

Final Examination 2016-2017

Form 4
Mathematics
Extended Part

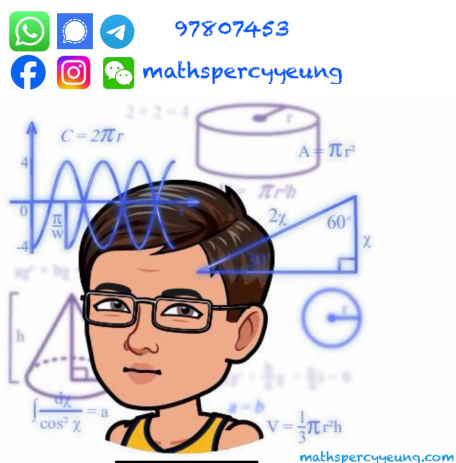
Module 2
(Algebra and Calculus)

Time allowed: 1 hour 30 minutes

Question – Answer Book

Instructions

1. Write your examination number in the spaces provided on the top right corner of the cover page.
2. The total mark of this paper is 60.
3. Answer ALL questions in this paper.
4. All working must be clearly shown.
5. Unless otherwise specified, numerical answers must be exact.
6. The diagrams in this paper are not necessarily drawn to scale



Exam no.			
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Question no.	Marks
1	(3)
2	(5)
3	(3)
4	(3)
5	(3)
6	(4)
7	(4)
8	(6)
9	(7)
10	(6)
11	(10)
12	(6)
Total	

Formulas for Reference

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$2\sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$2\cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$2\sin A \sin B = \cos(A-B) - \cos(A+B)$$

1. Expand $(1 + x - 2x^2)^{10}$ in ascending powers of x up to x^2 .

(3 marks)

[illegible]

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3. Find $\frac{d}{dx} x e^x$ from first principles. (3 marks)

(3 marks)

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4. Let $f(x) = \frac{\tan x}{x^2}$. Find $f'(\frac{\pi}{4})$.

(3 marks)

[The page contains faint horizontal lines and some illegible markings.]

5. Let $y^2 = x \ln x^3$. If $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{k}{x}$, find the value of k .

(3 marks)

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6. Let $y = \sqrt{x}^{\csc x}$

(a) Find $\frac{dy}{dx}$.

(b) Hence, find $\frac{dy}{dx}\bigg|_{x=\frac{\pi}{6}}$.

(4 marks)

[illegible]

(6 marks)

(b) $\lim_{x \rightarrow -\infty} x \left(\sqrt{2 - \frac{3}{x}} - \sqrt{2 + \frac{3}{x}} \right)$

- A blank sheet of lined paper with horizontal ruling lines.

10.(a) Let $t = \tan \frac{\theta}{2}$. Express $\sin \theta$ in terms of t . (3 marks)

(b) Hence, solve $\frac{1}{2} \sin \theta \sec^2 \frac{\theta}{2} - \sqrt{3} \cot \frac{\theta}{2} + \sqrt{3} - 1 = 0$ for $0 < \theta < 2\pi$. (3 marks)

[illegible]

11.(a) Given that $\cot(x - y) = k \cot x$.

Prove that $(k - 1) \sin(2x - y) = (k + 1) \sin y$.

(3 marks)

(b) Solve $\cot(x - 10^\circ) = 2 \cot x$ for $0 < x < 90^\circ$, giving the answer correct to 3 significant figures.

(3 marks)

(c) Given that $\cot(x - 30^\circ) = k \cot x$ has solutions in x .

Find the range of possible values of k .

(4 marks)

[illegible]

