2023-2024 S5
1st TERM EXAM
MATH CP
PAPER 1

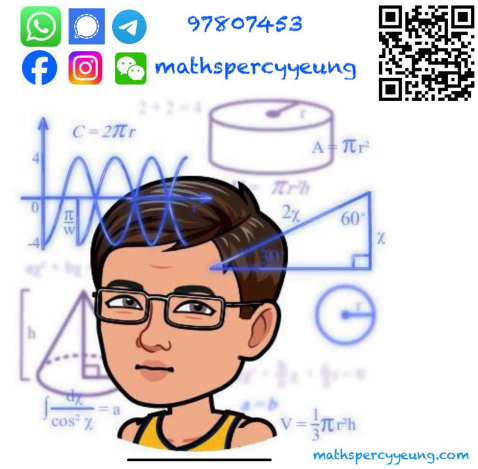
2023 – 2024
S5 First Term Examination

MATHEMATICS Compulsory Part

PAPER 1

Question–Answer Book

4th January, 2024
8:15 am – 10:00 am (1 hour 45 minutes)
This paper must be answered in English



Section	Marks
A (1 – 3)	/ 11
A (4 – 12)	/ 50
A Total	**/ 61**
B Total	**/ 24**
TOTAL	**/ 85**

INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.

2. This paper consists of THREE sections, A(1), A(2) and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

6. The diagrams in this paper are not necessarily drawn to scale.

Section A(1) (27 marks)

1. Simplify $\dfrac{(2024ab^7)^0}{3a^{-6}(2b^2)^3}$ and express your answer with positive indices. (3 marks)

2. Factorize

(a) x^3+8,

(b) x^2-x-6,

(c) x^3+x^2-x+2 .

(4 marks)

3. The angle and the area of a sector are 80° and 72π cm^2 respectively.

(a) Find the radius r of the sector.

(b) Find the perimeter of the sector in terms of π.

(4 marks)

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4. It is given that y is partly constant and partly varies directly as x^3. When $x = 1$, $y = 12$ and when $x = 2$, $y = 26$. Find the value of y when $x = -2$. (4 marks)

5. If the quadratic equation $3x^2 + (k+2)x + \frac{k}{2} + 1 = 0$ has real roots, where k is a constant, find the range of values of k. (4 marks)

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6. **(a)** Solve the compound inequality $\frac{1+x}{2} > \frac{2-x}{3} + 4$ and $3x - 21 \leq 0$.

(b) How many integers satisfy the inequality in **(a)**?

(4 marks)

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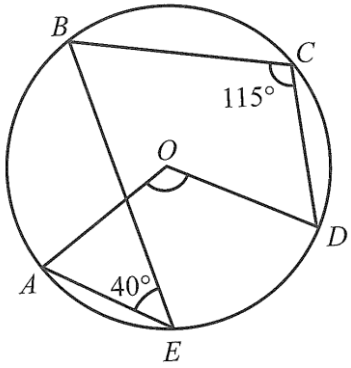
7. In the figure, O is the centre of the circle $ABCDE$. If $\angle AEB = 40°$ and $\angle BCD = 115°$, find $\angle AOD$. (4 marks)



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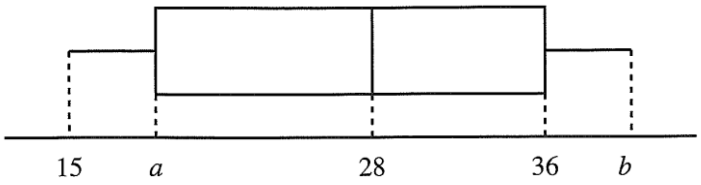
Section A(2) (34 marks)

8. The box-and-whisker diagram below shows the distribution of the typing speeds (in words per minutes) of a group of students in a school before training.



Typing speed (words per minute)

The inter-quartile range and the range of the distribution are 18 words per minute and 23 words per minute respectively.

(a) Find a and b. (2 marks)

(b) A datum equal to mean is removed from the data. The trainer claims that there is no change in the standard deviation. Do you agree? Explain your answer. (2 marks)

(c) After the training, the median of the distribution of the typing speeds of the group of students is 40 words per minute. The trainer claims that at least half of the students show improvement in the typing speed after the training. Do you agree? Explain your answer. (2 marks)

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9. Let $f(x) = x(2x+1)(3x+5) + px + q$, where p and q are constants. It is given that $x+2$ is a factor of $f(x)$. When $f(x)$ is divided by $x-2$, the remainder is $8-2q$.

(a) Find p and q. (3 marks)

(b) Betty claims that the equation $f(x) = 0$ has at least one irrational root. Do you agree? Explain your answer. (3 marks)

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10. The following stem-and-leaf diagram shows the ages of the players of a basketball team.

Stem (tens)	Leaf (units)
1	a 9
2	1 2 3 3 4 5 6 6 7 9
3	2 4

Given that the range of the ages of the players is 15.

(a) **(i)** Find the value of a.

(ii) Find the mean and the inter-quartile range of the ages of the players.

(3 marks)

(b) The three oldest players leave the team and the manager of the team recruits two new players to join the team. The manager finds that the mean age of the players is decreased by 1.

(i) Find the mean age of the two new players.

(ii) Furthermore, the manager finds that the inter-quartile range of the ages of players remains unchanged. Find a possible set of ages of the two new players.

(4 marks)

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11. A right circular cylindrical container of base radius 15 cm and height 2.5 cm and an inverted right circular conical vessel of base radius 10 cm and height 40 cm are held vertically. The container is fully filled with water. The water in the container is now poured into the vessel until the container is empty.

(a) Find the volume of water in the vessel. (Express your answer in terms of π.) (2 marks)

(b) Find the depth of water in the vessel. (3 marks)

(c) If a solid metal sphere of radius 6 cm is put into the vessel and the sphere is fully immersed into the water, what is the maximum number of spheres that can be put into the vessel before the water overflows? Explain your answer. (3 marks)

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12. A and B are two fixed points. P is a moving point such that $PA^2 + PB^2 = AB^2$. Denote the locus of P by Γ.

(a) Describe the geometric relationship between AB and Γ. (1 mark)

(b) A rectangular coordinate system is introduced such that A is a point lying on a straight line $L_1 : 4x - y + 3 = 0$. The equation of the straight line passing through A and B is $L_2 : 23x - 10y - 4 = 0$. $C(3, 6.5)$ is the mid-point of A and B.

(i) Find the equation of Γ.

(ii) Does Γ passes through the point (4, 19)? Explain your answer.

(6 marks)

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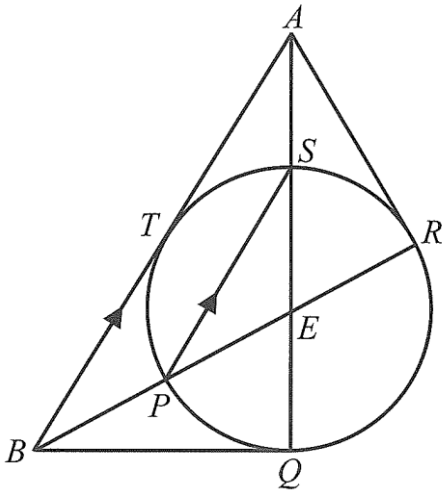
Section B (24 marks)

13. In the figure, E is the centre of a circle $PQRST$. BA and BQ are tangents to the circle at T and Q respectively. $ASEQ$ and $BPER$ are straight lines and BA // PS.



(a) Prove that B, Q, R and A are concyclic. (3 marks)

(b) Is AR the tangent to the circle at R ? Explain your answer. (2 marks)

(c) **(i)** Prove that ΔABQ and ΔBAR are congruent.

(ii) If $BA = 12$ cm, find the perimeter of the quadrilateral $ABQR$.

(5 marks)

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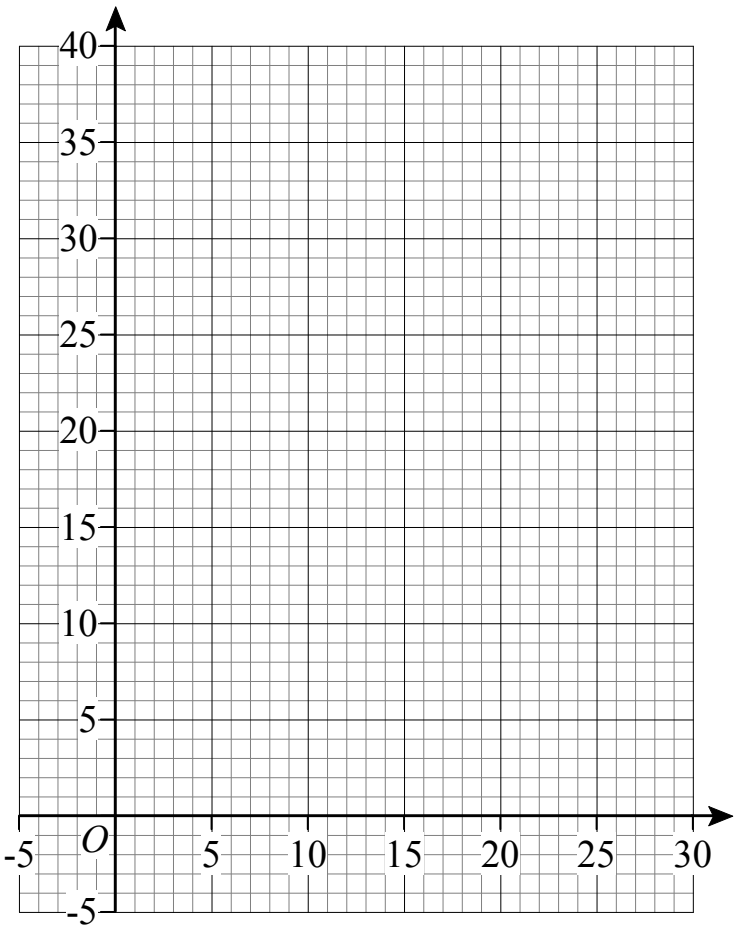
14. Mr. Cheung has to prepare at least 40 chicken wings, 60 cuttlefish-balls and 40 pieces of pork chop for a barbecue. A shop sells two types of barbecue food sets, *A* and *B*. There are 2 chicken wings, 6 cuttlefish-balls and 3 pieces of pork chop in each set *A*, and there are 4 chicken wings, 2 cuttlefish-balls and 2 pieces of pork chop in each set *B*. Suppose x sets *A* and y sets *B* are to be bought.

(a) Write down all the constraints about the number of sets *A* and the number of set *B* to be bought. (3 marks)

(b) Given that the selling prices of each set *A* and each set *B* are \$20 and \$16 respectively, how many sets of each type should Mr. Cheung buy, so that the amount payable is kept at the minimum? (5 marks)

(c) If the shop now offers a promotion deal of buying two sets *B* for \$25 only, how many sets of each type should Mr. Cheung buy, so that the amount payable is kept at the minimum? (2 marks)



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15. The weights of the students in a school are normally distributed. The mean and the standard deviation of the weights are 68 kg and 5 kg respectively. If there are 224 students with weight heavier than 73 kg, find the number of students with weight between 58 kg to 78 kg.

(4 marks)

END OF PAPER

Answers written in the margins will not be marked