S3 First Term Examination (2023-2024)

Mathematics

(2 hours)

Date: 16th January 2024 Name: _____

Time: 8:30 a.m. – 10:30 a.m. Class: _____ No.:____

Instructions to students:

1. This paper consists of THREE parts, Conventional Questions, Multiple-choice Questions and Bonus Question. There are Section A1, Section A2 and Section B in Conventional Questions. Section A1 carries 31 marks, Section A2 carries 27 marks, Section B carries 22 marks, Multiple-choice Questions carry 20 marks and Bonus Question carries 3 marks.

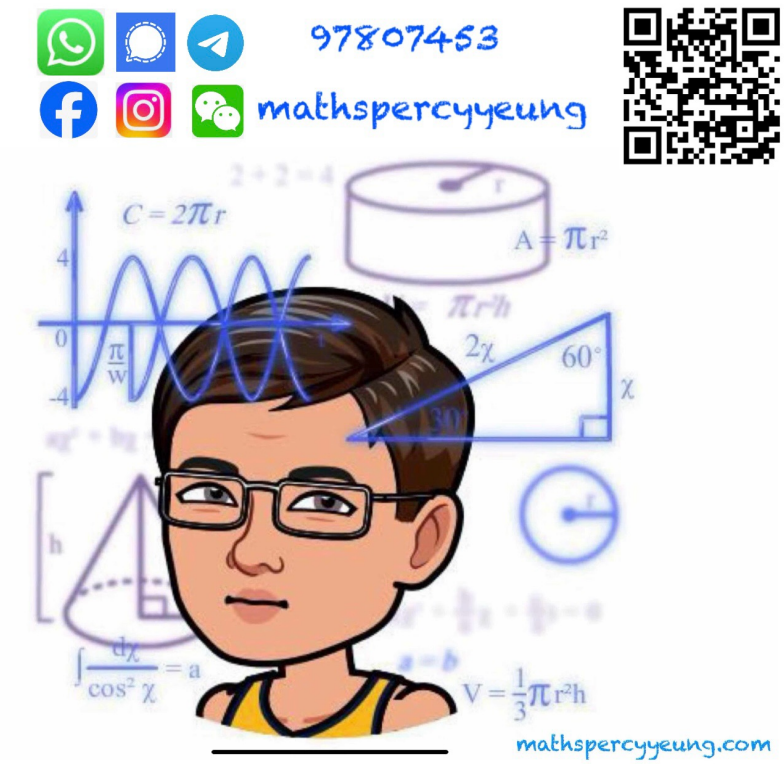
2. The maximum score of this paper is 100.

3. Attempt ALL questions in Conventional Questions and Multiple-choice Questions. Write your answers in the spaces provided in this Question / Answer Book.

4. Unless otherwise specified, show your workings clearly.

5. Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.

6. The diagrams in this paper are not necessarily drawn to scale.



Conventional Questions

Section A1 (31 marks)

1. Simplify $\frac{(p^2q^{-4})^3}{p^{-10}}$ and express the answer with positive indices. (3 marks)

2. Make h the subject of the formula $k(4h - k) = 5(h + 7k)$. (3 marks)

3. Factorize

(a) $xz - 3yz$,

(b) $x^2 + 11xy - 42y^2$,

(c) $2x^2 + 22xy - 84y^2 - xz + 3yz$.

(5 marks)

4. (a) Solve the inequality $x + 5 \leq -\frac{5x+7}{3}$.

(b) Write down the greatest integer satisfying the inequality in (a).

(4 marks)

5. The depreciation rate of a bicycle is 12% per year. If its present value is \$17 260, find its value 2 years ago. (*Give the answers correct to the nearest \$100.*) (2 marks)

6. In Figure 1, O is the incentre of $\triangle PQR$. $\angle POQ = 120°$ and $\angle OQR = 16°$. Find $\angle PRQ$. (4 marks)

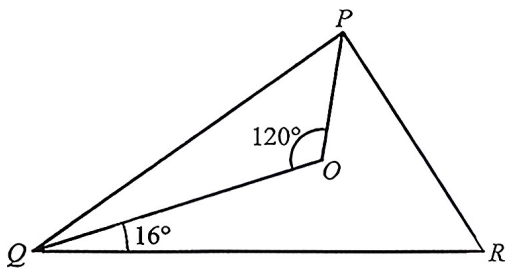


Figure 1

7. An alloy weighs 240 g, where 75% is gold and 25% is copper by weight. A new alloy is formed by decreasing the weight of gold by 30% and increasing the weight of copper by 50%.

   (a) Find the weight of gold in the new alloy.

   (b) Find the percentage change in the weight of the alloy.

(6 marks)

8. Simplify $\frac{2\cdot 3^{n+1}+8\cdot 3^{n}}{5\cdot 3^{n}-8\cdot 3^{n-1}}$, where n is a positive integer. (4 marks)

Section A2 (27 marks)

.. Jaden has a total of 25 \$5 coins and \$2 coins. If the total value of these coins is at most \$90. Find the maximum number of \$5 coins. (4 marks)

10. Michael wants to borrow \$80 000. He will repay the original sum of money together with the interest after 10 years. Bank *A* charges a simple interest rate of 8% p.a., while Bank *B* charges an interest rate of 6% p.a. compounded monthly.

(a) Find the interest Michael needs to pay if he borrows the money

(i) from Bank *A*,

(ii) from Bank *B*.

(*Give your answer correct to the nearest dollar.*)

(b) Which bank should Michael borrow the money from? Explain your answer.

(7 marks)

11. In Figure 2, $ABCD$ is a square and $ABEF$ is a rhombus. AD and EF intersect at point G. BF intersects AE and AD at I and H respectively. It is given that $\angle ABE = 52°$.

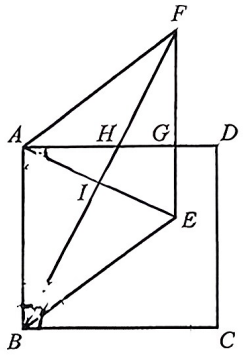


Figure 2

Find

(a) $\angle FAG$,

(b) $\angle IHG$.

(6 marks)

12. Chloe is single. She earns $40 000 each month. She lives with two dependent parents who are aged over 60.

Allowances	
Allowances	Amount ($)
• Basic	132 000
• Married person	264 000
• Child (For each of the 1st to 9th child)	120 000
• Each dependent parent/grandparent aged 60 or above and living with the taxpayer	100 000

Progressive rates		
Net chargeable income	Rate	Tax ($)
On the first $50 000	2%	1000
On the next $50 000	6%	3000
On the next $50 000	10%	5000
On the next $50 000	14%	7000
Remainder	17%	

(a) Find her net chargeable income.

(b) Chloe claims that her salaries tax payable according to the progressive rates exceeds 2% of her annual income. Is the claim correct? Explain your answer.

(4 marks)

1. In Figure 3, E and F are the mid-points of AD and BC respectively. It is given that $DF \parallel EB$. AC intersects DF and EB at H and G respectively.

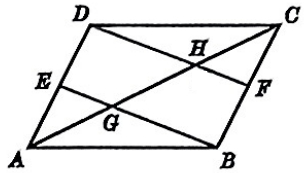


Figure 3

(a) If $CG = 12$, find AG.

(b) If $BG = 2EG$, prove that $BG = DH$.

(6 marks)

Section B (22 marks)

14. (a) Solve the inequality $\frac{3x}{2} < 14 - \frac{x}{4}$ and represent the solutions graphically.

(b) Hence, solve the inequality $\frac{5-2y}{2} < \frac{14}{3} + \frac{2y-5}{12}$ and represent the solutions graphically.

(7 marks)

15 It is given that p, q and k are non-zero integers.

(a) Simplify $\left(\frac{4}{25}\right)^p \times 2.5^q \times \left(\frac{8}{125}\right)^k$

(b) Suggest two sets of values of p, q and k such that $\left(\frac{4}{25}\right)^p \times 2.5^q \times \left(\frac{8}{125}\right)^k = 1$. Explain your answer.

(7 marks)

16. In Figure 4, $\triangle ABC$ and $\triangle ADE$ are equilateral triangles. BDC is a straight line. F is a point on AC such that $\angle BAD = \angle CBF$. BF and AD intersect at G.

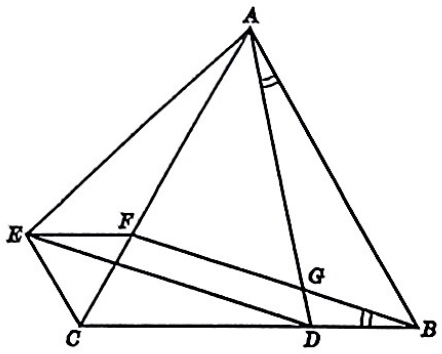


Figure 4

(a) Prove that $BF // DE$.

(b) Prove that $BDEF$ is a parallelogram.

(8 marks)

Multiple-choice Questions (20 marks)

Each question carries 2 marks. Put a ✓ for the correct answers in the boxes.

	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.
A										
B										
C										
D										

17. If $x < y$, which of the following inequalities are true?

I. $-2x - 5 > -2y - 5$

II. $\frac{x}{2} + 5 > \frac{y}{2} + 5$

III. $-2(5 + x) > -2(5 + y)$

A. I and II only
B. I and III only
C. II and III only
D. I, II and III

18. Which of the following inequalities has a solution $x = 3$ but does not have a solution $x = -7$?

A. $x - 7 < 0$
B. $x + 3 < 0$
C. $x - 3 > 0$
D. $x + 7 > 0$

19. Which of the following is/are in scientific notation?

I. -4×10^{-3}

II. 0.03×10^{-2}

III. 25×10^{8}

A. I only
B. III only
C. I and II only
D. II and III only

20. $27^{111} \cdot 4^{333} =$

A. 12^{333}.
B. 12^{444}.
C. 108^{333}.
D. 108^{444}.

21. $1110000011011_2 =$

A. $2^{12}+2^{11}+2^{10}+27$.

B. $2^{13}+2^{12}+2^{11}+27$.

C. $2^{12}+2^{11}+2^{10}+54$.

D. $2^{13}+2^{12}+2^{11}+54$.

22. In Figure 5, *PQRS* is a parallelogram. Which of the following is true?

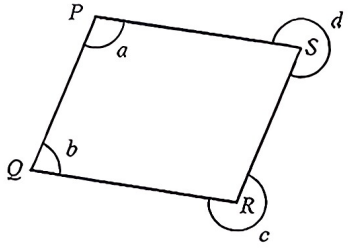


Figure 5

A. $a+b=c$

B. $a+c=360°$

C. $c+d=2(a+b)$

D. $a+b+c+d=540°$

23. In Figure 6, *BNC* is a straight line.

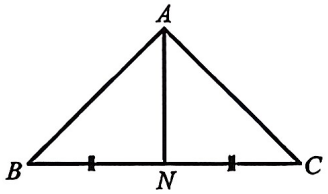


Figure 6

AN must be

A. a median of $\triangle ABC$.

B. an altitude of $\triangle ABC$.

C. an angle bisector of $\triangle ABC$.

D. a perpendicular bisector of $\triangle ABC$.

24. In Figure 7, $PQRS$ is a rectangle. N is a point on QR. It is given that $\triangle PQN \sim \triangle NRS$. Which of the following are true?

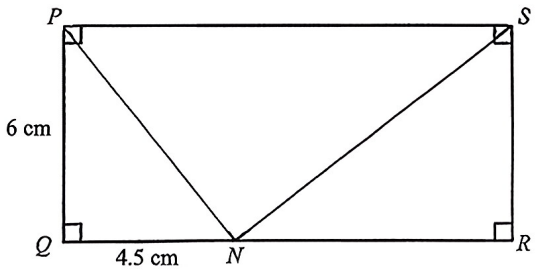


Figure 7

I. $PN \perp NS$

II. $\triangle PQN \sim \triangle SNP$

III. $PS = 14$ cm

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

25. Candice deposited \$50 000 in a bank at a simple interest rate of r% p.a. and received a simple interest of \$4500 after 18 months. Find the value of r.

A. 0.5

B. 5

C. 6

D. 8

26. In Figure 8, $ABCD$ is a parallelogram. DC is produced to a point G such that $CD = CG$. E is a point on AD such that $AD : AE = 5 : 1$. EG intersects BC at F. Find $BF : AE$.

A. 3 : 1

B. 3 : 2

C. 4 : 1

D. 5 : 2

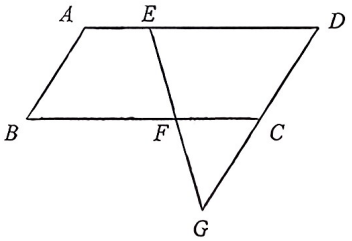


Figure 8

Bonus Question (3 marks)

27. How many zeros are there in the value of $25^{2020} \times 8^{2024} \times 5^{2022}$?

End of Paper