Final Examination 2023 – 2024
S.5 Mathematics
Paper 1

Name : _____ ( )

Class : S. 5 _____

Date : 11 June 2024

Time : 08:30 – 10:45 (2 hours and 15 minutes)

Past Paper Revision for 2526 S.5

Paper	Section	Marks
1	A(1)	/ 35
	A(2)	/ 35
	B	/ 35
	PP–_____	
	Total	/ 105
2		/ 45
Total		/ 100

INSTRUCTIONS

1. After the ar t write your name, class and cl

2. This paper

3. Attempt **Al** es provided in this Question-A

4. Graph pape . Write your name, class, class NSIDE this book.

5. Unless othe

6. Unless oth ct or correct to 3 significant f

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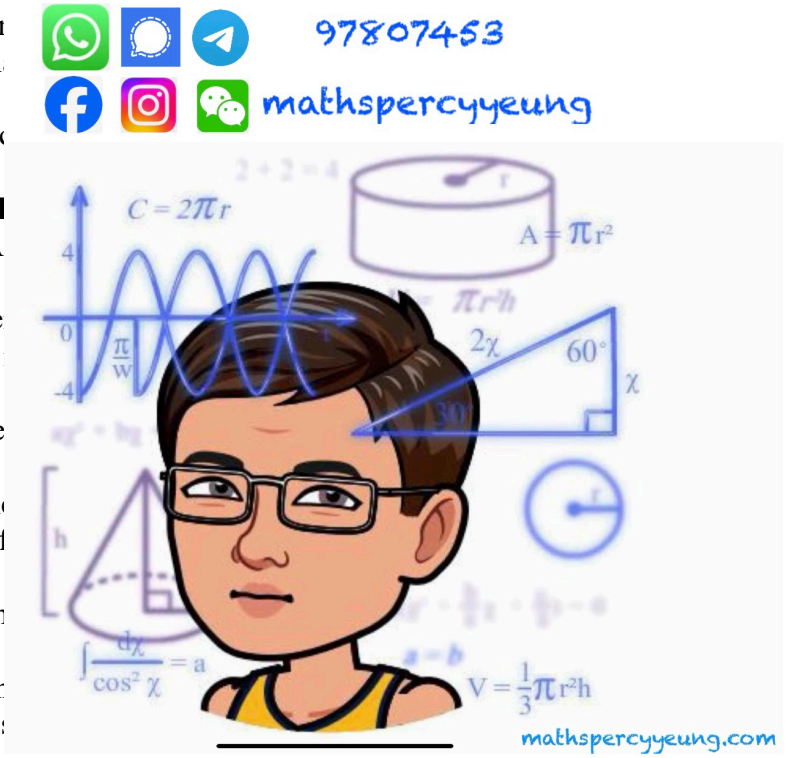


SECTION A(1) [35 marks]

1. Simplify $\frac{(ab^{-3})^4}{a^{-2}b^5}$ and express your answer with positive indices. **(3 marks)**

2. Make y the subject of the formula $\frac{x+2y}{3} = \frac{4-y}{x}$. **(3 marks)**

3. **(a)** Round up 1352.4976 to the nearest thousand.
(b) Round down 1352.4976 to 2 decimal places.
(c) Round off 1352.4976 to 4 significant figures.

(3 marks)

4. Factorize

(a) $4a^2-12a+9$,

(b) $\left(5a^2+1\right)^2-4a^2+12a-9$.

(4 marks)

5. Consider the compound inequality

$\frac{x+5}{6} \le -x+2$ and $-2x+1<5$ …… (*).

(a) Solve (*).

(b) How many non-negative integers satisfy (*)?

(4 marks)

6. In a hall, the number of boys is 25% less than the number of girls. If one girl leaves the hall, then the ratio of number of boys to that of girls is $7:9$. Find the number of boys in the hall. **(4 marks)**

7. The coordinates of the points A and B are $(8,3)$ and $(3,-6)$ respectively. A is rotated anticlockwise about O through $90°$ to A', where O is the origin. B' is the reflection image of B with respect to the x-axis.

(a) Write down the coordinates of A' and B'.

(b) Is $AB \perp A'B'$? Explain your answer.

(4 marks)

8. The marked price of a book is \$120 higher than its cost. A 20% profit is made by selling the book at a discount of 25% on its marked price. Find the cost and profit by selling the book.

(5 marks)

9. In **Figure 1**, $ABCD$ is a circle. AC and BD intersect at E.

(a) Prove that $\Delta ABE \sim \Delta DCE$.

(b) It is given that $AE = 9$ cm and $BE = 12$ cm, $CE = 16$ cm and $CD = 20$ cm. Someone claims that AC is the perpendicular bisector of BD. Do you agree? Explain your answer.

(5 marks)

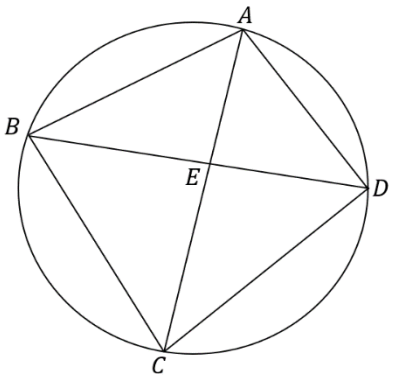


Figure 1

SECTION A(2) **[35 marks]**

10. It is given that h(x) partly varies as x and partly varies inversely as x. Suppose that h(1) = –2 and h(–6) = –5.5 .

(a) Find h(x). **(3 marks)**

(b) Solve the equation h(x) = 2. **(3 marks)**

11. The stem-and-leaf diagram below shows the distribution of the ages of a group of people.

Stem (10)	Leaf (1)						
2	8	9					
3	0	*a*	4	6	8		
4	0	1	1	3	4	7	*b*

It is given that the mean and the range of the above distribution are 38 and 21 respectively.

(a) (i) Find a and b. **(3 marks)**

(ii) Write down the median and the standard deviation of the above distribution. **(2 marks)**

(b) If a person is randomly selected from the group, find the probability that the age of the selected person is below 33. **(2 marks)**

12. The coordinates of the points A and B are $(4,4)$ and $(18,-44)$ respectively. AB is a diameter of circle C.

(a) Find the equation of C. **(2 marks)**

(b) The coordinates of D are $(-14,-20)$. Let P be a moving point on the same rectangular coordinate plane such that the perpendicular distance from P to the line joining A and D is equal to the perpendicular distance from P to the line joining B and D. Denote the locus of P by Γ. By considering the geometric relationship between Γ and $\angle ADB$, find the slope of Γ, where it is positive. **(4 marks)**

13. A thin metal sheet of the shape of a sector has an area of 175π cm^2 with radius 25 cm is folded into a right circular cone A. The inverted cone A is held vertically and is fully filled with water.

(a) Find the volume of the water. **(3 marks)**

(b) A right circular cone B is similar to cone A. The height of cone A is twice that of cone B. Some water is poured from cone A to cone B such that cone B is fully filled with water. A student claims that the decrease in the wet surface area of cone A is more than 50 cm^2. Do you agree? Explain your answer. **(4 marks)**

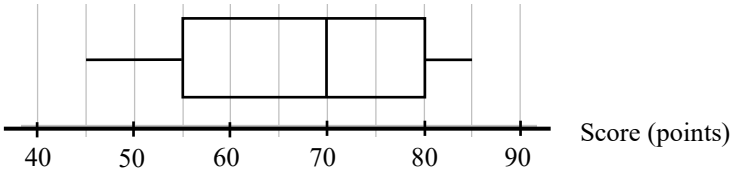
14. Let $p(x) = 6x^4 - 13x^3 + ax^2 + bx + c$, where a, b and c are constants. When $p(x)$ is divided by x, the remainder is -96. It is given that $p(x) = (3x^2 + rx - 24)(mx^2 - 5x + n)$, where r, m and n are constants.

(a) Find r, m and n. **(5 marks)**

(b) Let $q(x) = k(3x^2 + rx - 24)$, where k is a non-zero constant. Find the range of values of k such that $p(x) - q(x) = 0$ has at least 3 distinct real roots. **(4 marks)**

SECTION B [35 marks]

15. The box-and-whisker diagram below shows the distribution of the score (in points) of a group of players in a game. James gets the lowest score while Thomas gets 61 marks in the game. The standard scores of James and Thomas are -2.5 and -0.5 respectively.



(a) Find the mean of the distribution. **(2 marks)**

(b) Karina claims that the standard scores of at least half of the players in the game are positive. Do you agree? Explain your answer. **(2 marks)**

16. Let $f(x) = x^2 + (4k-4)x + 4k^2 - 6k + 4$, where k is a non-zero constant.

(a) Using the method of completing the square, express, in terms of k, the coordinates of the vertex U of the graph of $y = f(x)$. **(2 marks)**

(b) On the same rectangular coordinate system, let O be the origin and V be the vertex of the graph of $y = f(-x + 4)$.

(i) Write down the coordinates of V.

(ii) It is given that there is a point P on the x-axis such that $UVPO$ is a rhombus. Find the value(s) of k.

(3 marks)

17. Let $A(n)$ be the nth term of an arithmetic sequence. It is given that $A(4)=10$ and $A(1)+A(2)+A(3)+\cdots+A(20)=460$.

(a) Find A(1). **(3 marks)**

(b) Suppose that $\log_2 B(n)=A(n)$ for any positive integer n.

Find the least value of n such that $B(1)+B(2)+B(3)+\cdots+B(n)>99\ 000\ 000$. **(4 marks)**

18. In **Figure 2**, the acute-angled triangular metal frame ABC is held such that BC lies on the horizontal ground. It is given that $AB = 10\,\text{cm}$, $AC = 8\,\text{cm}$ and $\angle BAC = 50°$.

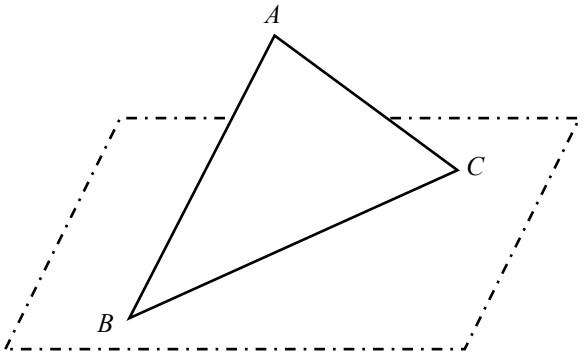


Figure 2

(a) Find

(i) the length of BC ,

(ii) $\angle ABC$.

(4 marks)

(b) A worker finds that the angle between AB and the horizontal ground is 30°. Let M be the mid-point of BC. The worker claims that the angle between AM and the horizontal ground exceeds 37°. Is the claim correct? Explain your answer. **(4 marks)**

19. R $(2k, k)$ is the centre of circle C, where k is a positive constant. Denote the origin by O. C cuts the y-axis at O and A.

(a) (i) Write down the coordinates of A in terms of k.

(ii) Find $\angle ARO$.

(3 marks)

(b) Suppose B is another point on circle C such that $OA = AB$ and $OB = 8\sqrt{5}$.

(i) Find the value of k.

(ii) Find the coordinates of B.

(iii) Denote the orthocentre and the centroid of ΔOAB by H and G respectively. Someone claims that the area of ΔGRB is half of that of the area of ΔGHB. Do you agree? Explain your answer.

(8 marks)

END OF PAPER