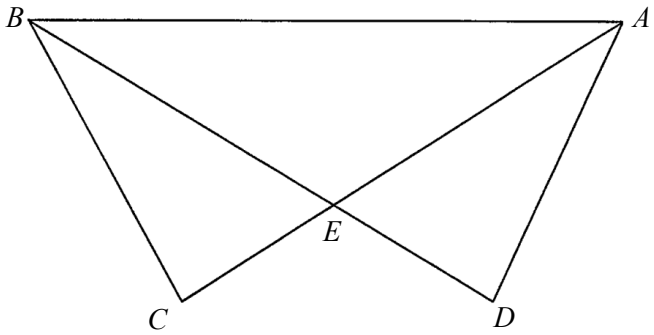

Figure 2

(a) Prove that $\Delta ABC \cong \Delta BAD$.

(b) Someone claims that the circumcentre of ΔABC and the circumcentre of ΔBAD are located at the same point. Do you agree? Explain your answer.

(c) If AD = 24 cm and DE = 18 cm, find the area of the pentagon $BADEC$.

(6 marks)

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9. A solid metal sphere with radius 6 cm is melted and recast into a solid right circular cone. The base radius of the cone is twice the height of the cone.

(a) Find the base radius of the cone.

(b) Someone claims that the total surface area of the solid has increased by more than 5×10^4 mm^2. Do you agree? Explain your answer.

(6 marks)

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Section B (40 marks)

10. Katie gets the following results for General Studies this term. The full mark of each item is 100. A student is considered "failed" if the student's weighted mean mark is lower than 50.

	Exam	**Form Test**	**Quiz**	**Test**	**Homework**
Marks	37	70	85	50	91
Weight	60	15	y	$20 - y$	5

It is given that y is a positive integer and Katie has failed in General Studies.

(a) Express the weighted mean mark of Katie in terms of y. Hence find the largest possible value of y.

(b) Find the lowest possible weighted mean mark of Katie.

(5 marks)

Answers written in the margins will not be marked.

11. Refer to Figure 3, denote the origin by O. The coordinates of A and B are $(k, 0)$ and $(k + 20, 0)$ respectively, where $k > 0$. C is a point vertically above A such that $AB = AC$. Let D be a point such that $ACDB$ is a square. (C and D are not shown in the figure.)

(a) Write down the coordinates of C and D in terms of k. (2 marks)

(b) Express the slope of OD in terms of k. (1 mark)

(c) E is a point obtained by translating B to the right by 5 units. It is given that $OD \perp DE$.

(i) Find the value of k.

(ii) Someone claims that the perimeter of the trapezium $OCDE$ is greater than 190 units. Do you agree? Explain your answer.

(5 marks)

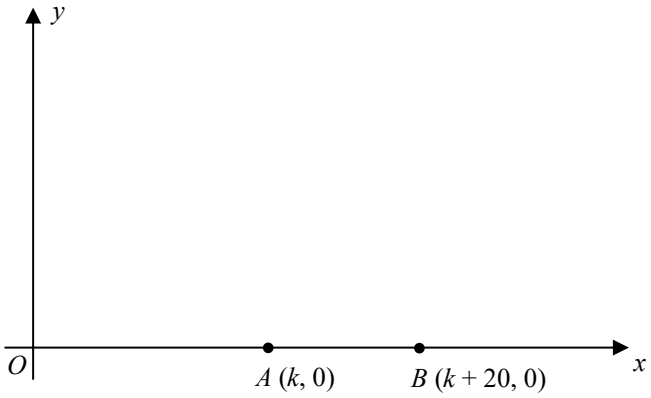

Figure 3

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12. A box contains 4 balls, of which two are gold and two are black. In a game, the particpant may choose Option *A* or Option *B*. For Option *A*, the participant draws one ball randomly from the box. For Option *B*, the participant draws two balls randomly without replacement from the box. The prizes won are shown in the table below.

Option *A* (Draws one ball):

Number of gold ball(s) drawn	Prize
0	$ 0
1	$ 90

Option *B* (Draws two balls without replacement):

Number of gold ball(s) drawn	Prize
0	$ 0
1	$ 30
2	$ 120

(a) If a participant chooses Option *B*, find the probability that two gold balls are drawn. (2 marks)

(b) Amber wants to choose an option with a higher expected value of the prize won. Which option should she choose? Explain your answer. (4 marks)

(c) Andrew wants to choose an option which has a higher probability to win a prize. Which option should he choose? Explain your answer. (2 marks)

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13. The base of a solid right pyramid is a square of area 1024 cm^2 . The height of the pyramid is 12 cm. The pyramid is divided into a frustum X and a smaller pyramid Y by a plane which is parallel to its base. It is given that the height of Y is 9 cm.

(a) Find the volume of the frustum X . (3 marks)

(b) Find the total surface area of the frustum X. (4 marks)

(c) The base of another solid right pyramid is a square. This pyramid is divided into a frustum Z and a pyramid by a plane which is parallel to its base. The height and the total surface area of Z are 6 cm and 3840 cm^2 respectively. Are X and Z similar? Explain your answer. (2 marks)

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14. Refer to Figure 4, at 10:02:05 a.m., the angle of depression of an airport A on the ground from a plane at P was 5°. The plane was flying horizontally at a constant speed of 840 km/h and it took 5 minutes to fly from P to Q. The angle of depression of A from Q was 10°. It is given that B is vertically above A such that B, Q and P lie on a straight line.

(a) Find the height of B above the ground. (4 marks)

(b) The plane started the landing procedure when the angle of depression of A from the plane became 20°.

(i) Find the time at which the plane started the landing procedure, correct to the nearest second.

(ii) After the start of the landing procedure, the plane reduced its speed and flew directly towards A. The landing procedure was completed when the plane arrived at A at 10:13:05 a.m. Find the average speed of the plane during the landing procedure. Give your answer in km/h.

(6 marks)

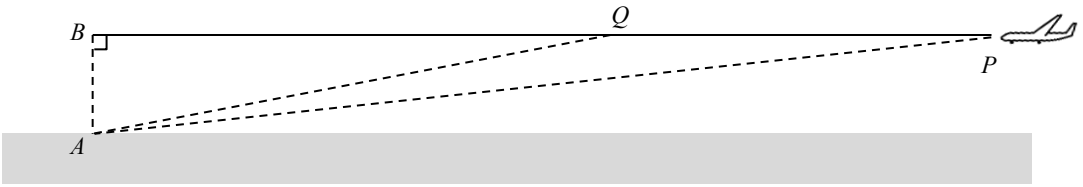

Figure 4

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