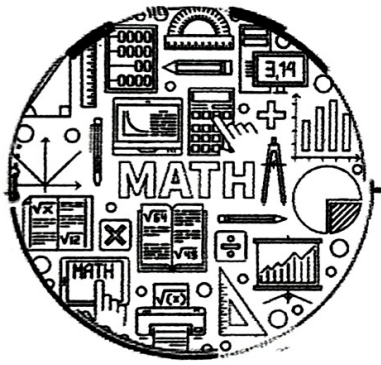
# S3 Mathematics Notes




## Areas and Volumes 3A

Content:

- Pyramids
- Circular Cones

# A Pyramids

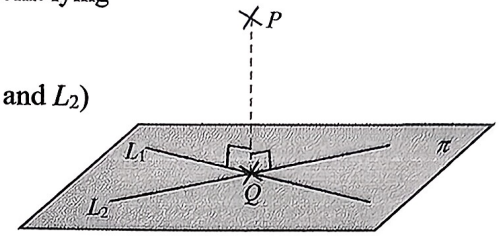
## Key Points: Projection of a Point on a Plane

In the figure, $P$ is a point not lying on the plane $\pi$ and $Q$ is a point lying on the plane $\pi$.

If $PQ$ is perpendicular to two straight lines or more (e.g. $L_1$ and $L_2$) on $\pi$ passing through $Q$, then

**(a)** $Q$ is called the *projection* of $P$ on the plane $\pi$,

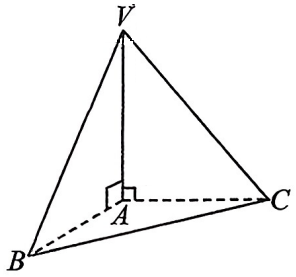
**(b)** $PQ$ is the distance between $P$ and the plane $\pi$.




## Let's Try

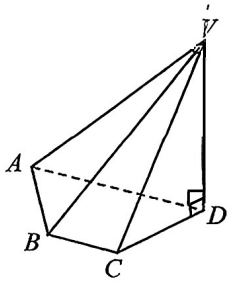
A solid is shown in each of the following. Fill in the blanks. **(1 – 4)**

**1.**



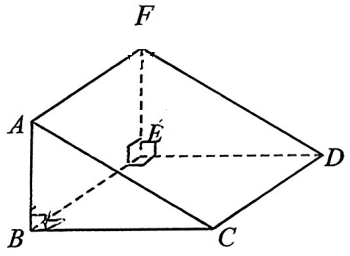

The projection of $V$ on the plane $ABC$ is _____.

**2.**




The projection of $V$ on the plane $ABCD$ is _____.

**3.**

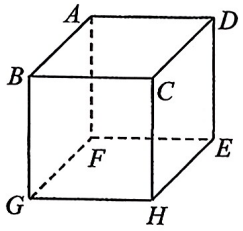



**(a)** The projection of $F$ on the plane $BCDE$ is _____.

**(b)** The projection of $C$ on the plane $ABEF$ is _____.

**(c)** The distance between $F$ and the plane $BCDE$ is _____.

**4.**




$ABCDEFGH$ is a cube.

**(a)** The projection of $D$ on the plane $FGHE$ is _____.

**(b)** The projection of $G$ on the plane $AFED$ is _____.

**(c)** The distance between $G$ and the plane $AFED$ is _____.

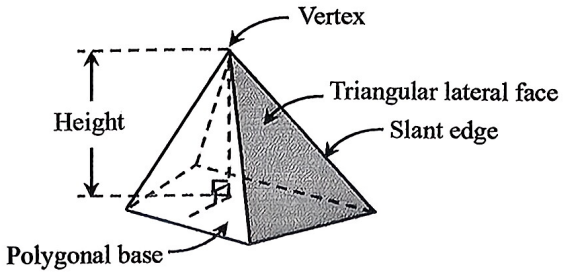
## Key Points: Pyramids

*Pyramids*

A **pyramid** is a polyhedron with

**(i)** one polygonal base and

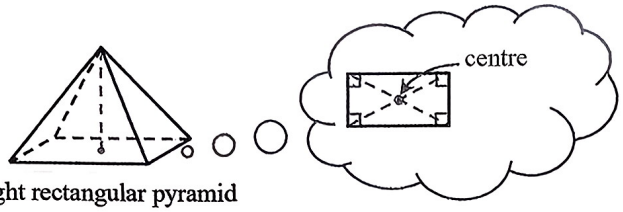
**(ii)** all the other triangular lateral faces meeting at the vertex.




*Right Pyramids*

A **right pyramid** is a pyramid whose vertex is vertically above the centre of its base.

e.g.

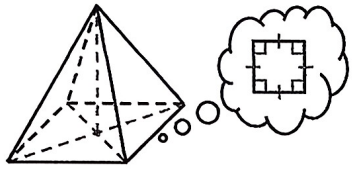



Right rectangular pyramid

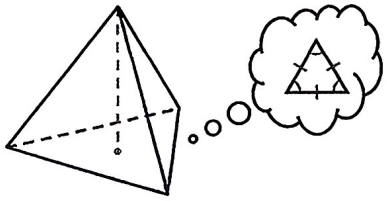
*Regular Pyramids*

A **regular pyramid** is a right pyramid whose base is a regular polygon.

e.g.



Right square pyramid

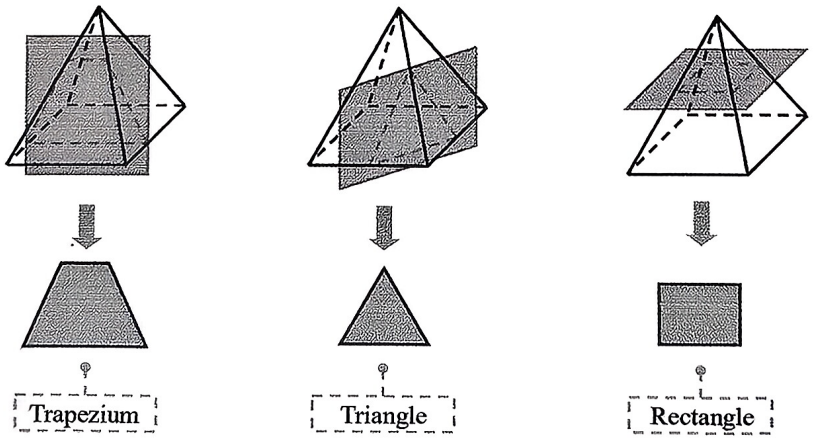


Regular tetrahedron
(Regular triangular pyramid)

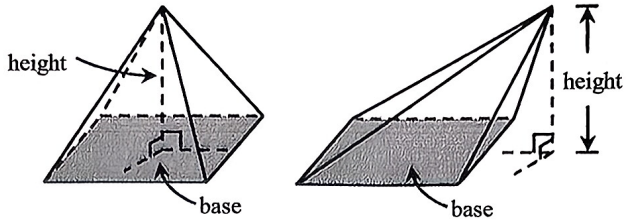
## Key Points: Cross Sections of Pyramids

When we cut a pyramid in different ways, the **cross sections** obtained may have different sizes and shapes.

The following shows some cross sections of a pyramid.

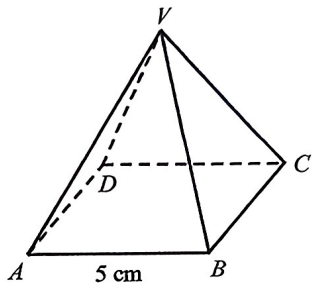


Volume of a pyramid $= \frac{1}{3} \times$ base area $\times$ height




## Example 1 (TB P.7.10)

The figure shows a right pyramid *VABCD*. Its base is a square of side 5 cm. If the volume of the pyramid is 50 $cm^3$, find the height of the pyramid.

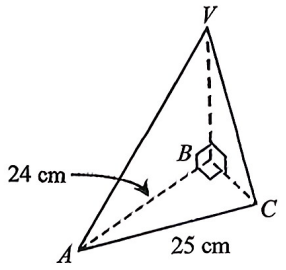



## Practice Exercise

1. The base of a pyramid is a rectangle of length 15 cm and width 8 cm. The volume of the pyramid is 280 $cm^3$. Find the height of the pyramid.

2. The volume and the height of a pyramid are 351 $cm^3$ and 13 cm respectively. The base of the pyramid is a square. Find the length of a side of the base of the pyramid.

**3.** The figure shows a pyramid $VABC$. Its base is a right-angled triangle, where $\angle ABC = 90°$, $AB = 24$ cm and $AC = 25$ cm. The volume of the pyramid is 280 cm$^3$. It is given that $\angle VBA = \angle VBC = 90°$. Find $VB$.



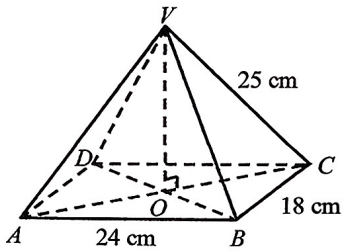

**Example 2 (TB P.7.11)**

The figure shows a right pyramid $VABCD$ of slant edge 25 cm. Its base is a rectangle of length 24 cm and width 18 cm. The diagonals of the base intersect at $O$.

**(a)** Find the length of $AC$.

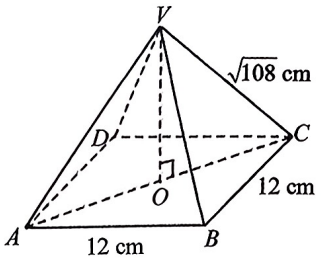
**(b)** Find the height $VO$ of the pyramid.

**(c)** Find the volume of the pyramid.



## Practice Exercise

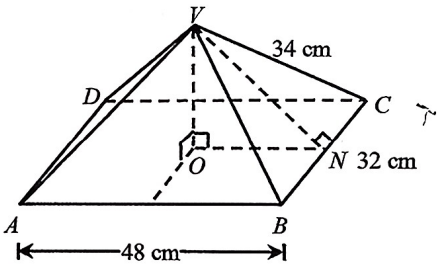
**4.** The figure shows a right pyramid $VABCD$ with a square base of side 12 cm. The length of slant edge $VC$ of the pyramid is $\sqrt{108}$ cm. $O$ is the projection of $V$ on the base. Find the volume of the pyramid.




**5.** The figure shows a right pyramid $VABCD$ of slant edge 34 cm. Its base is a rectangle of length 48 cm and width 32 cm.

**(a)** Find the lengths of $NC$ and $ON$.

**(b)** Hence, find the volume of the pyramid.

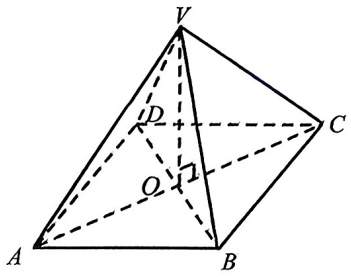


**6.** The figure shows a right pyramid *VABCD* with a square base. The slant edge is 19 cm and the diagonal of the base is $\sqrt{338}$ cm.

**(a)** Find the lengths of *VO* and *BC*.

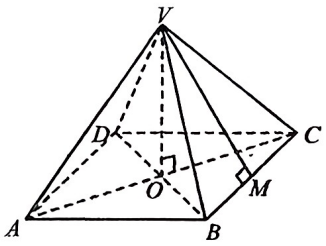
**(b)** Find the volume of the pyramid.

(*Give the answers correct to 3 significant figures if necessary.*)



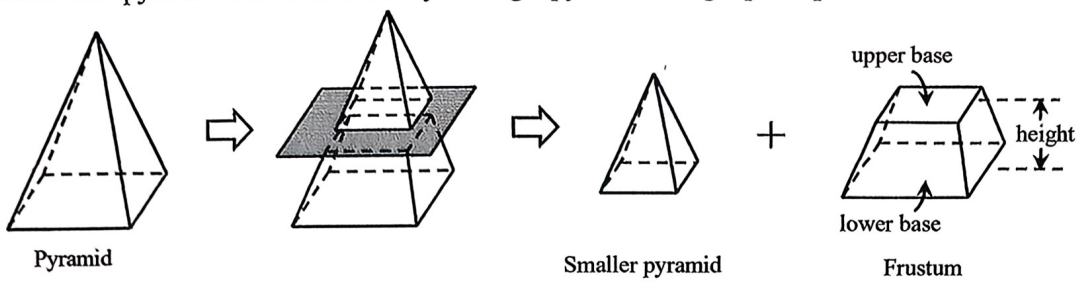

**7.** The height of a right pyramid with a square base is 8 cm. The diagonal of the base is $16\sqrt{2}$ cm. Find the volume of the pyramid.

**8.** The figure shows a right pyramid $VABCD$ with a square base. $M$ is a point lying on $BC$ such that $VM \perp BC$. $O$ is the projection of $V$ on the base. If $VC = 17$ cm and $VM = 15$ cm, find the volume of the pyramid correct to 3 significant figures.



## Key Points: Volumes of Frustums of Pyramids

A frustum of a pyramid can be obtained by cutting a pyramid along a plane parallel to its base.

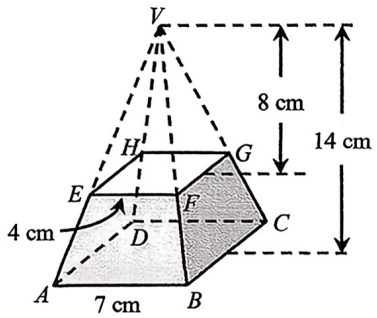



Volume of a frustum of a pyramid = volume of the original pyramid – volume of the smaller pyramid

## Let's Try

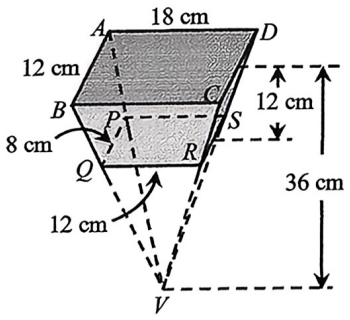
Find the volume of each of the following right frustums. **(1 – 2)**

**1.**




*ABCD* and *EFGH* are squares.

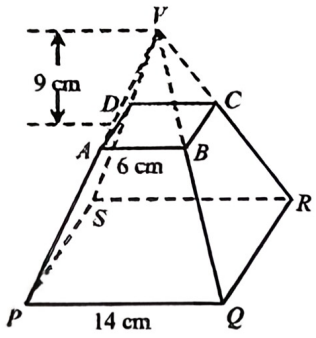
**2.**




*ABCD* and *PQRS* are rectangles.

**Example 3 (TB P.7.13)**

The figure shows a right frustum *ABCDSPQR*. The upper base and the lower base of the frustum are squares of sides 6 cm and 14 cm respectively. The height of pyramid *VABCD* is 9 cm.




**(a)** Find the height of the frustum.

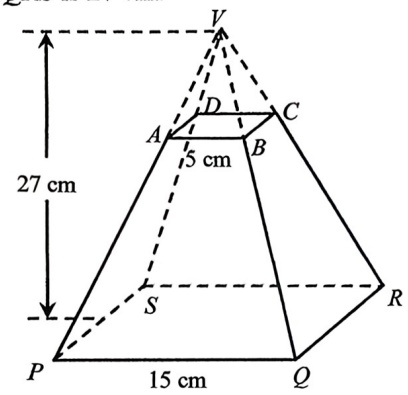
**(b)** Find the volume of the frustum.

## Practice Exercise

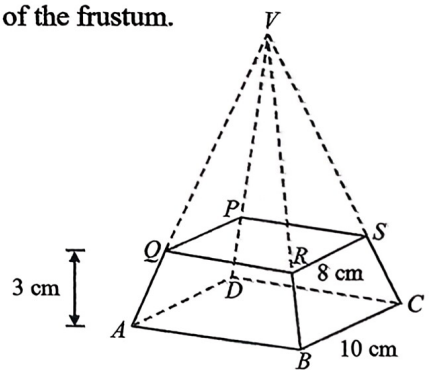
**10.** The figure shows a right frustum *ABCDSPQR*. The upper base and the lower base of the frustum are squares of sides 5 cm and 15 cm respectively. The height of pyramid *VPQRS* is 27 cm.

**(a)** Find the height of pyramid *VABCD*.

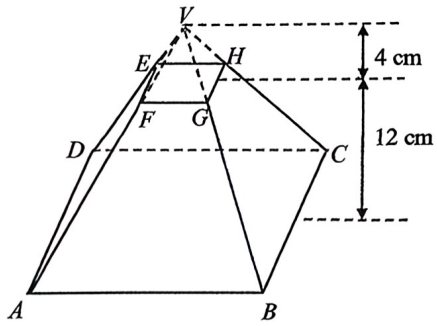
**(b)** Find the volume of the frustum.



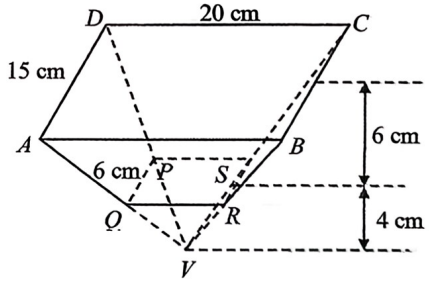

**11.** In the figure, the upper base and the lower base of a right frustum *ABCDPQRS* are squares of sides 8 cm and 10 cm respectively. The height of the frustum is 3 cm. Find the volume of the frustum.



**12.** The figure shows a right frustum *ABCDEFGH*. The upper base and the lower base of the frustum are squares. The height of the right pyramid *VEFGH* is 4 cm and the height of the right frustum *ABCDEFGH* is 12 cm. If the area of the lower base of the frustum is 144 $cm^2$, find the volume of the frustum.



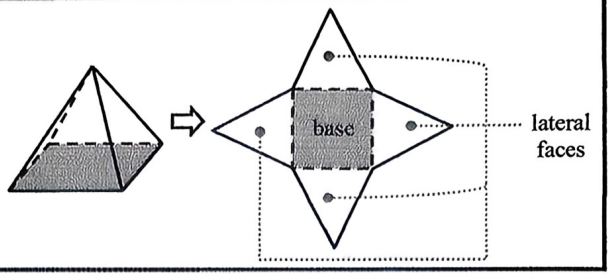

**13.** In the figure, *ABCDPQRS* is an inverted right frustum. Its upper base is a rectangle of length 20 cm and width 15 cm. Its lower base is a rectangle of width 6 cm. The height of pyramid *VPQRS* is 4 cm. Find the volume of the frustum.



## Key Points: Total Surface Areas of ...

Total surface area of a pyramid

= base area + total area of all the lateral faces

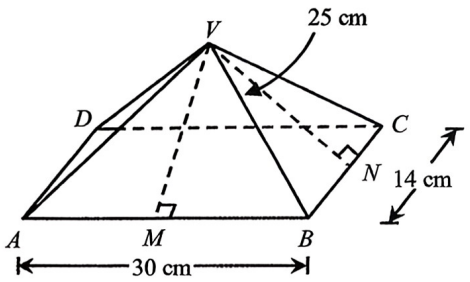



### Example 4 (TB P.7.16)

The figure shows a right pyramid *VABCD* of slant edge 25 cm. Its base is a rectangle of length 30 cm and width 14 cm.

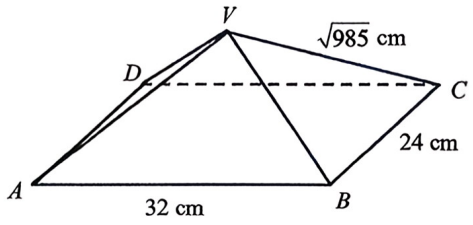
**(a)** Find the lengths of *VM* and *VN*.

**(b)** Find the total surface area of the pyramid.



## Practice Exercise

**14.** The figure shows a right pyramid $VABCD$ of slant edge $\sqrt{985}$ cm. Its base is a rectangle of length 32 cm and width 24 cm.

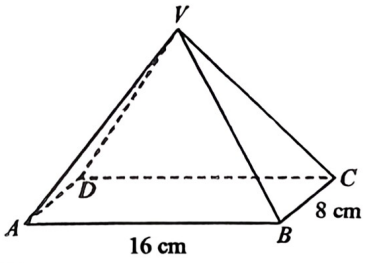



**(a)** Find the height of $\triangle VAB$ with $AB$ as the base.

**(b)** Find the height of $\triangle VBC$ with $BC$ as the base.

**(c)** Find the total surface area of the pyramid.

**15.** In the figure, the height of the right pyramid *VABCD* is 10 cm. Its base is a rectangle of length 16 cm and width 8 cm.




**(a)** Find the area of $\triangle VBC$.

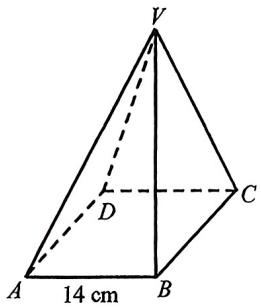
(*Leave the answer in surd form.*)

**(b)** Find the area of $\triangle VAB$.

(*Leave the answer in surd form.*)

**(c)** Find the total surface area of the pyramid correct to 3 significant figures.

**16.** The figure shows a right pyramid $VABCD$ with a rectangular base, where $AB = 14$ cm. The area of $\triangle VAB$ is 133 cm$^2$ and the area of $\triangle VBC$ is 187 cm$^2$. If the total surface area of the pyramid is 948 cm$^2$, find the length of $BC$.

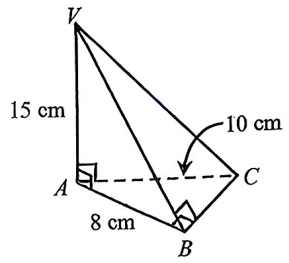



**17.** The total surface area of a right pyramid is 532 cm$^2$. Its base is a square of side 14 cm. Find the height of the pyramid correct to 3 significant figures.

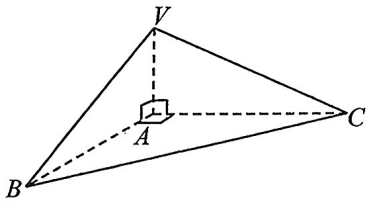
**18.** The figure shows a pyramid $VABC$ with a triangular base, where $AB = 8$ cm, $AC = 10$ cm, $VA = 15$ cm and $\angle ABC = \angle VAB = \angle VAC = \angle VBC = 90°$.

**(a)** Find the lengths of $VB$ and $BC$.

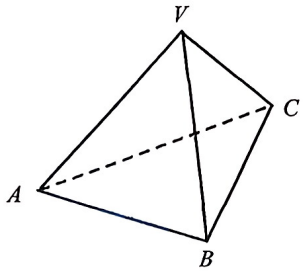
**(b)** Find the total surface area of the pyramid.



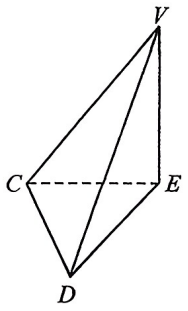

**19.** In the figure, the base $ABC$ of the pyramid $VABC$ is an isosceles right-angled triangle with $\angle BAC = 90°$. $VA$ is the height of the pyramid. If $AB = 12$ cm and $VA = 5$ cm, find the total surface area of the pyramid correct to 3 significant figures.



**20.** The figure shows a regular tetrahedron $VABC$ of side 16 cm. Find the total surface area of the regular tetrahedron. Leave the answer in surd form.




**21.** The figure shows a pyramid $VCDE$ whose base $CDE$ is an isosceles right-angled triangle with $\angle CDE = 90°$. $VE$ is the height of the pyramid. It is given that $\angle VDC = 90°$, the length of the longest side of the base is $20\sqrt{2}$ cm and the volume of the pyramid is 600 $cm^3$. Find the total surface area of the pyramid correct to 3 significant figures.



# B Circular Cones

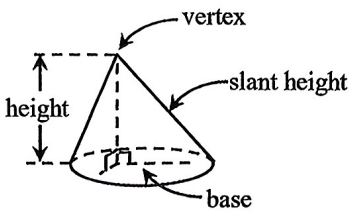
## Key Points: Circular Cones

*Circular Cones*

A **circular cone** is a non-polyhedron with

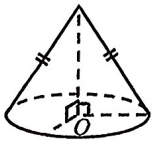
**(i)** a curved surface and

**(ii)** a circular base.




*Right Circular Cones*

A **right circular cone** is a circular cone whose vertex is vertically above the centre $O$ of its base.

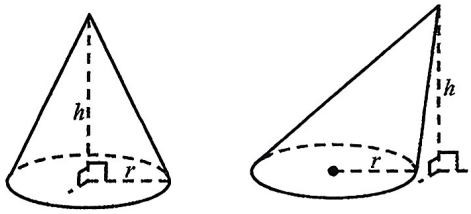



Right circular cone

## Key Points: Volume of Circular Cones

For a circular cone of base radius $r$ and height $h$,

Volume of a circular cone $= \frac{1}{3}\pi r^2 h$

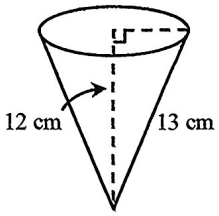



### Example 5 (TB P.7.29)

The figure shows an inverted right circular cone of height 12 cm and slant height 13 cm.

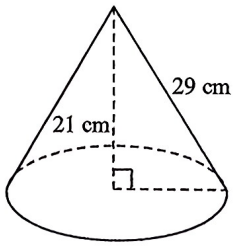
**(a)** Find the base radius of the circular cone.

**(b)** Find the volume of the circular cone in terms of $\pi$.



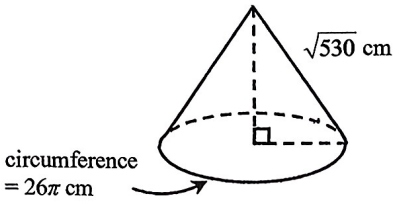
## Practice Exercise

**22.** The figure shows a right circular cone of height 21 cm and slant height 29 cm. Find the volume of the circular cone correct to 3 significant figures.




**23.** The base diameter and the slant height of a right circular cone are 16 cm and 10 cm respectively. Find the volume of the circular cone in terms of $\pi$.

**24.** The figure shows a right circular cone of slant height $\sqrt{530}$ cm. The circumference of its base is $26\pi$ cm. Find the volume of the circular cone correct to 3 significant figures.



**25.** Five identical cubes each of side 8 cm are melted and recast into some identical right circular cones. It is given that the slant height of each right circular cone is 5 cm and the base radius of each circular cone is 3 cm. Find the maximum number of right circular cones that can be recast.

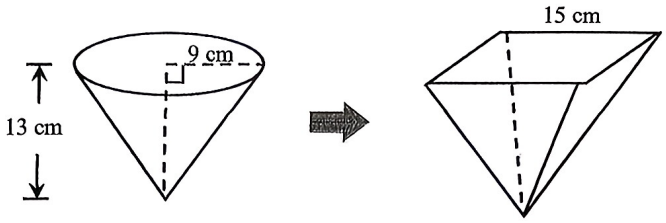
**Example 6 (TB P.7.30)**

The height and the base radius of a metal right circular cylinder are 10 cm and 15 cm respectively. The circular cylinder is melted and recast into a right circular cone of base radius 30 cm. Find the height of the circular cone.

**Practice Exercise**

**26.** The height and the base radius of a metal circular cone are 12 cm and 8 cm respectively. The circular cone is melted and recast into a circular cylinder of base radius 6 cm. Find the height of the circular cylinder correct to 3 significant figures.

**27.** A solid metal right circular cone of base radius 9 cm and height 13 cm is melted and recast into a solid right pyramid with a square base of side 15 cm. Find the height of the pyramid correct to 3 significant figures.

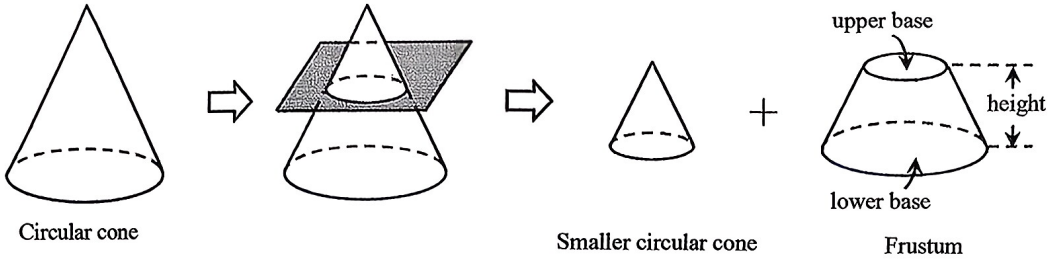



**28.** The height of a metal circular cone is 10 cm. The circular cone is melted and recast into a circular cylinder of base radius 12 cm and height 30 cm. Find the base radius of the circular cone.

**29.** The height and the base radius of a metal circular cone are 15 cm and 10 cm respectively. The circular cone is melted and recast into some identical circular cylinders each of base radius 4 cm and height 6 cm. At most how many circular cylinders can be recast?

## Key Points: Volumes of Frustums of Circular Cones

A frustum of a circular cone can be obtained by cutting the circular cone along a plane parallel to its base.




Volume of a frustum of a circular cone =

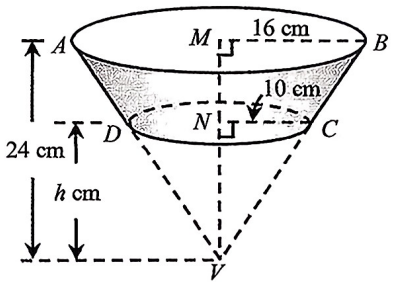
volume of the original circular cone – volume of the smaller circular cone

### Example 7 (TB P.7.32)

The figure shows an inverted right frustum $ABCD$. The radii of the upper base and the lower base are 16 cm and 10 cm respectively. The height of circular cone $VAB$ is 24 cm.

**(a)** Find the value of $h$.

**(b)** Find the volume of the frustum.



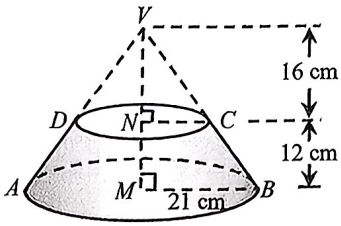
## Practice Exercise

**30.** The figure shows a right frustum $ABCD$ of height 12 cm. The radius of the lower base is 21 cm and the height of circular cone $VDC$ is 16 cm.

**(a)** Find the base radius of circular cone $VDC$.

**(b)** Find the volume of the frustum.

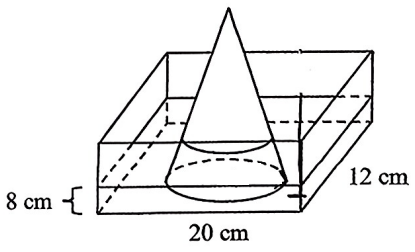
(*Give the answer correct to 3 significant figures.*)



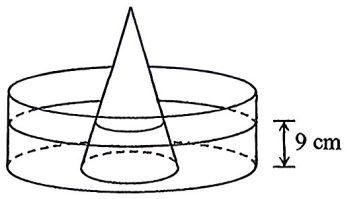

**31.** In the figure, a solid metal right circular cone is put into a rectangular tank and the depth of water in the tank is 8 cm. The height and the base radius of the circular cone are 20 cm and 5 cm respectively.

**(a)** Find the volume of the frustum immersed in water in terms of $\pi$.

**(b)** If the circular cone is taken away, find the depth of water in the tank correct to 2 decimal places.

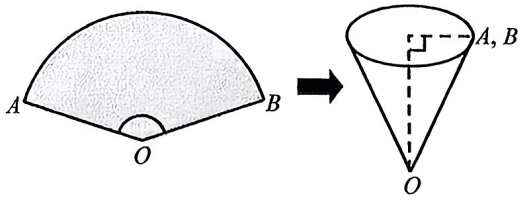


**32.** In the figure, a solid metal right circular cone of height 24 cm and base radius 8 cm is put into a cylindrical tank of base radius 15 cm. The depth of water in the tank is 9 cm. If the circular cone is taken away, find the depth of water in the tank.




**33.** A cylindrical tank of base radius 16 cm is placed on the horizontal ground and it contains some water. Now, a solid metal right circular cone of height 20 cm and base radius 10 cm is placed vertically inside the tank without water overflow. If the radius of the upper base of the frustum immersed in water is 6 cm, find the height of water risen in the cylindrical tank. (*Give the answer correct to 3 significant figures.*)

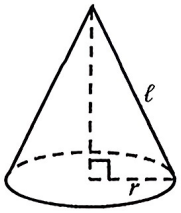
## Key Points: Nets of Right Circular Cones




## Key Points: Total Surface Areas of Right Circular Cones

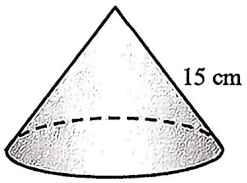
For a right circular cone of base radius $r$ and slant height $\ell$,

1. Curved surface area $= \pi r \ell$
2. Total surface area $= \pi r^2 + \pi r \ell$



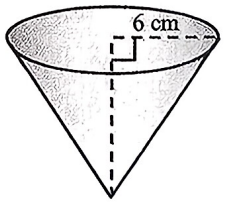

**Example 8**

The slant height of a right circular cone is 15 cm. If the curved surface area of the circular cone is $135\pi$ cm$^2$, find the base radius of the circular cone.

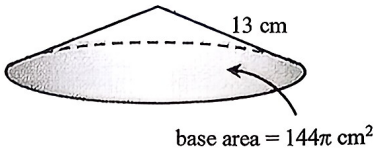



**Practice Exercise**

**34.** The base radius of an inverted right circular cone is 6 cm. If the total surface area of the circular cone is $120\pi$ cm$^2$, find the slant height of the circular cone.



**35.** The figure shows a right circular cone of slant height 13 cm. The area of the base is $144\pi$ cm$^2$. Find the total surface area of the circular cone.




## Example 9 (TB P.7.35)

Figure (1) shows a sector $OAB$ with radius 15 cm and $\angle AOB = 144°$. $O$ is the centre of sector $OAB$. By joining $OA$ and $OB$ together, the sector is folded to form a right circular cone as shown in Figure (2).

**(a)** Find the base radius of the circular cone.

**(b)** Find the volume of the circular cone correct to 3 significant figures.

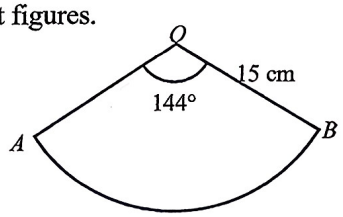



Figure (1)

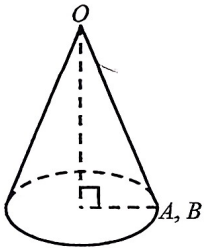


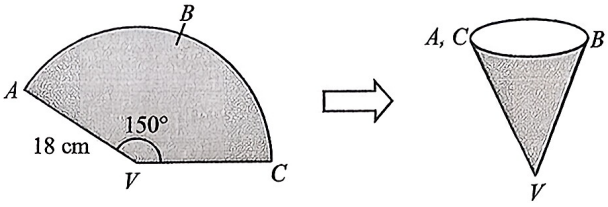

Figure (2)

## Practice Exercise

**36.** In the figure, a piece of paper in the shape of sector $VABC$ is folded to form an inverted right circular cone by joining $VA$ and $VC$.

**(a)** Find the base radius of the circular cone.

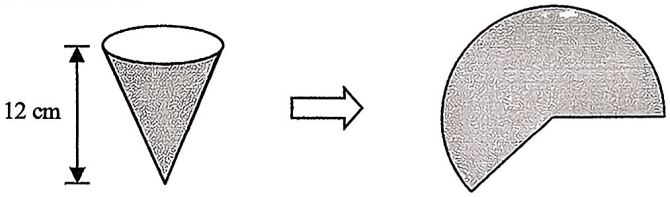
**(b)** Find the capacity of the circular cone correct to 2 decimal places.




**37.** In the figure, the capacity of an inverted right circular cone is $324\pi$ cm$^3$. The circular cone is cut along its slant height and unfolded to form a sector.

**(a)** Find the curved surface area of the circular cone in terms of $\pi$.

**(b)** Find the angle of the sector.



**38.** In the figure, a piece of paper in the shape of sector $VABC$ is folded to form an inverted right circular cone by joining $VA$ and $VC$.

**(a)** Express the height of the circular cone in surd form.

**(b)** Are 8000 circular cones formed in **(a)** enough to hold 0.8 $m^3$ of juice? Explain your answer.