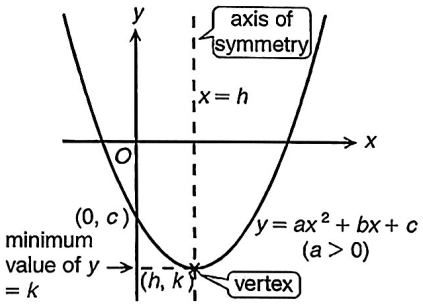
S.4 Mathematics
Chapter 05 – Quadratic Functions – Notes

Name:_____ Class:_____( ) Date:_____

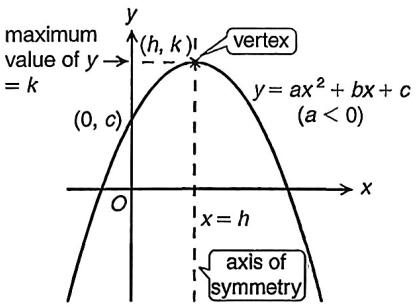
Section 5.1 – Features of the Graphs of Quadratic Function

1. Please refer to the textbook P.2 - 7.
2. Video from Identity:
   07:02 – Graph of Quadratic Function

Features of the graphs of quadratic functions $y = ax^2 + bx + c$



The graph opens upwards, and its vertex is the minimum point.



The graph opens downwards, and its vertex is the maximum point.

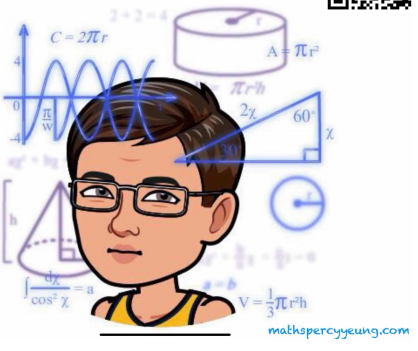
Note: (a) The y-intercept of the graph is c.

(b) The x-intercept(s) of the graph is/are the root(s) of $ax^2 + bx + c = 0$.

A. Pre-lesson:

1. Determine the direction of opening, y-intercept and the number of x-intercept(s) of the following quadratic functions.

	Direction of Opening	y-intercept	Number of x-intercept(s)
$y = 2x^2 - 4x + 3$			
$y = -3x^2 - 5x - 6$			
$f(x) = -10x + 5 + 5x^2$			
$f(x) = 5x - x^2$			



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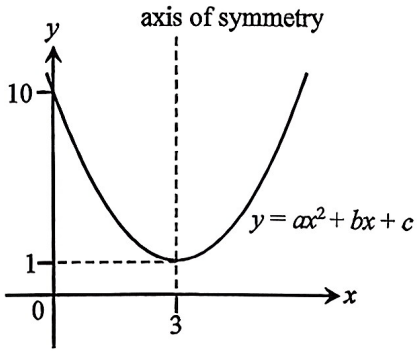
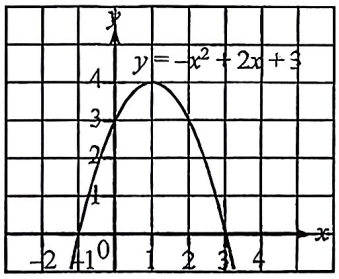
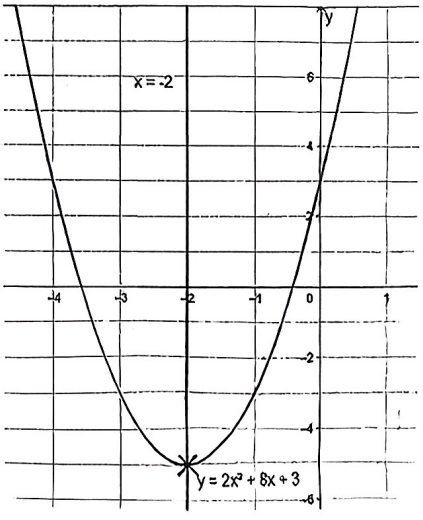
2. The figure shows the graph of $y = ax^2 + bx + c$. Write down
   (a) the y-intercept of the graph,
   (b) the equation of the axis of symmetry of the graph,
   (c) the coordinates of the vertex of the graph,
   (d) the minimum value of the function.



3. The figure shows the graph of $y = -x^2 + 2x + 3$. Find
   (a) the y-intercept of the graph,
   (b) the equation of the axis of symmetry of the graph,
   (c) the coordinates of the vertex of the graph,
   (d) the maximum value of the function.



4. The figure shows the graph of $y = 2x^2 + 8x + 3$.
   (a) State and draw the axis of symmetry of the graph.
   (b) Find the coordinates of the vertex and mark it on the
   (c) Find the maximum or minimum value of the function graphically.



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B. Teaching Examples and Classwork

5. Find the direction of opening, the y-intercept and the x-intercepts of the graph of the following functions.

   (a) $y = x^2 + 7x + 6$

   (b) $y = -x^2 + 5x - 4$

6. Find the direction of opening, the y-intercept and the x-intercepts of the graph of the function.

   (a) $y = 2x^2 - 5x$

   (b) $y = 12 - x^2 - x$

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Consider the graphs of the following functions. (7 – 8)

(a) Determine the directions of opening.

(b) Find the x-intercepts and the y-intercepts.

(c) Find the axis of symmetry of graph of the functions

7. $y = x^2 + 4x + 3$

8. $y = -3x^2 + 9x$

9. It is given that $R(a, 9)$ is the vertex of the graph of the function $y = -x^2 - 2x + 8$. Find

(a) the value of a,

(b) the axis of symmetry of the graph,

(c) the maximum or minimum value of y.

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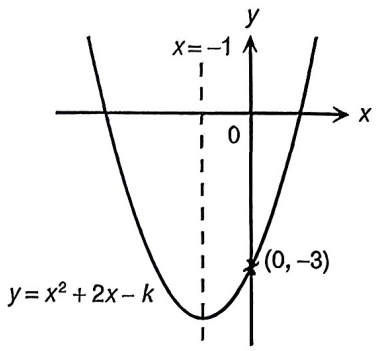
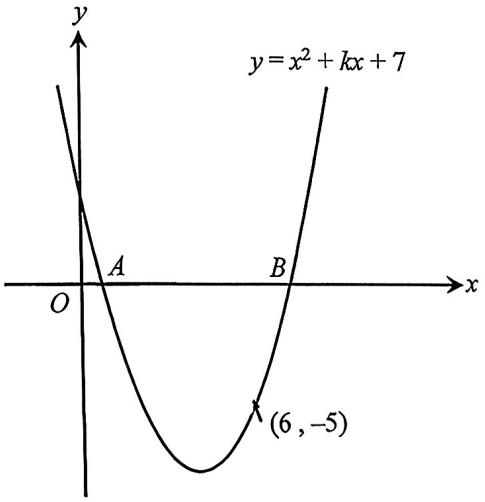
10. The figure shows the graph of $y = x^2 + 2x - k$ which cuts the y-axis at $(0, -3)$. The axis of symmetry is $x = -1$.
    (a) Find the value of k.
    (b) Find the coordinates of the minimum point of the graph.



11. The figure shows the graph of $y = x^2 + kx + 7$, where k is a constant. The graph cuts the x-axis at A and B, and passes through $(6, -5)$.
    (a) Find k.
    (b) Find the coordinates of A and B.



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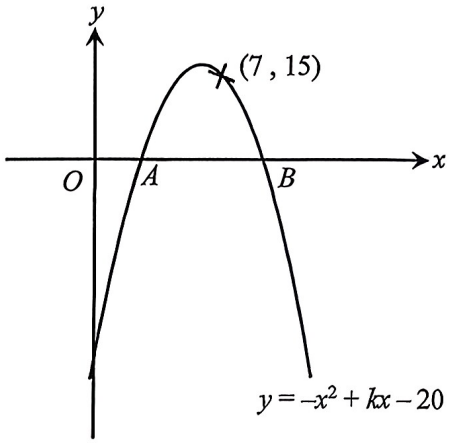
12. The figure shows the graph of $y = -x^2 + kx - 20$, where k is a constant. The graph passes through $(7, 15)$ and cuts the x-axis at A and B.

(a) Find k.

(b) Find the coordinates of A and B.

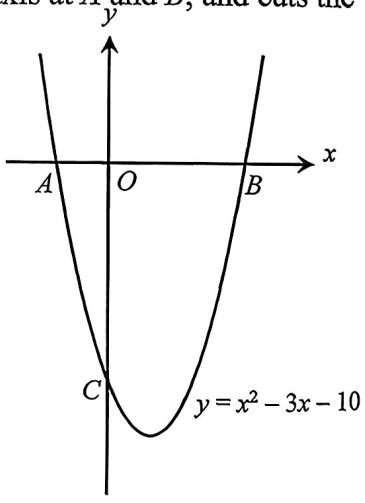


Level 2

13. The figure shows the graph of $y = x^2 - 3x - 10$. The graph cuts the x-axis at A and B, and cuts the y-axis at C.

(a) Find the coordinates of A, B and C.

(b) Find the area of $\triangle ABC$.



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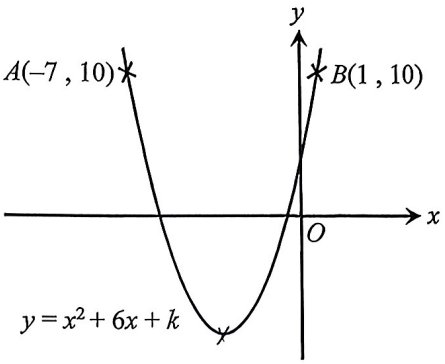
14. The figure shows the graph of $y = x^2 + 6x + k$, where k is a constant. $A(-7, 10)$ and $B(1, 10)$ are two points on the graph.

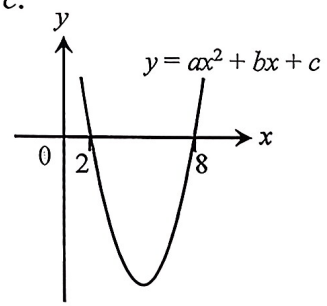
(a) Find the value of k.

(b) **(i)** Find the equation of the axis of symmetry of the graph.

(ii) Find the coordinates of the vertex of the graph.



15. The figure shows the graph of $y = ax^2 + bx + c$, where a, b and c are constants. It is given that the minimum value of the function is –9. Find the values of a, b and c.



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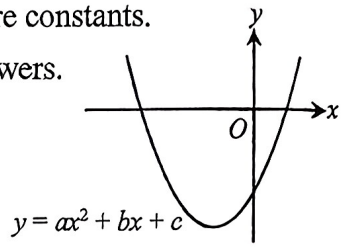
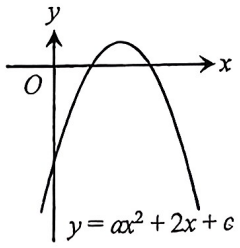
16. The figure shows the graph of $y = ax^2 + bx + c$, where a, b and c are constants.
Determine whether each of the following is true. Explain your answers.



(a) $c > 0$

(b) $ac < 0$

17. The figure shows the graph of $y = ax^2 + 2x + c$, where a and c are constants.
Determine whether $ac < 1$ is true. Explain your answer.



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18. It is given that the graph of $y = (x - 3)(x + 5)$ cuts the y-axis at B and cuts the x-axis at A and C, where C is on the right of A. $D(4 , k)$ is a point on the graph.

(a) Find the coordinates of A, B and C.

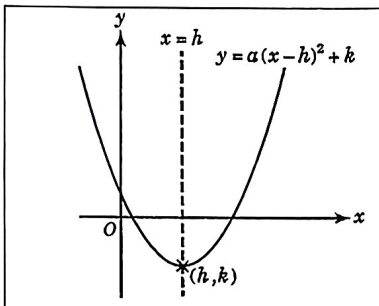
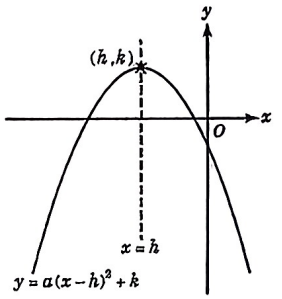
(b) Find the value of k.

(c) Find the area of the quadrilateral $ABCD$.

Section 5.2a – Maximum or Minimum Values of Quadratic Function

1. Please refer to the textbook P.19 - 20.
2. Video from Identity: _____

Features of the Quadratic Function $y = a(x - h)^2 + k$ and Its Graph

		
Range of a	$a > 0$	$a < 0$
Direction of opening	upwards	downwards
Maximum or minimum value	k, minimum value	k, maximum value
Vertex	(h, k), the lowest point	(h, k), the highest point
Axis of symmetry	$x = h$	

00:00 – 02:21 – Relationship between $y = a(x-h)^2 + k$ and vertex.

A. Pre-lesson:

1. For each of the following functions, write down
   - **(i)** the direction of opening, the coordinates of the vertex and the equation of the axis of symmetry of its graph,
   - **(ii)** the maximum or minimum value.

   (a) $y = (x - 3)^2 + 5$

   (b) $y = -(x + 2)^2 - 6$

B. Teaching Examples and Classwork

For each of the following quadratic functions, (2 – 5)

find (a) the direction of opening of its graph,

(b) the coordinates of the vertex its graph,

(c) the axis of symmetry its graph,

(d) its optimum value and state whether the value is a maximum value or a minimum value.

2. $y = x^2 + 1$

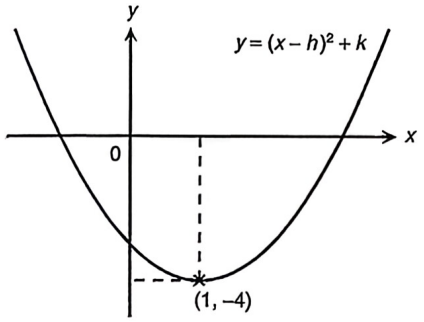
3. $y = -3(x - 2)^2 - 2$

4. $y = (x + 3)^2 + 5$

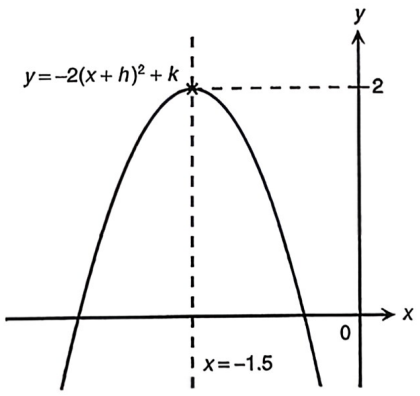
5. $y = -(x + 2)^2 - 1$

The following figures show the graphs of the quadratic functions. Find the values of h and k. (6 – 7)

6.



7.



8. For each of the following quadratic functions, find its optimum value and the corresponding value of x, and state whether the optimum value is a maximum or a minimum.

(a) $y = 3(x-2)^2 - 7$ (b) $y = -2(x+5)^2 + 10$

(c) $y = \frac{1}{2}\left(x+\frac{3}{4}\right)^2 - \frac{5}{6}$ (d) $y = -\frac{1}{10}\left(x-\frac{8}{5}\right)^2 + \frac{1}{8}$

Section 5.2b – The method of Completing the Square

1. Please refer to the textbook P.22 – 23.
2. Video from Identity:

   02:21 – Completing the square

   07:37 – Optimum values of quadratic functions

   13:02 – Summary (with a shortcut of finding the vertex)

1. Completing the square

To complete the square for expressions $x^2 + kx$ and $x^2 - kx$, add $\left(\frac{k}{2}\right)^2$ to each expression.

Then $x^2 + kx + \left(\frac{k}{2}\right)^2 = x^2 + 2\left(\frac{k}{2}\right)x + \left(\frac{k}{2}\right)^2 = \left(x+\frac{k}{2}\right)^2$

and $x^2 - kx + \left(\frac{k}{2}\right)^2 = x^2 - 2\left(\frac{k}{2}\right)x + \left(\frac{k}{2}\right)^2 = \left(x-\frac{k}{2}\right)^2$

2. Optimum values of quadratic functions

By completing the square, any quadratic function $y = ax^2 + bx + c$ can be converted to the form $y = a(x-h)^2 + k$, where $h = -\frac{b}{2a}$ and $k = -\frac{b^2 - 4ac}{4a}$.

(a) If $a > 0$, then the minimum value of y is k when $x = h$.

(b) If $a < 0$, then the maximum value of y is k when $x = h$.

A. Pre-lesson:

For each of the following quadratic functions, (9 – 12)

(a) rewrite it into the form $y = a(x - h)^2 + k$ by using the method of completing the square.

(b) find its optimum value and state whether the value is a maximum or minimum,

(c) find the coordinates of the vertex and the axis of symmetry of its graph.

9. $y = x^2 - 2x + 1$

(a) $y = x^2 - 2x + 1$

$= x^2 - 2x + \left(-\right)^2 - \left(-\right)^2 + 1$

$= (x - \_\_)^2 + \_\_\_$

10. $y = x^2 + 8x + 4$

(b) $y = x^2 + 8x + 4$

$= x^2 + 8x + \left(-\right)^2 - \left(-\right)^2 + 4$

$= \_\_\_\_\_$

11. $y = -x^2 + 4x + 7$

(a) $y = -x^2 + 4x + 7$

$= -(x^2 - 4x) + 7$

$= -\left(x^2 - 4x + \left(-\right)^2 - \left(-\right)^2\right) + 7$

$=$

$=$

12. $y = -4x^2 - 12x - 10$

(a) $y = -4x^2 - 12x - 10$

$= -4(\quad\quad) - 10$

$=$

$=$

$=$

B. Teaching Examples and Classwork

13. Given that the minimum value of the function $y = x^2 + 20x + k$ is 7, find the value of k.

Level 2

14. The y-intercept of the graph of $y = (x + 1)(x + k) + 4k$ is 10.
    (a) Find the value of k.
    (b) Find the coordinates of the vertex and the axis of symmetry of the graph.

15. It is given that $V(2, 5)$ is the vertex of the graph of a quadratic function $y = f(x)$, and its y-intercept is 29.
    (a) Find $f(x)$.
    (b) Find the direction of opening of the graph.
    (c) Does the graph have any x-intercepts? Explain your answer.

16. (a) The maximum value of the function $y = -x^2 - 2x + c$ is 6, where c is a constant. Find the value of c.
    (b) The minimum value of the function $y = x^2 - 2kx$ is -9, where k is a constant. Find the values of k.

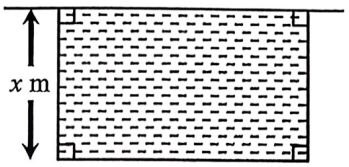
17. If the sum of two real numbers is 6, find the maximum value of their product.

18. The profit of producing n bottles of hand sanitizer by a production line is $\$P$ per hour, where $P = -2n^2 + 280n - 9\,000$.

(a) How many bottles of hand sanitizer should be produced by the production line per hour such. that the profit is a maximum?

(b) Find the maximum profit.

19. In the figure, a fence is used to surround a rectangular garden in front of a wall. The total length of the fence is 40 m. Let x m be the length of the side of the garden which is perpendicular to the wall and y m^2 be the area of the garden.



(a) Express y in terms of x.

(b) Someone claims that the area of the garden can exceed 200m^2. Do you agree? Explain your answer.

Level 3

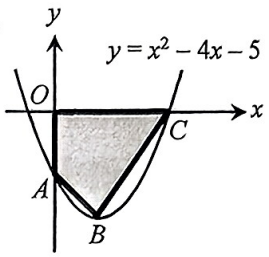
20. The maximum value of the function $y = -3x^2 + 12hx + 8h^2$ is 5, where h is a constant. Find the values of h.

21. The base of a parallelogram is $(x - 6)$ cm and the height is $(18 - x)$ cm, where $6 < x < 18$. Can the area of the parallelogram be greater than 38 cm^2? Explain your answer.

22. In the figure, B is the vertex of the graph of the function $y = x^2 - 4x - 5$.
It is given that the graph cuts the y-axis at the point A and cuts the positive x-axis at the point C.

(a) Find the coordinates of A, B and C.

(b) Find the area of the quadrilateral $OABC$, where O is the origin.



F. Past Paper

Paper I

1. Let $f(x)=2x^2-4kx+3k^2+5$, where k is a real constant.

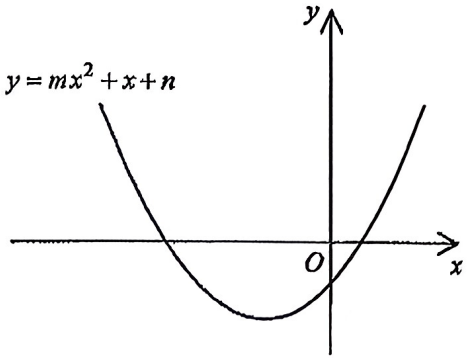
   (a) Does the graph of $y=f(x)$ cut the x-axis? Explain your answer. (2 marks)

   (b) Using the method of completing the square, express, in terms of k, the coordinates of the vertex of the graph of $y=f(x)$. (3 marks)

[HKDSE 15 #18ab]

Paper II

2. The figure shows the graph of $y = mx^2 + x + n$, where m and n are constants. Which of the following is true?



A. $m < 0$ and $n < 0$
B. $m < 0$ and $n > 0$
C. $m > 0$ and $n < 0$
D. $m > 0$ and $n > 0$

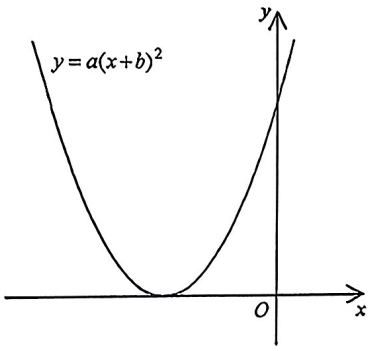
[HKDSE 14 #5]

3. Let $f(x) = 3x^2 - 6x + k$, where k is a constant. If the y-coordinate of the vertex of the graph of $y = f(x)$ is 7, then $k =$

A. 1.
B. 3.
C. 4.
D. 10.

[HKDSE 14 #35]

4. The figure shows the graph of $y = a(x+b)^2$, where a and b are constants. Which of the following is true?

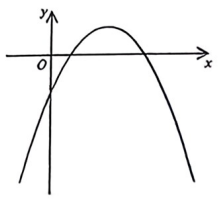


A. $a < 0$ and $b < 0$
B. $a < 0$ and $b > 0$
C. $a > 0$ and $b < 0$
D. $a > 0$ and $b > 0$

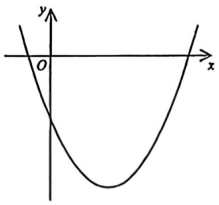
[HKDSE 15 #8]

5. If $-1<a<0$, which of the following may represent the graph of $y=(ax+1)^2+a$?

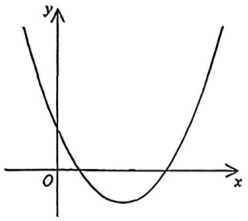
A.



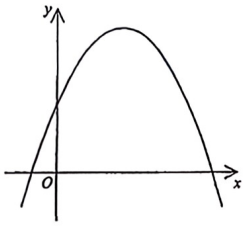
B.



C.



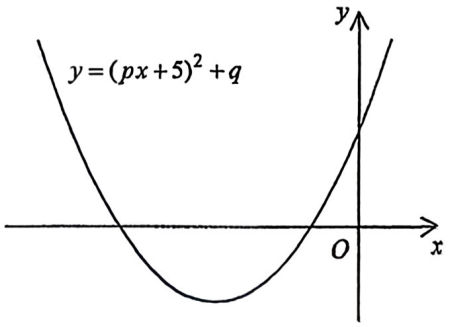
D.



[HKDSE 16 #9]

6. The figure shows the graph of $y=(px+5)^2+q$, where p and q are constants. Which of the following is true?

A. $p<0$ and $q<0$
B. $p<0$ and $q>0$
C. $p>0$ and $q<0$
D. $p>0$ and $q>0$



[HKDSE 17 #9]

7. Which of the following statements about the graph of $y=16-(x-6)^2$ is true?

A. The graph cuts the x-axis.
B. The graph opens upwards.
C. The y-intercept of the graph is 16 .
D. The graph passes through the origin.

[HKDSE 18 #5]

8. Which if the following statements about the graph of $y=(3-x)(x+2)+6$ is/are true?

I. The graph opens downwards.
II. The graph passes through the point $(1,10)$.
III. The x-intercepts of the graph are -2 and 3.

A. I only
B. II only
C. I and III only
D. II and III only.

[HKDSE 19 #10]

9. Let m and n be real constants. Which of the following statements about the graph of $y=(m-x)^2+n$ must be true?

I. The graph opens upwards.
II. The y-intercept of the graph is positive.
III. The graph passes through the point (n,m).

A. I only
B. II only
C. I and III only
D. II and III only.

[HKDSE 21 #14]

10. Let h and k be real constants such that $hk<0$. Which of the following statements about the graph of $y=(h-x)(k-x)$ are true?

I. The graph opens upwards.
II. The graph has two x-intercept.
III. The y-intercept of the graph is positive.

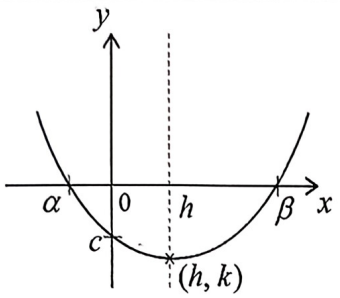
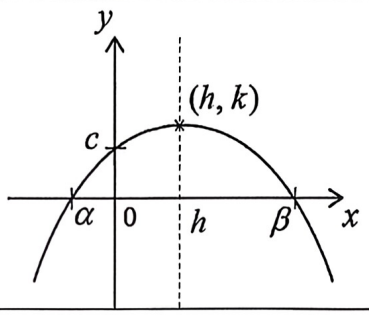
A. I and II only
B. I and III only
C. II and III only
D. I, II and III

[HKDSE 22 #10]

Appendix

Summary of Features of Graphs of Quadratic Functions

For a quadratic function $y = ax^2 + bx + c$ (or $y = a(x-h)^2 + k$) (or $y = a(x-\alpha)(x-\beta)$),

$a > 0$	$a < 0$
	

Common Features:

1) y-intercept $= c$

2) x-intercepts $= \alpha$ and β, which are the real roots of $ax^2 + bx + c = 0$

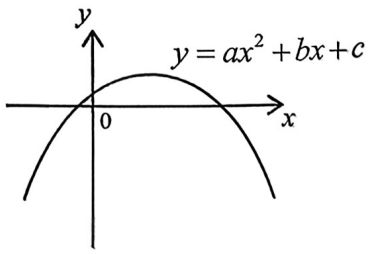
3) Axis of symmetry: $x = h$, where $h = \dfrac{\alpha+\beta}{2} = -\dfrac{b}{2a}$.

4) Coordinates of Vertex $= (h,k) = \left(\dfrac{\alpha+\beta}{2}, -\dfrac{\Delta}{4a}\right)$ where $\Delta = b^2 - 4ac$.

Differences:

1) The graph opens <u>upwards</u>.	1) The graph opens <u>downwards</u>.
2) For the vertex (h, k), k is the <u>minimum</u> value of y and h is the corresponding value of x.	2) For the vertex (h, k), k is the <u>maximum</u> value of y and h is the corresponding value of x.
3) No <u>maximum</u> value.	3) No <u>minimum</u> value.

Example 1: Determine the sign of a, b and c in the following graph.



Solution:

1) $a < 0$ because the graph opens downwards.

2) $c > 0$ because y-intercept > 0.

3) Because x-coordinate of the vertex > 0,

$-\dfrac{b}{2a} > 0$, $b > 0$ as $a < 0$.