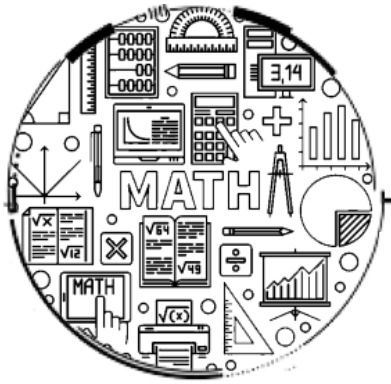
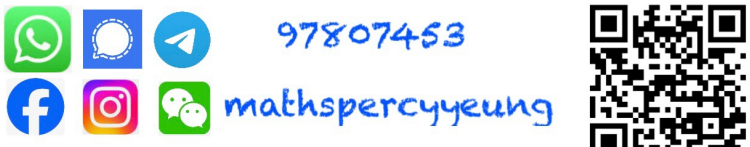
# FKS F3 Ch7 Areas and Volumes 3B

## S3 Mathematics Notes




## Areas and Volumes 3B

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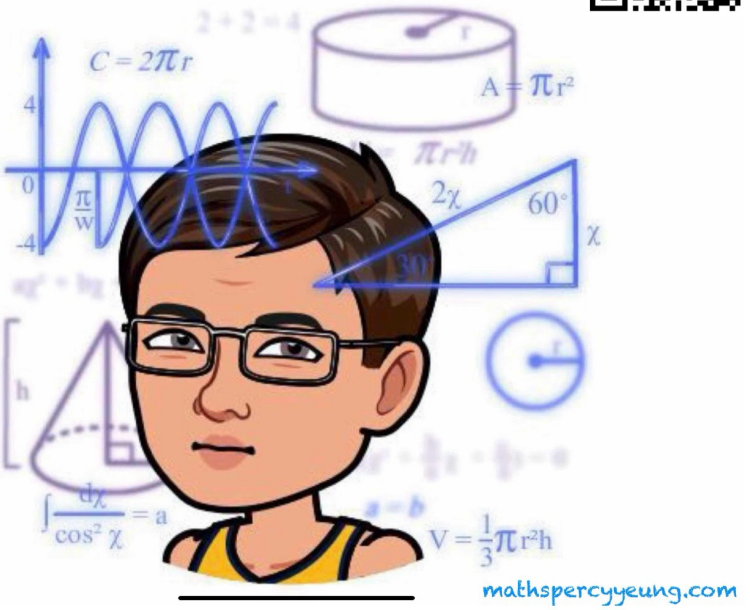
- Spheres
- Similar Figures



# A Spheres

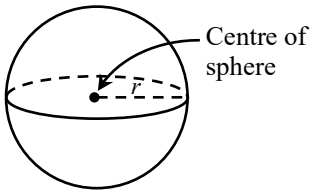
## Key Points: Spheres

*Spheres*

For a **sphere**,

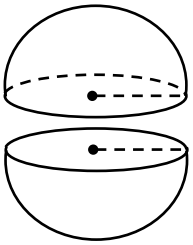
**(i)** the fixed point which has the same distance to any point on the surface of the sphere is called the **centre**,

**(ii)** the distance ($r$) between the centre of the sphere and any point on the surface is the **radius**.




*Hemispheres*

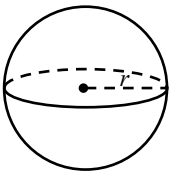
If a sphere is cut along a plane passing through the centre, then the sphere will be divided into two equal parts. Each part is called a hemisphere.



## Key Points: Volume of Spheres

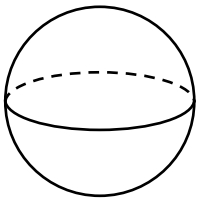
For a sphere of radius $r$,

Volume of a sphere = $\frac{4}{3}\pi r^3$



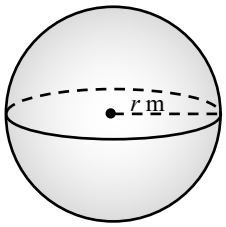

### Example 1

The figure shows a sphere with volume $972\pi$ cm$^3$. Find the radius of the sphere.

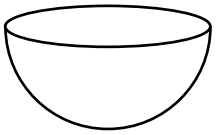


## Practice Exercise

**1.** The figure shows a sphere with volume $288\pi\ \text{m}^3$. Find the value of $r$.



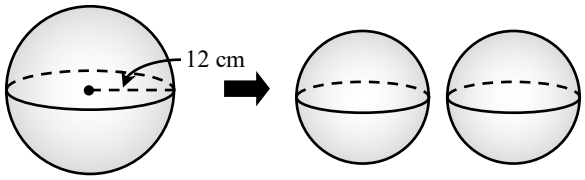

**2.** The volume of a hemisphere is $144\pi\ \text{cm}^3$. Find the diameter of the hemisphere.



**3.** A solid metal sphere of radius 12 cm is melted and recast into 2 identical small spheres.

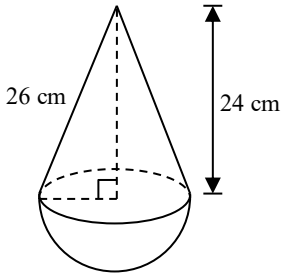
**(a)** Express the total volume of the 2 small spheres in terms of $\pi$.

**(b)** Find the radius of each small sphere.



## Example 2 (TB P.7.44)

The figure shows a solid which consists of a hemisphere and a right circular cone with the same base. Find the volume of the solid correct to 3 significant figures.

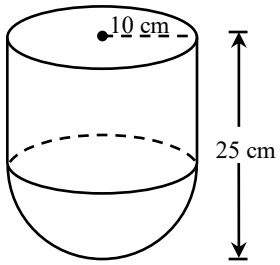



## Practice Exercise

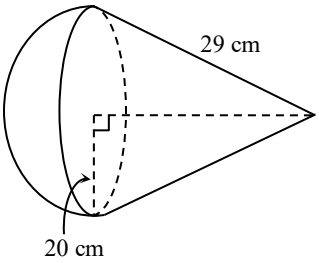
**4.** The figure shows a solid which consists of a hemisphere and a right circular cylinder with the same base. The base radius and the height of the solid are 10 cm and 25 cm respectively.

**(a)** Find the height of the cylinder.

**(b)** Find the volume of the solid correct to 3 significant figures.

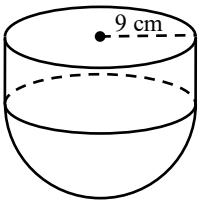


**5.** The figure shows a solid which consists of a right circular cone and a hemisphere with the same base. Find the volume of the solid in terms of $\pi$.



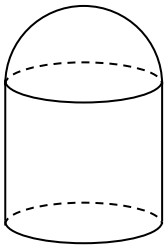

**Example 3 (TB P.7.44)**

The figure shows a solid which consists of a right circular cylinder and a hemisphere with the same base. The base radius of the circular cylinder is 9 cm. If the volume of the solid is $972\pi$ cm$^3$, find the height of the circular cylinder.

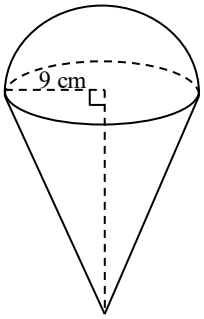


## Practice Exercise

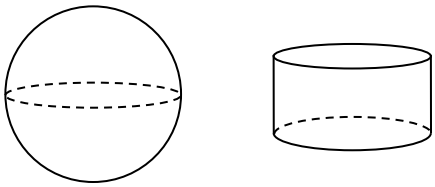
**6.** The figure shows a solid which consists of a hemisphere and a right circular cylinder with the same base. The height of the circular cylinder is twice its base radius. If the volume of the solid is $1944\pi$ cm$^3$, find the diameter of the hemisphere.



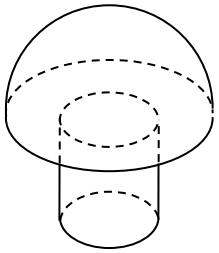
**7.** The figure shows a solid which consists of a hemisphere and an inverted right circular cone with the same base. If the base radius of the circular cone is 9 cm and the volume of the whole solid is $1026\pi$ cm$^3$, find the height of the whole solid.



**8.** In the figure, the sphere and the right circular cylinder have the same radii. The height and the base radius of the circular cylinder are the same. If the total volume of the two solids is $4032\pi$ cm$^3$, find the radius of the sphere.

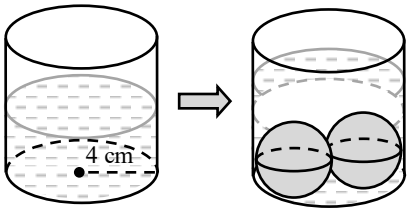


**9.** The figure shows a solid which consists of a hemisphere and a right circular cylinder. The radius of the hemisphere is twice the base radius of the circular cylinder. The base radius of the circular cylinder is half of its height. If the volume of the solid is 600 cm$^3$, find the radius of the hemisphere, correct to 3 significant figures.



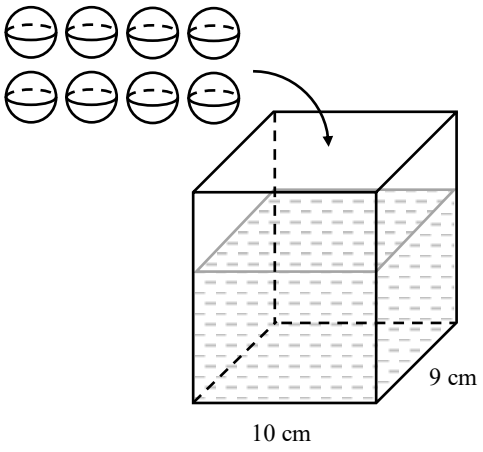
## Example 4 (TB P.7.45)

The figure shows a right cylindrical container of base radius 4 cm with some water inside. Two identical metal spheres of radius 2 cm each are totally immersed in the water and no water overflows. Find the rise in the water level.




## Practice Exercise

**10.** The figure shows a rectangular tank of length 10 cm and width 9 cm. 8 identical metal spheres of radius 1.5 cm each are put into the tank such that they are totally immersed in the water and no water overflows. Find the rise in the water level, correct to 3 significant figures.



**11.** A right circular cylindrical container contains some water. Five identical metal spheres of radii 1.2 cm are totally immersed in water and the water level rises by 0.3 cm. Find the base radius of the container correct to 3 significant figures.

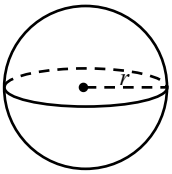
**12.** A right circular cylindrical container of base radius 8 cm contains some water. Some identical metal spheres of radii 2 cm are totally immersed in water and the water level rises by 4 cm. Find the number of metal spheres put in the container.

**13.** A right circular cylindrical container of base radius 18 cm contains some water. A metal ball of radius 4.5 cm is put into the container and it is just covered by the water. Find the depth of water in the container originally.

## Key Points: Surface Areas of Spheres

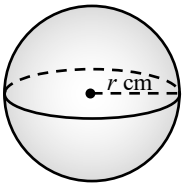
For a sphere of radius $r$,

Surface area = $4\pi r^2$



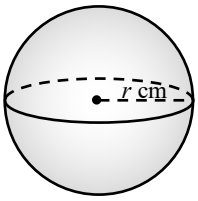

### Example 5

The figure shows a sphere of radius $r$ cm. If the surface area of the sphere is $64\pi\,\text{cm}^2$, find $r$.

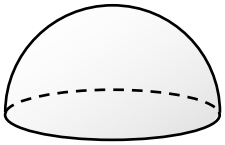



### Practice Exercise

**14.** The figure shows a sphere of radius $r$ cm. If the surface area of the sphere is $50\pi\,\text{cm}^2$, find $r$.

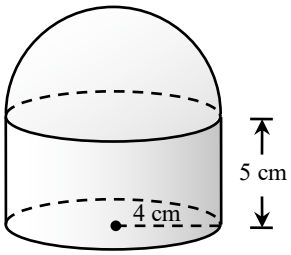



**15.** In the figure, the total surface area of a hemisphere is $507\pi\,\text{cm}^2$. Find the diameter of the hemisphere.



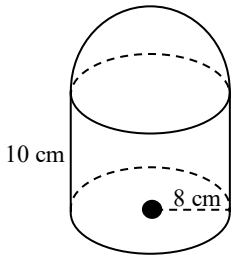
## Example 6 (TB P.7.47)

The figure shows a solid which consists of a hemisphere and a right circular cylinder with the same base. The base radius and the height of the circular cylinder are 4 cm and 5 cm respectively. Find the total surface area of the solid.

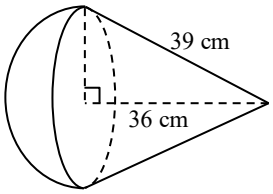



## Practice Exercise

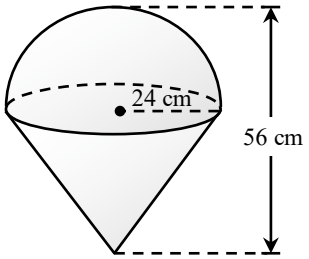
**16.** The figure shows a solid which consists of a hemisphere and a right circular cylinder with the same base. Find the total surface area of the solid in terms of $\pi$.



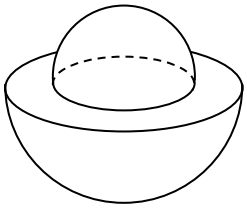

**17.** The figure shows a solid which consists of a hemisphere and a right circular cone with the same base. Find the total surface area of the solid in terms of $\pi$.



**18.** The figure shows a solid which consists of a hemisphere and a right circular cone with the same base. The base radius of the circular cone is 24 cm and the height of the solid is 56 cm. Find the total surface area of the solid in terms of $\pi$.

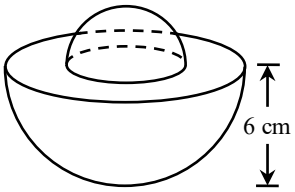



**19.** The figure shows a solid formed by two hemispheres. The radii of the hemispheres are 13 cm and 20 cm respectively. Find the total surface area of the solid correct to 3 significant figures.



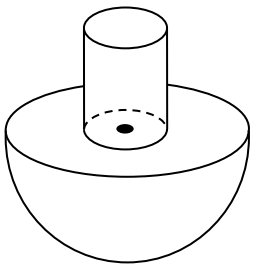
## Example 7 (TB P.7.48)

The figure shows a sculpture which consists of two hemispheres. If the radius of the larger hemisphere is 6 cm and the volume of the sculpture is $162\pi$ cm$^3$, find the total surface area of the sculpture in terms of $\pi$.

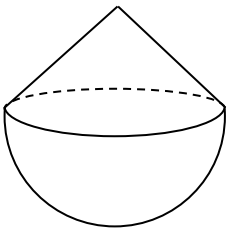



## Practice Exercise

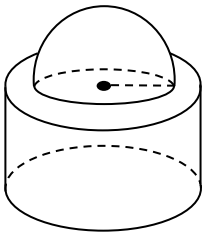
**20.** The figure shows a solid which consists of a hemisphere and a right circular cylinder. The radius of the hemisphere is triple the base radius of the circular cylinder. The height of the circular cylinder is twice its base radius. The volume of the solid is $4320\pi$ cm$^3$. Find the total surface area of the solid in terms of $\pi$.



**21.** In the figure, a solid is formed by a hemisphere and a right circular cone with the same base. The height of the circular cone is two-thirds its base radius. The volume of the solid is $648\pi$ cm$^3$. Find the total surface area of the solid correct to 3 significant figures.



**22.** In the figure, a solid is formed by a hemisphere and a right circular cylinder. The radius of the hemisphere is $\frac{3}{4}$ of the base radius of the circular cylinder. The height of the circular cylinder is the same as its base radius. The volume of the solid is $2214\pi$ cm$^3$. Find the total surface area of the solid in terms of $\pi$.



# B Similar Figures

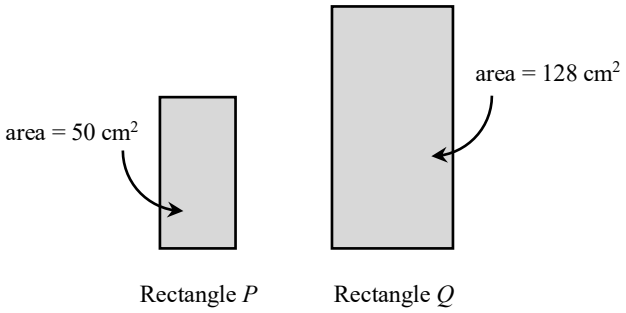
## Key Points: Property of Similar Plane Figures

If a pair of corresponding **lengths** of two similar 2-D figures are $\ell_1$ and $\ell_2$ while their **areas** are $A_1$ and $A_2$ respectively, then we have

$$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

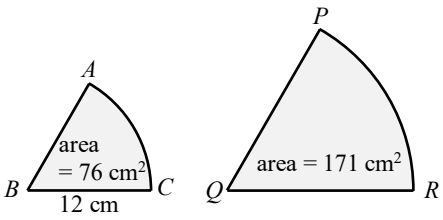
## Example 8 (TB P.7.57)

The figure shows two similar rectangles $P$ and $Q$. The areas of rectangle $P$ and rectangle $Q$ are 50 cm$^2$ and 128 cm$^2$ respectively. The perimeter of rectangle $P$ is 30 cm. Find the perimeter of rectangle $Q$.




## Practice Exercise

**23.** The figure shows two similar sectors $ABC$ and $PQR$. The areas of sectors $ABC$ and $PQR$ are 76 cm$^2$ and 171 cm$^2$ respectively. If the radius of sector $ABC$ is 12 cm, find the radius of sector $PQR$.



**24.** The ratio of the perimeters of two similar rhombuses is 3 : 8. If the area of the smaller rhombus is 162 $cm^2$, find the area of the larger rhombus.

**25.** The heights of two similar triangles are 24 cm and 30 cm respectively. If the area of the larger triangle is 131.25 $cm^2$, find the area of the smaller triangle.

**26.** The areas of two similar rectangles are in the ratio 81 : 49. If the length of a diagonal of the smaller rectangle is 147 cm, find the length of a diagonal of the larger rectangle.

**27.** *X* and *Y* are two similar rectangles. The ratio of the area of *X* to the area of *Y* is 9 : 16. It is given that the area of rectangle *Y* is 96 $cm^2$ and the ratio of its length to its width is 3: 2. Find the perimeter of *X*.

## Key Points: Scale Drawings

In scale drawing, the scale 1 : $n$ means that the length of 1 unit in the drawing represents an actual length of $n$ units.

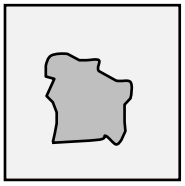
### Example 9 (TB P.7.57)

The scale of a map is 1 : 2500. If the actual area of a stadium is 0.025 $km^2$, find the area of the stadium on the map in $cm^2$.

### Practice Exercise

**28.** The scale of a map is 1 : 1200. If the area of a house on the map is 7 $cm^2$, find the actual area of the house in $m^2$.

**29.** The figure shows a map of a camp site. The scale of the map is 1 : 800. If the actual area of the camp site is 128 $m^2$, find the area of the camp site on the map in $cm^2$.



**30.** The actual area of a market is 4800 $m^2$. If the area of the market on a map is 12 $cm^2$, find the scale of the map in the form 1 : $n$.

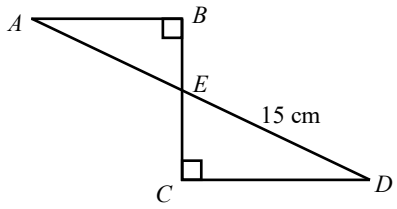
**31.** The area of a park on a map of scale 1 : 500 is 36 $cm^2$. If the scale of the map is changed to 1 : 800, someone claims that the area of the park on the new map will be greater than 22 $cm^2$. Do you agree? Explain your answer.

## Example 10 (TB P.7.58)

In the figure, $AED$ and $BEC$ are straight lines and $\angle ABE = \angle DCE = 90°$.

**(a)** Prove that $\triangle ABE \sim \triangle DCE$.

**(b)** If the areas of $\triangle ABE$ and $\triangle DCE$ are 48 cm$^2$ and 75 cm$^2$ respectively, find $AE$.

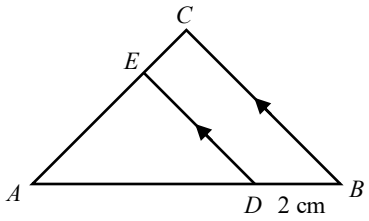



## Practice Exercise

**32.** In the figure, $D$ and $E$ are points lying on $AB$ and $AC$ respectively such that $DE$ // $BC$.

**(a)** Prove that $\triangle ADE \sim \triangle ABC$.

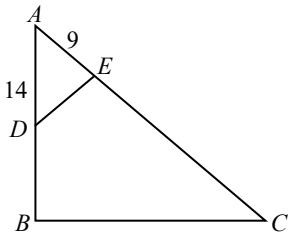
**(b)** If the areas of $\triangle ADE$ and quadrilateral $DBCE$ are 45 cm$^2$ and 35 cm$^2$ respectively, find $AD$.



**33.** In the figure, $D$ and $E$ are points lying on $AB$ and $AC$ respectively such that $\angle ADE = \angle ACB$.

**(a)** Prove that $\triangle ADE \sim \triangle ACB$.

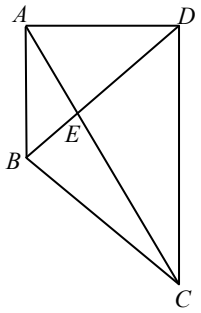
**(b)** If the areas of $\triangle ADE$ and quadrilateral $DBCE$ are 32 cm$^2$ and 96 cm$^2$ respectively, find $CE$.




**34.** In the figure, $ABCD$ is a trapezium with $AB$ // $DC$. The diagonals $AC$ and $BD$ intersect at point $E$. It is given that $AC = 45$ cm.

**(a)** Write down a pair of similar triangles and give the proof.

**(b)** If the areas of $\triangle ABE$ and $\triangle CDE$ are 48 cm$^2$ and 108 cm$^2$ respectively, find $CE$.

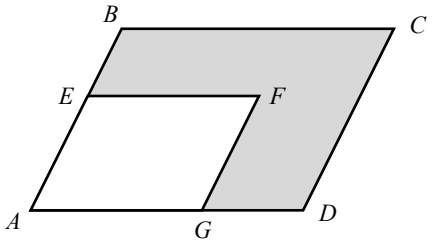


## Example 11 (TB P.7.60)

In the figure, *AEB* and *AGD* are straight lines. *ABCD* and *AEFG* are two similar parallelograms. The area of *AEFG* is 20 cm$^2$ and *AG* : *GD* = 5 : 3.

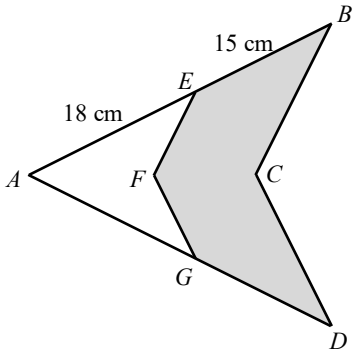
**(a)** Find the area of parallelogram *ABCD*.

**(b)** Find the area of the shaded region.

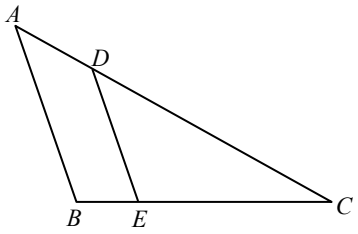



## Practice Exercise

**35.** In the figure, *AEB* and *AGD* are straight lines. *ABCD* and *AEFG* are two similar quadrilaterals. *AE* = 18 cm and *EB* = 15 cm. If the area of *AEFG* is 72 cm$^2$, find the area of the shaded region.



**36.** In the figure, $D$ and $E$ are points lying on $AC$ and $BC$ respectively such that $DE \parallel AB$. If $BE : EC = 2 : 5$ and the area of $\triangle CDE$ is 50 cm$^2$, find the area of quadrilateral $ABED$.

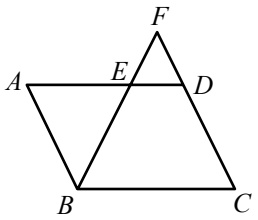



**37.** In $\triangle PQR$, $S$ and $T$ are points lying on $PQ$ and $PR$ respectively such that $ST \parallel QR$. If $PS : SQ = 5 : 4$ and the area of quadrilateral $SQRT$ is 112 cm$^2$, find the area of $\triangle PST$.

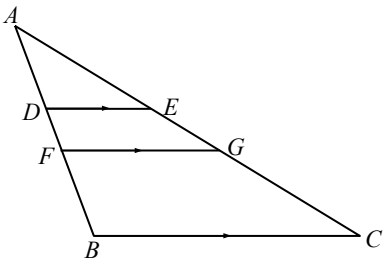
**38.** In the figure, $ABCD$ is a parallelogram. $CD$ is produced to a point $F$ such that $AD$ and $BF$ intersect at $E$. It is given that $BE : EF = 3 : 2$ and the area of $\triangle DEF$ is 12 cm$^2$.

**(a)** Find the area of quadrilateral $BCDE$.

**(b)** Find the area of parallelogram $ABCD$.



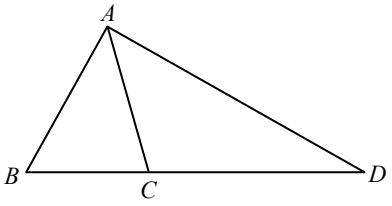

**39.** In the figure, $D$ and $F$ are points lying on $AB$ while $E$ and $G$ are points lying on $AC$. It is given that $AE : EG : GC = 2 : 1 : 2$. Find the ratio of the area of $DFGE$ to the area of $FBCG$.



## Key Points: Triangles with same Height

It is given that $BCD$ is a straight line, we have

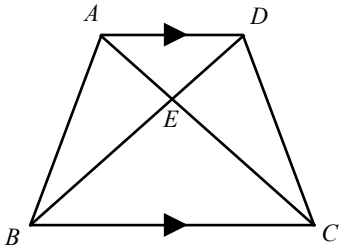
$$\frac{\text{area of } \triangle ABC}{\text{area of } \triangle ACD} = \frac{BC}{CD}.$$




## Key Points: Triangles with same Height

Let $A_1$, $A_2$, $A_3$ and $A_4$ be the area of $\triangle ADE$, $\triangle ABE$, $\triangle BCE$ and $\triangle CDE$ respectively.

1. $\frac{A_1}{A_3} = \left(\frac{AD}{BC}\right)^2$
2. $A_2 = A_4$
3. $A_1 \times A_3 = A_2 \times A_4$

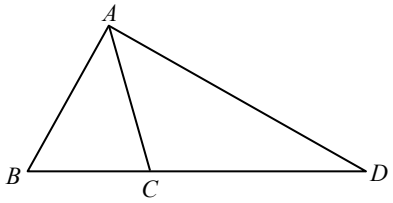



## Let's Try

**1.** In the figure, $BCD$ is a straight line. If $BC : CD = 3 : 4$ and the area of $\triangle ABC$ is 6 cm$^2$, find the area of $\triangle ACD$.

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle ACD} = \frac{(\qquad\qquad)}{(\qquad\qquad)}$$

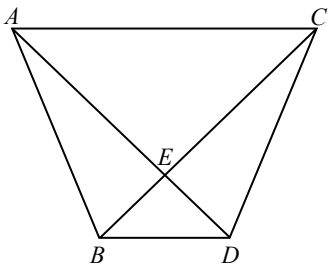
$\therefore$ Area of $\triangle ACD$ =




**2.** In the figure, $ABDC$ is a trapezium with $AC$ // $BD$. The diagonals $AD$ and $BC$ intersect at point $E$. If $AC : BD = 8 : 3$ and the area of $\triangle BDE$ is 18 cm$^2$, find the area of

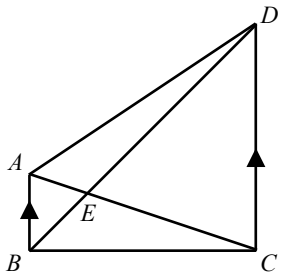
**(a)** $\triangle ACE$,

**(b)** $\triangle ABE$.



## Example 12 (TB P.7.61)

In the figure, $ABCD$ is a trapezium, where $AB$ // $DC$. $AC$ and $BD$ intersect at $E$. $CD = 3AB$ and the area of $\triangle ABE$ is 16 cm$^2$. Find the areas of the following triangles.

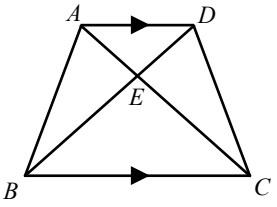



**(a)** $\triangle CDE$

**(b)** $\triangle ADE$

## Practice Exercise

**40.** In the figure, $ABCD$ is a trapezium with $AD$ // $BC$. $AC$ and $BD$ intersect at $E$. The area of $\triangle ADE$ is 10 cm$^2$ and $BC = 2AD$. Find the areas of the following triangles.



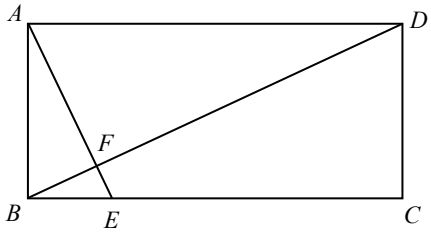

**(a)** $\triangle CBE$

**(b)** $\triangle ABC$

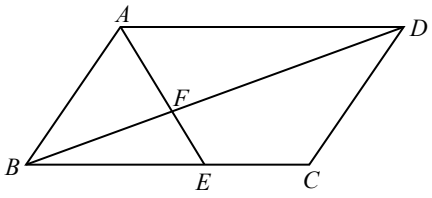
**41.** In the figure, $ABCD$ is a rectangle. $E$ is a point lying on $BC$ such that $BE : EC = 1 : 4$. $AE$ and $BD$ intersect at point $F$. It is given that the area of $\triangle BEF$ is 20 cm$^2$.

**(a)** Find the areas of $\triangle DAF$ and $\triangle ABF$,

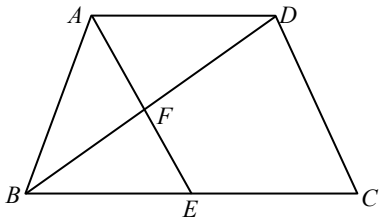
**(b)** Find the area of quadrilateral $CDFE$.



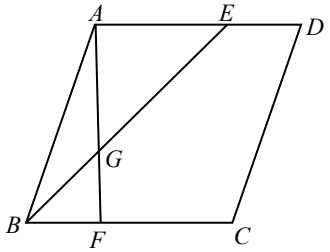

**42.** In the figure, $ABCD$ is a parallelogram. $E$ is a point lying on $BC$ such that $BE : EC = 4 : 3$. $AE$ and $BD$ intersect at point $F$. If the area of quadrilateral $FECD$ is 122 cm$^2$, find the area of $\triangle BEF$.



**43.** In the figure, $ABCD$ is a trapezium with $AD$ // $BC$ and $AD : BC = 2 : 3$. $E$ is a point lying on $BC$ such that $BE : EC = 1 : 1$. $BD$ and $AE$ intersect at point $F$. If the area of $\triangle BEF$ is 45 $cm^2$, find the area of quadrilateral $FECD$.




**44.** In the figure, $ABCD$ is a parallelogram. $E$ and $F$ are points lying on $AD$ and $BC$ respectively such that $AE : ED = 3 : 2$ and $BF : CF = 2 : 3$. $AF$ and $BE$ intersect at point $G$. If the area of $\triangle ABG$ is 24 $cm^2$, find the area of pentagon $CDEGF$.



## Key Points: Property of Similar Solids

If a pair of corresponding **lengths** of two similar 3-D figures are $\ell_1$ and $\ell_2$,

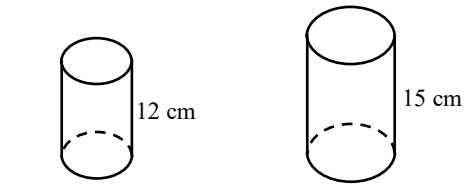
the **areas** of corresponding surfaces are $A_1$ and $A_2$ respectively, and

their **volumes** are $V_1$ and $V_2$ respectively, then we have

$$\frac{A_1}{A_2}=\left(\frac{\ell_1}{\ell_2}\right)^2 \quad \text{and} \quad \frac{V_1}{V_2}=\left(\frac{\ell_1}{\ell_2}\right)^3.$$

### Example 13

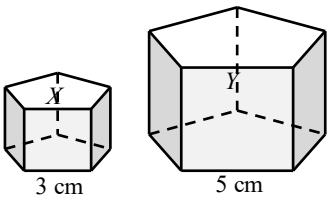
The figure shows two similar right circular cylinders $P$ and $Q$. The heights of circular cylinder $P$ and circular cylinder $Q$ are 12 cm and 15 cm respectively. If the volume of circular cylinder $Q$ is 250 cm$^3$, find the volume of circular cylinder $P$.



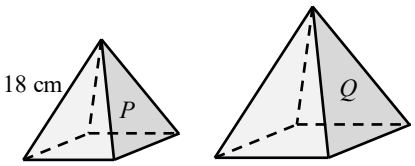

Circular cylinder $P$ Circular cylinder $Q$

### Practice Exercise

**45.** The figure shows two similar prisms $X$ and $Y$. The bases of the prisms are regular pentagons. The lengths of a side of the bases are 3 cm and 5 cm respectively. If the volume of prism $Y$ is 250 cm$^3$, find the volume of prism $X$.



**46.** The figure shows two similar right pyramids $P$ and $Q$. The total surface areas of pyramids $P$ and $Q$ are 432 $cm^2$ and 675 $cm^2$ respectively. If the length of each slant edge of pyramid $P$ is 18 cm, find the length of each slant edge of pyramid $Q$.

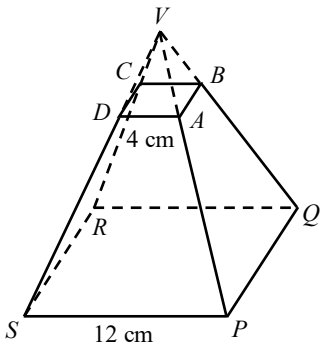



**Example 14 (TB P.7.63)**

The figure shows a right frustum $ABCDSPQR$. The upper base and the lower base of the frustum are squares of sides 4 cm and 12 cm respectively. The volume of pyramid $VABCD$ is 24 $cm^3$.

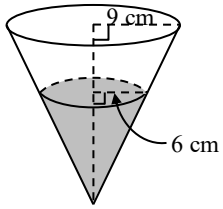
**(a)** Find the volume of pyramid $VPQRS$.

**(b)** Find the volume of the frustum.

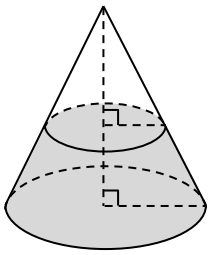


## Practice Exercise

**47.** In the figure, an inverted right circular conical vessel stands vertically and contains 96 $cm^3$ of water. The radius of the water surface is 6 cm and the base radius of the vessel is 9 cm. How much water should be added to fill up the vessel?




**48.** A sealed right circular conical container that contains some water is placed on a horizontal table as shown. The area of the wet curved surface of the container is 220 $cm^2$. The radius of the upper water surface and the lower water surface are 8 cm and 14 cm respectively. Find the area of the dry surface in the container correct to 3 significant figures.



**49.** An inverted right circular conical container stands vertically and contains $256\pi$ $cm^3$ of water. The wet surface area and dry surface area of the container are $32\pi\sqrt{13}$ $cm^2$ and $256\pi\sqrt{13}$ $cm^2$ respectively. Find the volume of the dry part of the container in terms of $\pi$.

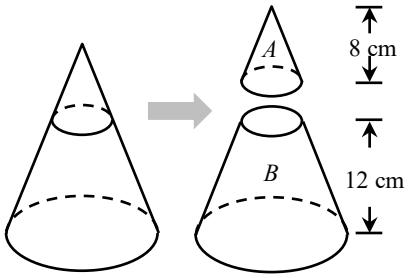
## Example 15 (TB P.7.63)

In the figure, a right circular cone is cut along a plane parallel to its base to obtain a smaller cone $A$ and a frustum $B$. The heights of circular cone $A$ and frustum $B$ are 8 cm and 12 cm respectively.

**(a)** Find the ratio of the radius of the upper base to that of the lower base of frustum $B$.

**(b)** Find the ratio of the curved surface area of circular cone $A$ to that of frustum $B$.

**(c)** Find the ratio of the volume of circular cone $A$ to that of frustum $B$.



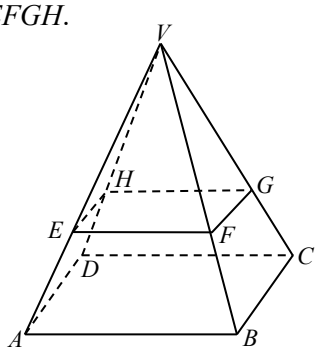

## Practice Exercise

**50.** A pyramid is cut along a plane parallel to its base to obtain a small pyramid and a frustum. It is given that the small pyramid and the original pyramid are similar pyramids. The ratio of the height of the original pyramid to the height of frustum obtained is 7 : 3. Find the ratio of the volume of the small pyramid to that of the frustum.

**51.** In the figure, *VEFGH* and *VABCD* are similar pyramids. The lengths of a pair of corresponding sides of the two pyramids are 10 cm and 16 cm respectively.

**(a)** Find the ratio of the area of $\triangle VFG$ to the area of quadrilateral *FBCG*.

**(b)** Find the ratio of the volume of frustum *EFGHDABC* to that of pyramid *VEFGH*.




**52.** A pyramid is cut along a plane parallel to its base to obtain a small pyramid and a frustum. It is given that the small pyramid and the original pyramid are similar pyramids. The ratio of the total area of the lateral faces of the small pyramid to that of the frustum obtained is 49 : 32. Angela claims that the volume of the small pyramid must be larger than the volume of the frustum. Do you agree? Explain your answer.

## Example 16 (TB P.7.65)

When a sphere is heated, it expands to a larger sphere and its surface area increases by 21%.

**(a)** Find the ratio of the radii of the spheres before and after heating.

**(b)** Find the percentage increase in the volume of the sphere after heating.

## Practice Exercise

**53.** When a circular cone is cooled, it contracts accordingly and its volume decreases by 48.8%.

**(a)** Find the percentage change in the base radius of the circular cone.

**(b)** Find the percentage change in the total surface area of the circular cone.

**54.** When a hemisphere is heated, it expands to a larger hemisphere and its volume increases by 15.7625%.

**(a)** Find the percentage increase in the base circumference of the hemisphere.

**(b)** Find the percentage increase in the total surface area of the hemisphere.

**55.** When a spherical ice ball with original volume 3375 $cm^3$ melts, it shrinks to a smaller spherical ice ball and its surface area decreases by 36%. If the melting rate of the ice ball remains constant and is 27 $cm^3$/min by volume, how long does it take the smaller ice ball to melt completely?