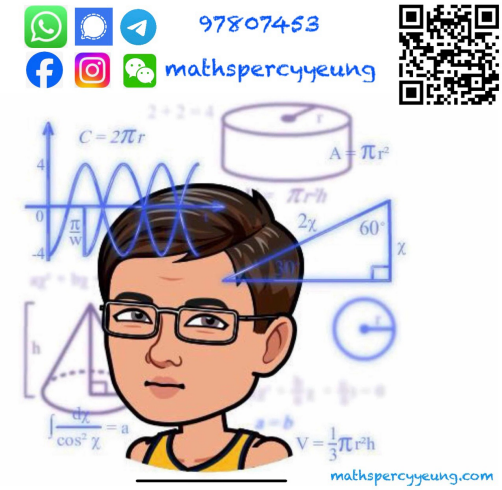
2023-2024 S4
2nd TERM UT
MATH CP

2023 – 2024
S4 Second Term Uniform Test

MATHEMATICS Compulsory Part

Question–Answer Book

8th April, 2024
9:45 am – 11:00 am (1 hour 15 minutes)
This paper must be answered in English



INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.

2. Answer ALL questions in Section A. You should use an HB pencil to mark all the answers on the Answer Sheet, so that wrong marks can be completely erased with a clean rubber. You must mark the answers clearly; otherwise you will lose marks if the answers cannot be captured. You should mark only ONE answer for each question. If you mark more than one answer, you will receive NO MARKS for that question.

3. Attempt ALL questions in Sections B and C. Write your answers in the spaces provided in this Question – Answer Book.

4. Unless otherwise specified, all working must be clearly shown and numerical answers should be either exact or correct to 3 significant figures.

5. The diagrams in this paper are not necessarily drawn to scale.

Sections	Marks
A Total	**/28**
B (15 – 16)	
B (17 – 20)	
B Total	**/28**
C Total	**/14**
TOTAL	**/70**

Section A (28 marks)

Choose the best answer for each question.

1. $(4^{2n+2} \cdot 64)^2 =$

   A. 2^{4n+10}.
   B. 2^{4n+16}.
   C. 2^{8n+10}.
   D. 2^{8n+20}.

2. Simplify $\frac{3}{x+1}+\frac{3}{x-1}$.

   A. $\frac{6}{x^2-1}$
   B. $\frac{6x}{x^2-1}$
   C. $\frac{3}{x^2-1}$
   D. $\frac{3x}{x^2-1}$

3. If $f(x)=3x+10$, then $f(x+1)-f(x-1)=$

   A. 0.
   B. 6.
   C. x.
   D. $3x$.

4. Let $f(x)=-2x^3+9x^2+k$. If $f(x)$ is divisible by $2x-1$, find the remainder when $f(x)$ is divided by $2x+1$.

   A. $-\frac{1}{2}$
   B. -2
   C. $\frac{1}{2}$
   D. $\frac{9}{2}$

5. Which of the following statements about the graph of the quadratic function $y = 3(x + 4)^2 – 18$ is true?

   A. The coordinates of the vertex of the graph are $(4 , 18)$.
   B. The axis of symmetry of the graph is $y = 18$.
   C. The graph and the straight line $y = –19$ intersect.
   D. The y-intercept of the graph is 30.

6. If $125^{3x} = 5$, then $x =$

   A. $\frac{1}{6}$.
   B. $\frac{2}{3}$.
   C. $\frac{4}{5}$.
   D. $\frac{1}{9}$.

7. Given that $\cos\theta = \frac{3}{4}$ and $0^o < \theta < 90^o$, find the value of $\tan\theta$.

A. $\frac{3}{5}$

B. $\frac{\sqrt{7}}{3}$

C. $\frac{4}{3}$

D. $\frac{5}{3}$

8. The straight line L_1: $ax + 9y + 6 = 0$ is perpendicular to the straight line L_2: $3x - 5y - 6 = 0$, where a is a non-zero real number. Find the value of a.

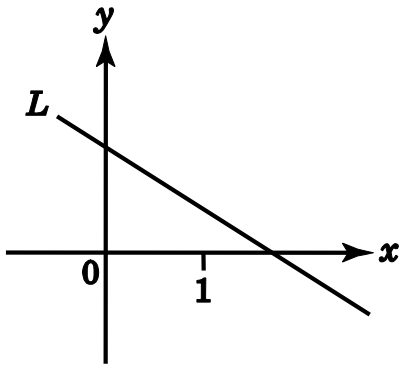
A. –15

B. –3

C. 3

D. 15

9. The figure shows the graph of the straight line L: $ax + y + b = 0$. Which of the following are true?



I. $a > 0$

II. $b < 0$

III. $a + b > 0$

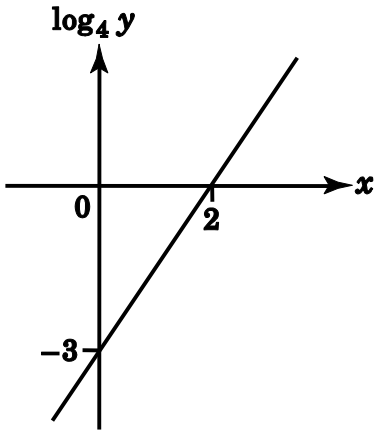
A. I and II only

B. I and III only

C. II and III only

D. I, II and III

10. The graph in the figure shows the linear relation between x and $\log_4 y$. If $y = mn^x$, then $n =$



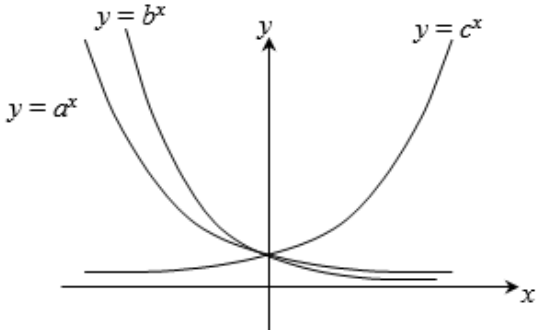
A. $\frac{1}{64}$.

B. $\frac{1}{8}$.

C. 8.

D. 64.

11. The figure shows the graph of $y = a^x$, $y = b^x$ and $y = c^x$, where a, b and c are constants. Which of the following must be true?



I. $a < b$
II. $b < c$
III. $a < 1$

A. I and II only
B. I and III only
C. II and III only
D. I, II and III

12. Solve $5^{2x+1} - 1 + 25^x = 3749$.

A. $x = 2$
B. $x = 2.5$
C. $x = 4$
D. $x = 5$

13. Solve
$[\log (x + 10)]^2 - 3 \log (x + 10) - 4 = 0$.

A. $x = -1$ or 4
B. $x = -\frac{99}{10}$ or 9 990
C. $x = -\frac{1}{10}$ or 10 000
D. $x = \frac{1}{10}$ or 10 000

14. Which of the following has the least value?

A. 500^{200}
B. 400^{300}
C. 300^{400}
D. 200^{500}

Section B(1) (7 marks)

15. Peter spent \$80 on some \$5 stamps and \$3 stamps. If the total number of stamps is 22 , find the numbers of \$5 and \$3 stamps that Peter bought respectively. (4 marks)

16. Prove that $\cos^2\theta - 3\sin^2\theta \equiv (1+2\sin\theta)(1-2\sin\theta)$. (3 marks)

Section B(2) (21 marks)

17. Simplify $\left(\frac{\sqrt[3]{a^2}}{\sqrt[6]{a}}\right)^{-\frac{4}{3}}$ and express your answer with positive indices. (3 marks)

18. Solve $\log_5(3x+8) = \log_5 x + 1$. (4 marks)

19. The monthly salary \$$S$ of Martin in the t-th year after 2023 can be represented by the following formula:

$$S = 13\,500(1 + k)^t \text{ , where } k \text{ is a positive constant.}$$

It is given that the monthly salary of Martin is \$14 040 in 2024 .

(a) Find the value of k .

(b) In which year will his monthly salary start to exceed that in 2023 by 60%?

(7 marks)

20. Let $f(x)=8x^3-3tx-t$. It is given that $f(x)$ is divisible by $x-1$.

(a) Find the value of t .

(b) Factorize $f(x)$.

(c) Using the result of (b) or otherwise, solve $f(x)=0$.

(7 marks)

Section C (14 marks)

21. Simplify $\dfrac{\log_9 x^5}{\log_3 x^4}$, where $x > 0$ and $x \neq 1$. (3 marks)

22. The graph in Figure 1 shows the linear relation between $\sqrt{y}$ and x^3 . The intercept on the horizontal axis and vertical axis of the graph are -4 and 1 respectively. Find the value of y when $x=2$.

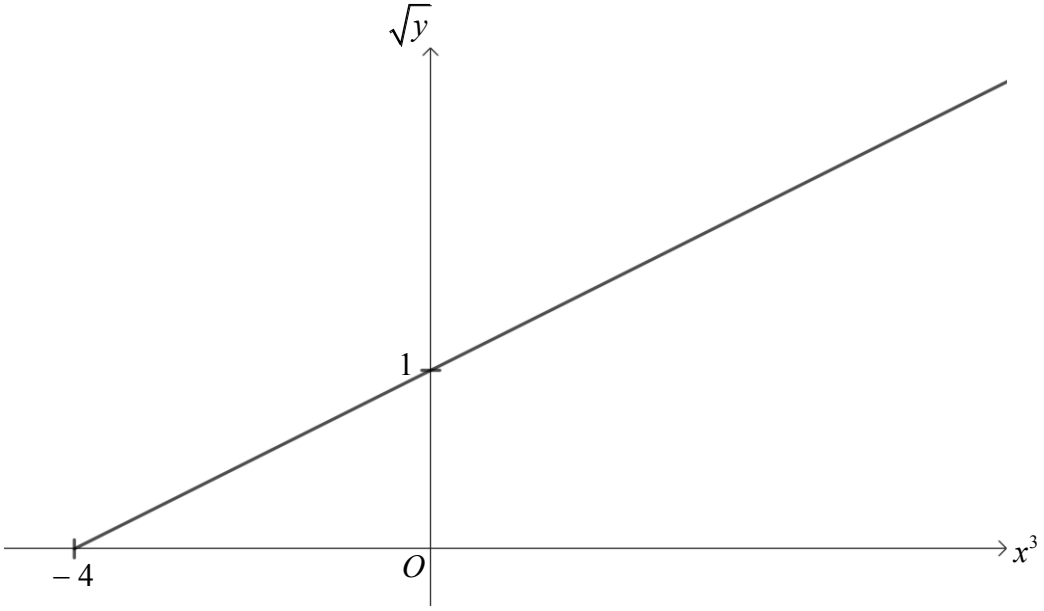


Figure 1

(4 marks)

23. It is given that $y = k - 4x$ and $y = x^2 - 12x + 18$ intersect at A and B.

(a) Find the range of values of k. (4 marks)

(b) Express the coordinates of the mid-point of AB in terms of k. (3 marks)

END OF PAPER