S.4 Mathematics
Ch 7 Exponential Functions

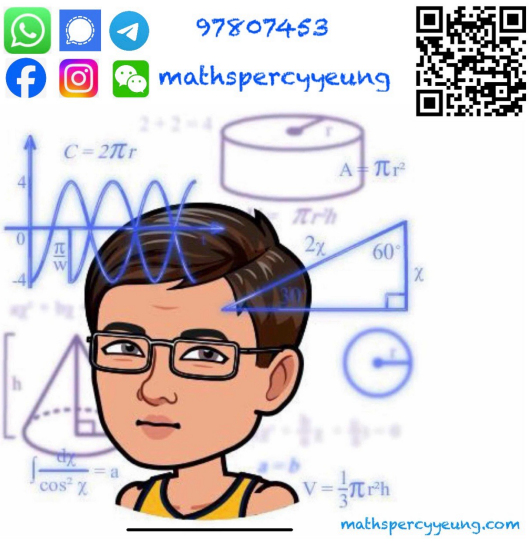
A. Section 7.0 Review (Laws of integral Indices)

I. Pre-lesson

1.	$a^n = a\times a\times a\times ...\times a$ (n times)	5.	$a^m \div a^n = a^{m-n}$
2.	$a^0 = 1$	6.	$(a^m)^n = a^{mn}$
3.	$a^{-n} = \frac{1}{a^n}$	7.	$(ab)^n = a^n b^n$
4.	$a^m \times a^n = a^{m+n}$	8.	$(\frac{a}{b})^n = \frac{a^n}{b^n}$

Please refer to the textbook P.2 – 3, simplify each of the following and express the answers with **positive indices.**

1. **(a)** $\frac{c^6}{(cd)^3}$ **(b)** $\frac{(a^{-2}b^2)^4}{b^6}$



2. **(a)** $\frac{(x^{-2}y^3)^{-3}}{x^4y^{-7}}$ **(b)** $\frac{(h^4k^{-5})^3}{(h^{-2}k^{-1})^{-2}}$

3. **(a)** $\dfrac{(2x^3y^{-4})^6}{x^4(y^3)^{-2}}$ **(b)** $\dfrac{(-3m^{-1}n^2)^{-3}}{(m^{-2}n^{-3})^2}$

5. **(a)** $2^{511}\left(\dfrac{1}{8^{100}}\right)$ **(b)** $27^{299}\left(\dfrac{1}{3^{196}}\right)$

6. **(a)** $3^{997}\cdot 5^{997}$ **(b)** $4^{199}\cdot 7^{199}$

C. ***The n-th Root***

If $x^n = y$, then $x = \sqrt[n]{y}$ we call x is nth root of y.

e.g. $\because (-3)^5 = -243$

$\therefore$ -3 is a 5^{th} root of -243.

e.g. $\because (-2)^4 = 16$ and $(2)^4 = 16$

$\therefore$ -2 and 2 are 4^{th} root of 16.

- For n is an **odd** number, there is ONE answer for x
- For n is an **even** number, there are TWO answers for x, one is positive and one is negative.

B. Section 7.1 Rational Indices

1. Please refer to the textbook P.4 - 6.
2. Video from Identity: Definition of Rational Indices https://youtu.be/R63mKSA93Wk

   Law of Rational Indices https://youtu.be/TcRGILzsfUs

I. Pre-lesson

For $y \neq 0$ and where n is a positive integer, we define the rational indices as follows:

$y^{\frac{1}{n}} = \sqrt[n]{y}$ and $y^{\frac{m}{n}} = \sqrt[n]{y^m}$

1. Express the following in the form $a^{\frac{m}{n}}$, where m and n are positive integers.

(a) $\sqrt{a}$ = _____

(b) $\sqrt[5]{a}$ = _____

(c) $\sqrt{a^3}$ = _____

(d) $\sqrt[6]{a^2}$ = _____

(e) $\sqrt[5]{a^2}$ = _____

(f) $\frac{1}{\sqrt[3]{a}}$ = _____

(g) $\frac{1}{(\sqrt[5]{a})^3}$ = _____

(h) $\frac{1}{(\sqrt[3]{a^2})^{-2}}$ = _____

2. Find the values of the following without using calculator.

(a) $\sqrt[4]{256}$ = _____

(b) $\sqrt[3]{8}$ = _____

(c) $\sqrt[3]{-64}$ = _____

(d) $-\sqrt[4]{81}$ = _____

(e) $125^{\frac{1}{3}}$ = _____

(f) $(\frac{64}{125})^{-\frac{2}{3}}$ = _____

(g) $81^{-\frac{1}{2}}$ = _____

II. Teaching example and classwork

3. Simplify the following expressions and express your answers with positive indices.

(a) $\left(a^{\frac{3}{4}}\right)^{8}$ **(b)** $(a^{-9})^{\frac{2}{3}}$ **(c)** $\left(\frac{1}{a^{\frac{1}{2}}}\right)^{-5}$

4. Simplify the following expressions and express your answers with positive indices.

(a) $x^2 \cdot \sqrt{x}$ **(b)** $\frac{\sqrt[4]{a}}{\sqrt[8]{a^3}}$

(c) $\sqrt[3]{x} \cdot \sqrt[3]{x^2}$ **(d)** $\sqrt{x^4 y^2}$

(e) $\frac{\sqrt[3]{a^{-2}}}{\sqrt[5]{a^{-3}}}$ **(f)** $\sqrt[4]{x^3} \cdot \frac{x^{-2}}{\sqrt{x}}$

(g) $\dfrac{b^4}{(a^6b^3)^{\frac{1}{3}}}$

(h) $\dfrac{\sqrt[4]{a^4b^8}}{\sqrt{ab}}$

Level 2

5. Simplify the following expressions and express your answers with positive indices.

(a) $\left(a^{-\frac{1}{6}} \times \sqrt{a^3}\right)^3$

(b) $(mn)^{\frac{1}{3}} \div \dfrac{\sqrt[3]{m^4}}{(\sqrt[3]{n})^2}$

(c) $\sqrt[3]{8a} \div 4\sqrt[3]{a^2}$

(d) $\dfrac{\sqrt[3]{-8x^2}}{\sqrt{x}}$

(e) $\dfrac{(81x^2y^3)^{\frac{1}{4}}}{\sqrt{xy^{-\frac{1}{2}}}}$

(f) $\dfrac{(\sqrt[3]{27t})^2}{\sqrt[3]{t}} \div 9t^{\frac{1}{6}}$

(g) $2p^{\frac{1}{2}}q \times \left(p^{-2}q^{\frac{1}{3}}\right)^{-2} \div \sqrt{\dfrac{p}{9}}$

(h) $\sqrt[3]{a^2 \times \sqrt{a^3}}$

6. Simplify the following expressions and express your answers with positive indices.

(a) $\dfrac{49^n}{7^{n+1}}$ **(b)** $8 \cdot 4^{n+1}$ **(c)** $\dfrac{(3^{2n})(2^{3n})}{12^n}$

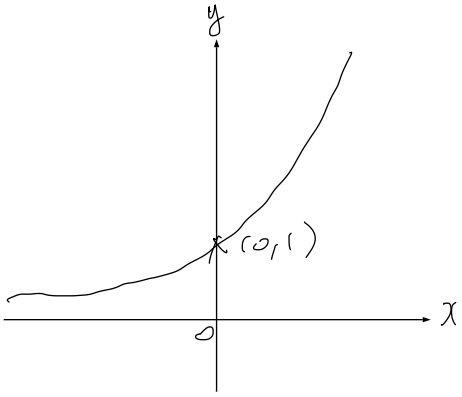
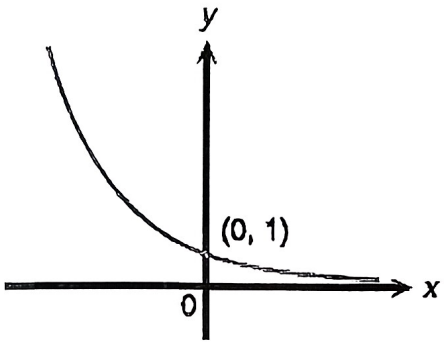
Level 3

7. Simplify $\dfrac{4^{2n}+4^n}{2^{3n}+2^n}$ and express your answers with positive indices.

C. Section 7.2 Exponential Functions and Their Graphs

1. Please refer to the textbook P.13 – 15.
2. Video from Identity: https://youtu.be/W1N72t37LFA

I. Pre-lesson

<table>
<tr><th>Range of a</th><th>$a > 1$</th><th>$0 < a < 1$</th></tr>
<tr><td>Graph of $y = a^x$</td><td> 
</td><td> 
</td></tr>
<tr><td>Common features</td><td colspan="2">1. The graph cuts the y-axis at _____.
2. The graph never cuts the _____. It lies above the x-axis.
3. The graph has neither a maximum point, a minimum point nor an axis of symmetry.</td></tr>
<tr><td>Differences</td><td>1. The value of y _____ as x increases.
2. The value of y gets closer and closer to _____ as x decreases indefinitely.
3. As x increases, the rate of increase of y becomes greater.</td><td>1. The value of y _____ as x increases.
2. The value of y gets closer and closer to _____ as x increases indefinitely.
3. As x increases, the rate of decrease of y becomes smaller.</td></tr>
</table>

Note: The graphs of $y = a^x$ and _____ show reflectional symmetry with each other about the y-axis.

1. Which of the following are exponential functions of real numbers x? (Put a '✓' in the appropriate box.)

(a) $f(x)=x^4$ ☐ **(b)** $f(x)=0.001^x$ ☐

(c) $f(x)=(-5)^x$ ☐ **(d)** $f(x)=4^x$ ☐

2. In each of the following, determine whether the graphs of two given functions have reflectional symmetry about the y-axis, and put a '✓' in the appropriate box.

		Yes	**No**
(a)	$y=0.9^x$ and $y=-0.9^x$	☐	☐
(b)	$y=\left(\frac{5}{7}\right)^x$ and $y=\left(\frac{7}{5}\right)^x$	☐	☐
(c)	$y=8^x$ and $y=8^{-x}$	☐	☐
(d)	$y=0.4^x$ and $y=2.5^x$	☐	☐

II. Teaching example and classwork

3. Given that $f(x)=2(3^x)$ and $g(x)=\dfrac{0.5^x}{5}$, find the values of the following expressions.

(Give your answers correct to 3 significant figures if necessary.)

(a) $f(1)+g(1)$ **(b)** $2f(0)-g(-1)$ **(c)** $f(1.5)+3g(-0.5)$

4. If $f(x) = a\left(\frac{3}{7}\right)^x$ and $f(2) = 9$, find the value of a.

5. It is given that $f(x) = 0.25(k^x)$ (where $k > 0$) and $f(2) = 25$. Find the values of

(a) k, **(b)** $\frac{100f(0.5)}{f(3)}$.

(Leave your answers in surd form if necessary.)

6. It is given that $f(x) = 2(3^x)$ and $g(x) = \left(\frac{4}{5}\right)^x$.

(a) Find the values of the following expressions.

(i) $5f(2) \cdot 5g(2)$ **(ii)** $\frac{[f(0.5)]^2}{g(0.5)}$

(Leave your answers in surd form if necessary.)

(b) If $2f(k) \cdot g(-1) = 45$, find the value of k.

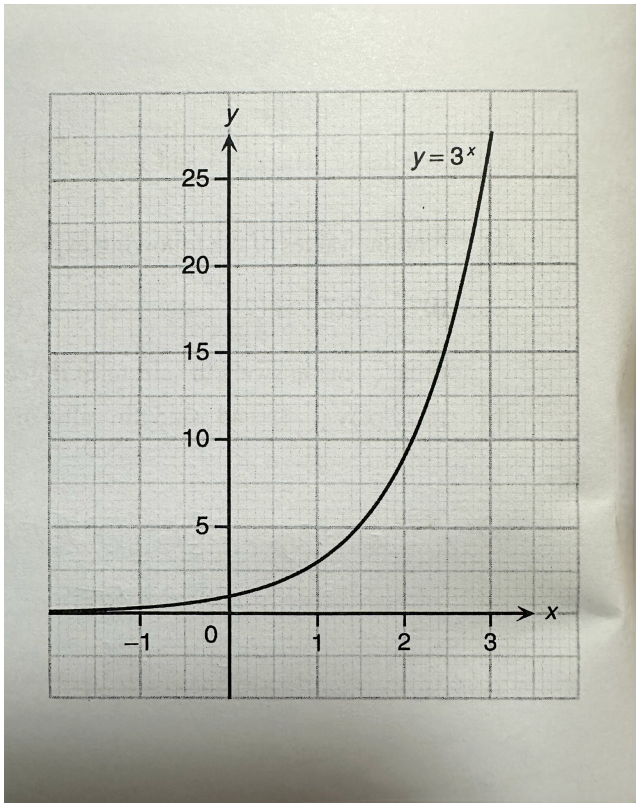
7. The figure shows the graph of $y = 3^x$.

(a) Using the graph, find the values of

(i) $3^{1.7}$, **(ii)** $\left(\frac{1}{3}\right)^{-0.8}$.

(b) Solve the following equations graphically.

(i) $3^x = 10$ **(ii)** $3^x = 24$



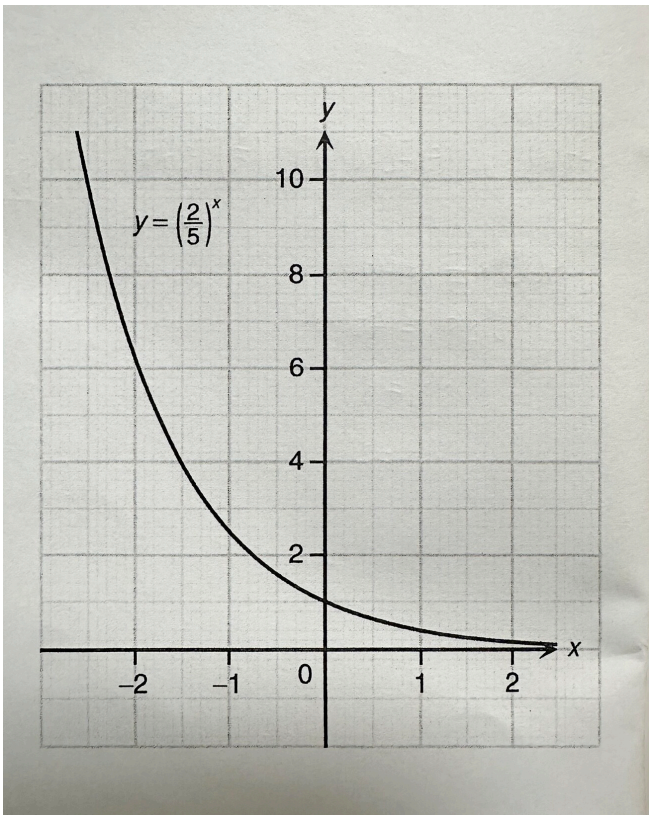
8. The figure shows the graph of $y = \left(\frac{2}{5}\right)^x$.

(a) Using the graph, find the values of

(i) $\left(\frac{2}{5}\right)^{-0.4}$, **(ii)** $\left(\frac{5}{2}\right)^{\frac{5}{2}}$.

(b) Solve the following equations graphically.

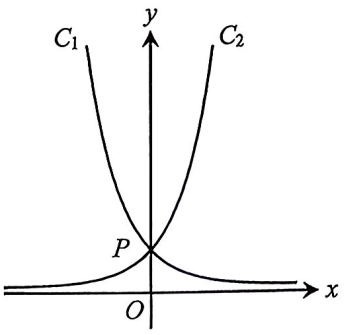
(i) $\left(\frac{2}{5}\right)^x = 4$ **(ii)** $0.4^x = 7$



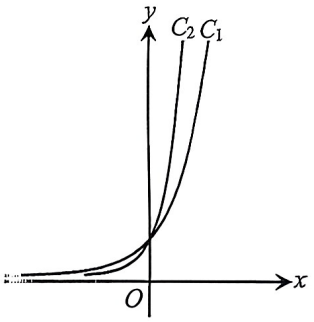
9. The two curves C_1 and C_2 in the figure represent the graphs of the exponential functions $y = 0.25^x$ and $y = b^x$ respectively, where b is a positive constant. C_1 and C_2 intersect at P. C_1 is the reflection image of C_2 with respect to the y-axis.

(a) Find the coordinates of P.

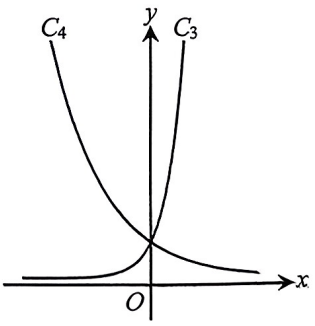
(b) Find the value of b.



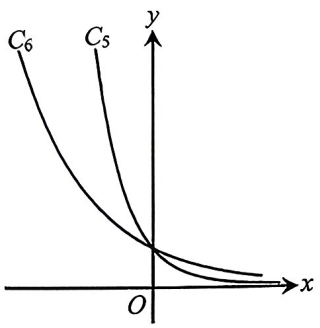
10. The two curves in the figure represent the graphs of the exponential functions $y = 3^x$ and $y = 7^x$. Write down the corresponding exponential functions for C_1 and C_2.



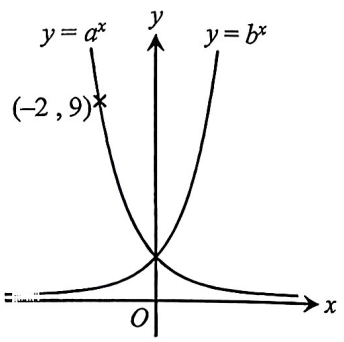
11. The two curves in the figure represent the graphs of the exponential functions $y = \left(\frac{1}{2}\right)^x$ and $y = 8^x$. Write down the corresponding exponential functions for C_3 and C_4.



12. The two curves in the figure represent the graphs of the exponential functions $y = 0.3^x$ and $y = \left(\frac{3}{5}\right)^x$. Write down the corresponding exponential functions for C_5 and C_6.



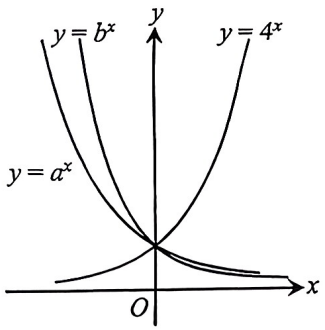
13. The figure shows the graphs of $y = a^x$ and $y = b^x$, where a and b are positive constants. It is given that the graph of $y = a^x$ passes through $(-2, 9)$.



(a) Find the value of a.

(b) If the graph of $y = b^x$ is the reflection image of the graph of $y = a^x$ with respect to the y-axis, find the value of b.

14. The figure shows the graphs of the exponential functions $y = a^x$, $y = b^x$ and $y = 4^x$, where a and b are positive constants. The graph of $y = a^x$ is the reflection image of the graph of $y = 4^x$ with respect to the y-axis.



(a) Find the value of a.

(b) Is the value of b greater than $\frac{1}{4}$? Explain your answer.

Applications of Exponential Functions

Exponential Functions are common in real life. It is widely used in mathematics and Science, for example, the growth of population, compound interest, the reproduction of bacteria etc.

15. The number of visitors in a city is 56 000 at present. The number N of visitors in the city after t months can be represented by the following formula:

$$N = k(1.03)^t, \text{ where } k \text{ is a constant.}$$

(a) Find the value of k.

(b) Find the number of visitors in the city after one year, correct to the nearest 1 000.

16. A country starts a new policy for car taxation. The number N of cars sold by a car company in the mth year after the start of the new policy can be estimated by the following formula:

$$N = 5\,000k^m, \text{ where } k \text{ is a positive constant.}$$

It is known that the number of cars sold by the company in the 4th year after the start of the new policy is 2 048.

(a) Find the value of k.

(b) Is it possible for the company to sell over 5 000 cars in a certain year after the start of the new policy? Explain your answer.

17. A bank offers an investment plan. If a customer invests \$$P$ for the plan, the total amount \$$A$ received by the customer after n years can be represented by the following formula:

$$A = Pk^n, \text{ where } k \text{ is a constant.}$$

Peter invests \$20 000 for the plan. It is known that the total amount received by him will be \$26 620 after 3 years.

(a) Find the value of k.

(b) Peter claims that he will earn by more than 60% after 5 years from his investment amount. Do you agree? Explain your answer.

D. Past Paper

[HKCEE 2007(1)]

1. If n is a positive integer, then $3^{2n} \cdot 4^{n} =$

   A. 6^{2n}.

   B. 6^{3n}.

   C. 12^{2n}.

   D. 12^{3n}.

[HKCEE 2008(1)]

2. $(\frac{1}{2})^{888}(-2)^{887} =$

   A. -2.

   B. -0.5.

   C. 0.

   D. 0.5.

[HKCEE 2009(1)]

3. $2^{n} \bullet 3^{n} =$

   A. 5^{n}.

   B. 6^{n}.

   C. 8^{n}.

   D. 9^{n}.

[HKCEE 2010(2)]

4. $(\frac{1}{9})^{500}(3^{500})^{3} =$

   A. 0.

   B. 3^{500}.

   C. 6^{500}.

   D. 18^{500}.

[HKCEE 2010(39)]

5. If a and b are positive, then $\frac{1}{\sqrt{a^3}} \div \frac{\sqrt{b}}{a} =$

   A. $\frac{\sqrt{b}}{ab}$.

   B. $\frac{\sqrt{ab}}{b}$.

   C. $\frac{\sqrt{ab}}{ab}$.

   D. $\frac{\sqrt{a^3b}}{b}$.

[HKCEE 2011(1)]

6. $(5)^{334}(\frac{-1}{5})^{333} =$

   A. -5.

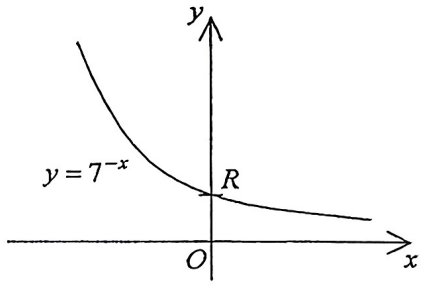
   B. -0.2.

   C. 0.

   D. 5.

[HKCEE 2011(38)]

7. The figure shows the graph of $y = 7^{-x}$.



   The coordinates of R are

   A. (1, 0).

   B. (0, 1).

   C. (7, 0).

   D. (0, 7).

[DSE 2012SP(1)]

8. $(3a)^{2} \bullet a^{3} =$

   A. $3a^{5}$.

   B. $6a^{6}$.

   C. $9a^{5}$.

   D. $9a^{6}$.

[DSE 2012 PP(1)]

9. $x^{3}(2x+x) =$

   A. $3x^{4}$.

   B. $2x^{5}$.

   C. $3x^{5}$.

   D. $2x^{6}$.

[DSE 2012(1)]

10. $\dfrac{(2x^4)^3}{2x^5} =$

A. $3x^2$.
B. $3x^7$.
C. $4x^7$.
D. $4x^{59}$.

[DSE 2013(1)]

11. $(27 \cdot 9^{n+1})^3 =$

A. 3^{6n+12}.
B. 3^{6n+15}.
C. 3^{9n+12}.
D. 3^{9n+18}.

[DSE 2014(1)]

12. $(2n^3)^{-5} =$

A. $\dfrac{1}{32n^2}$.
B. $\dfrac{1}{32n^{15}}$.
C. $\dfrac{1}{10n^{125}}$.
D. $\dfrac{1}{10n^{243}}$.

[DSE 2015(2)]

13. $\dfrac{(3y^6)^4}{3y^2} =$

A. $4y^5$.
B. $4y^8$.
C. $27y^{12}$.
D. $27y^{22}$.

[DSE 2016(1)]

14. $8^{222} \cdot 5^{666} =$

A. 10^{666}.
B. 10^{888}.
C. 40^{666}.
D. 40^{888}.

[DSE 2017(2)]

15. $\left(\dfrac{1}{9^{555}}\right)3^{444} =$

A. 0.
B. $\dfrac{1}{3^{111}}$.
C. $\dfrac{1}{3^{222}}$.
D. $\dfrac{1}{3^{666}}$.

[DSE 2018(1)]

16. $\dfrac{8^{2n+1}}{4^{3n+1}} =$

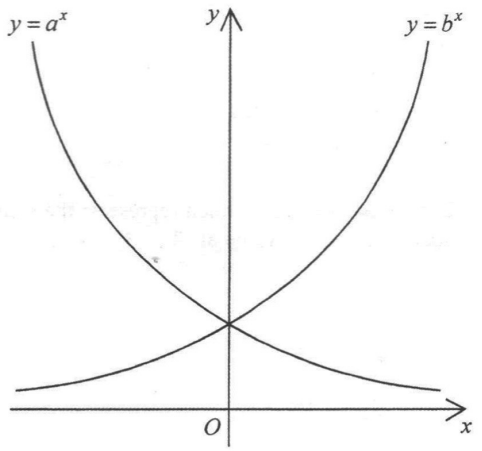
A. 1.
B. 2.
C. 2^n.
D. 2^{-n}.

[DSE 2020(33)]

17. The figure shows the graph of $y = a^x$ and $y = b^x$ on the same rectangular coordinate system, where a and b are positive constants. If the graph $y = a^x$ is the reflection image of the graph of $y = b^x$ with respect to the y-axis, which of the following are true?

I. $a < 1$
II. $b > 1$
III. $ab = 1$

A. I and II only
B. I and III only
C. II and III only
D. I, II and III



Note: