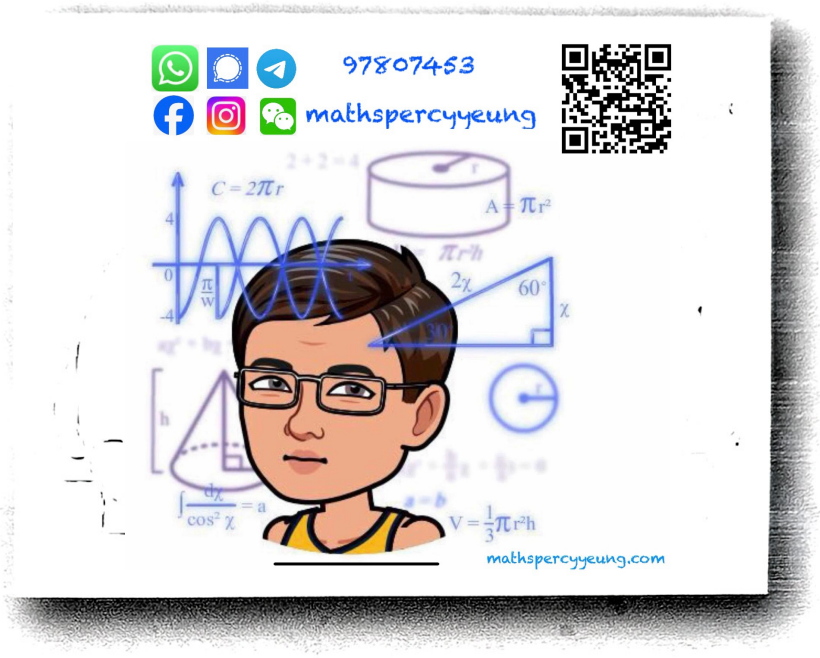
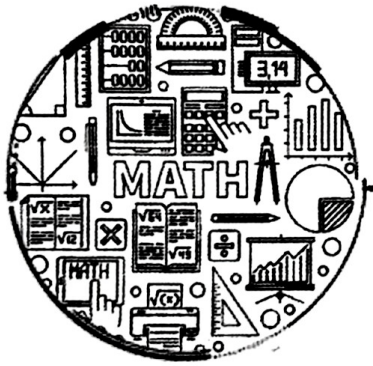
S3 Mathematics Notes





More about Geometric Proofs

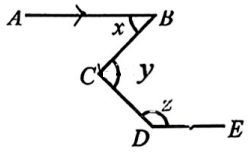
Content:

- *Proofs Relating to Straight Lines and Triangles*
- *Proofs Relating to Quadrilaterals*

A Proofs Relating to Straight Lines and Triangles

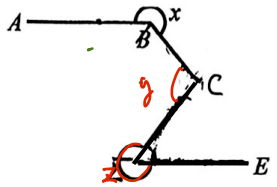
Example 1

In the figure, $AB \parallel DE$. Prove that $y = 180° + x - z$.

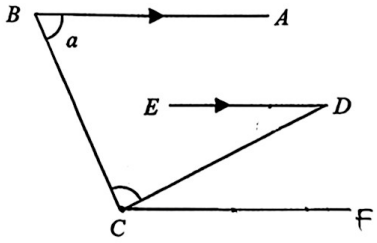


Practice Exercise

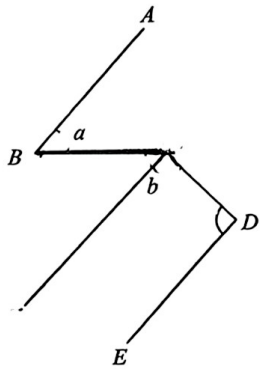
1. In the figure $AB \parallel DE$. Prove that $z = 180° - y + x$.



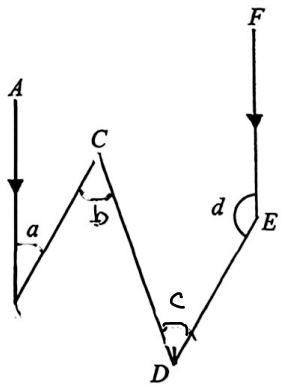
2. In the figure, $BA \parallel ED$. Prove that $c = 180° - a - b$.



3. In the figure, $BA \parallel ED$. Use two methods to prove that $a = b + c - 180°$.

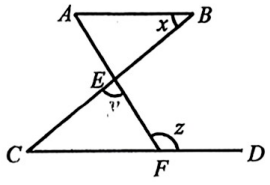


4. In the figure, $AB \parallel FE$. Prove that $d = 180° - a + b - c$.



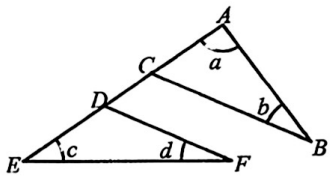
Example 2

In the figure, *AEF*, *BEC* and *CFD* are straight lines. If *AB* // *CD*, Brian claims that $z = x + y$. Do you agree? Explain your answer.

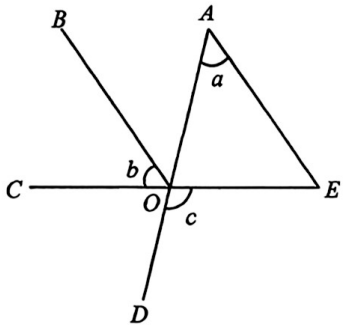


Practice Exercise

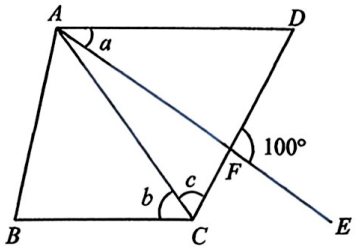
5. In the figure, *ACDE* is a straight line. If *CB* // *DF*, Jason claims that $a + b + c + d = 180°$. Do you agree? Explain your answer.



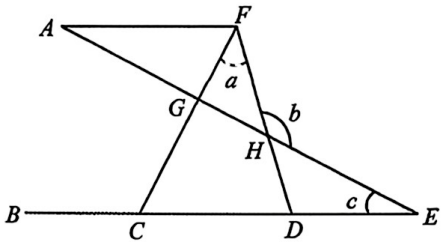
6. In the figure, *AD* and *CE* intersect at point *O*. It is given that *AE* // *BO*. Sharon claims that $a + b = c$. Do you agree? Explain your answer.



7. In the figure, AE and CD intersect at point F. It is given that $AD \,//\, BC$. Daniel claims that $b - a + c = 100°$. Do you agree? Explain your answer.

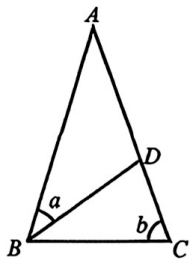


8. In the figure, C and D are points lying on BE. AE cuts CF and DF at points G and H respectively. If $AF \,//\, BE$, express $\angle BCF$ in terms of a, b and c.



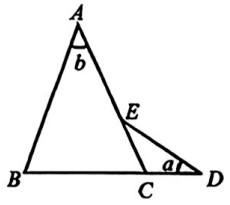
Example 3

In the figure, $\triangle ABC$ and $\triangle BCD$ are isosceles triangles with $AB = AC$ and $BC = BD$. Prove that $3b - a = 180°$.

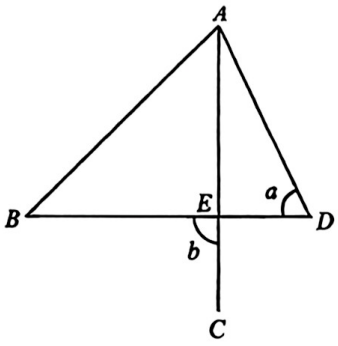


Practice Exercise

9. In the figure, $\triangle ABC$ and $\triangle CDE$ are isosceles triangles with $AB = AC$ and $CD = CE$. Prove that $4a + b = 180°$.

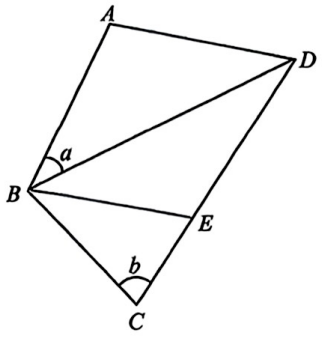


10. In the figure, AC and BD intersect at point E. It is given that $AB = BD$ and $AE = BE$. Prove that $4a + b = 360°$.



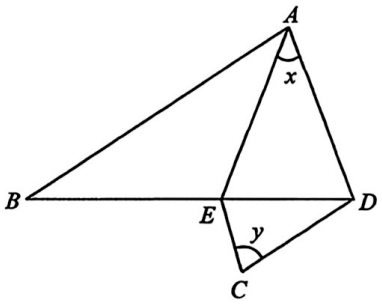
11. In the figure, E is a point lying on CD such that $AD \parallel BE$. It is given that $AB = AD$ and $BD = CD$.

Express $\angle BEC$ in terms of a and b.



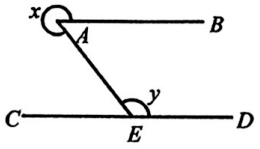
12. In the figure, E is a point lying on BD such that $AE = AD = BE$. It is given that $CD = ED$ and $AB \parallel DC$.

What is the relation between x and y? Explain your answer.



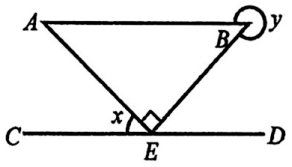
Example 4

In the figure, $x - y = 180°$. Determine whether AB is parallel to CD.

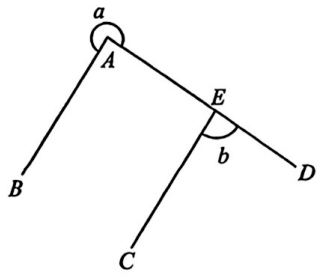


Practice Exercise

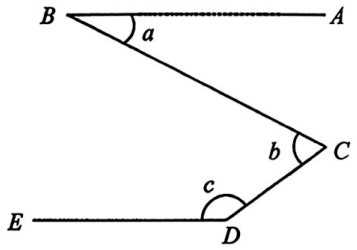
13. In the figure, $y - x = 270°$. Determine whether AB is parallel to CD.



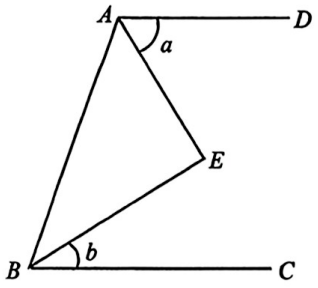
14. In the figure, E is a point lying on AD such that $a + b = 360°$. Determine whether AB is parallel to EC.



15. In the figure, $b - a + c = 180°$. Prove that $BA \parallel ED$.

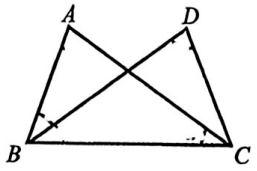


16. In the figure, AE and BE bisect $\angle BAD$ and $\angle ABC$ respectively. If $AE \perp BE$, prove that $AD \parallel BC$.



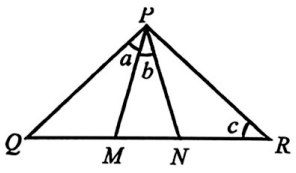
Example 5

In the figure, $\triangle ABC \cong \triangle DCB$. Prove that $2a + b + c = 180°$.

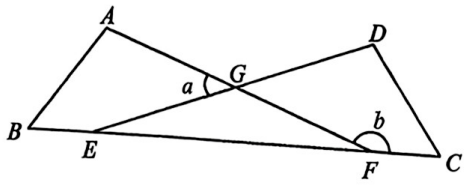


Practice Exercise

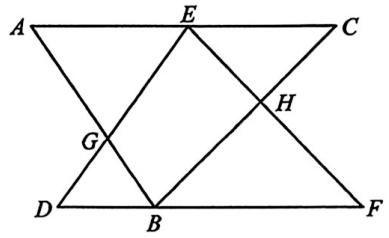
17. In the figure, $QMNR$ is a straight line. $\triangle PQN \cong \triangle PRM$. Prove that $2a + b + 2c = 180°$.



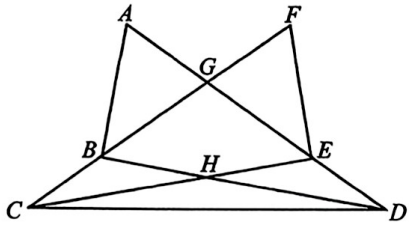
18. In the figure, E and F are points lying on BC such that $\triangle ABF \cong \triangle DCE$. AF and DE intersect at point G. Prove that $a + 2b = 360°$.



19. In the figure, B and E are points lying on DF and AC respectively such that $\triangle ABC \cong \triangle DEF$. AB and DE intersect at point G while BC and EF intersect at point H. It is given that $AC \parallel DF$. Prove that $\triangle AGE$ is an isosceles triangle.



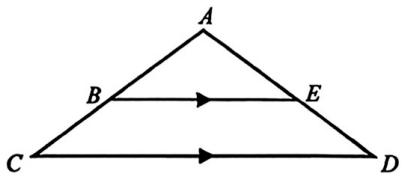
20. In the figure, $AGED$, $FGBC$, BHD and EHC are straight lines. It is given that $\triangle ABD \cong \triangle FEC$ and $BC = ED$. Prove that $\triangle HCD$ is an isosceles triangle.



Mathematical Skills: Three pairs of Common Similar Triangles

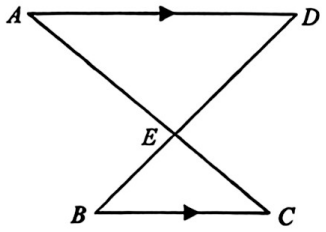
Type 1

In the figure, $BE \parallel CD$. ABC and AED are straight lines.



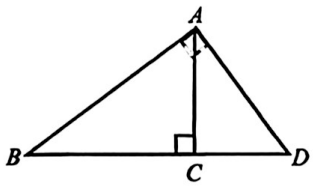
Type 2

In the figure, AC and BD intersect at point E. It is given that $AD \parallel BC$.



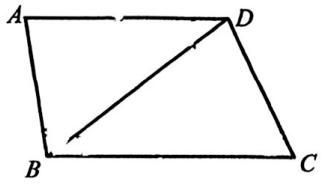
Type 3

In the figure, BCD is a straight line.



Example 6

In the figure, $AD \parallel BC$ and $\angle ABD = \angle BCD$.

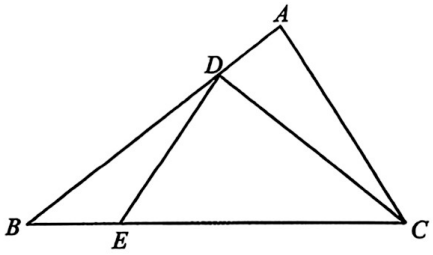


(a) Prove that $\triangle ABD \sim \triangle DCB$.

(b) Prove that $AB \times BD = AD \times DC$.

Practice Exercise

21. In the figure, D and E are points lying on AB and BC respectively such that CD bisects $\angle ADE$ and $AC = CD = BD$. It is given that $\angle ACD = \angle BDE$.



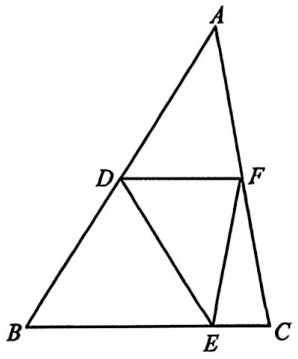
(a) Prove that $\triangle ABC \sim \triangle DCE$.

(b) Prove that $AB \times CE = BC \times DC$.

22. In the figure, D, E and F are points lying on AB, BC and AC respectively. It is given that $DF \parallel BC$, DF bisects $\angle ADE$ and $\angle AFE$, and $CE : DF : BE = 1 : 2 : 3$.

(a) Prove that $\triangle ABC \sim \triangle EDF$.

(b) Hence, prove that D is the mid-point of AB.

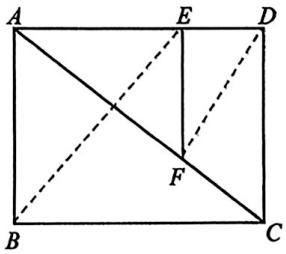


23. In the figure, $ABCD$ is a rectangle. E and F are points lying on AD and AC respectively such that $AB \parallel EF \parallel DC$.

(a) Prove that $\triangle ABC \sim \triangle FEA$.

(b) Join BE and DF. Are the areas of $\triangle ABE$ and $\triangle ADF$ the same? Explain your answer.

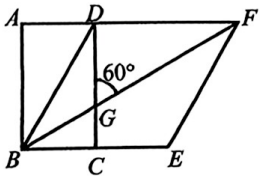
(a)



B Proofs relating to Quadrilaterals

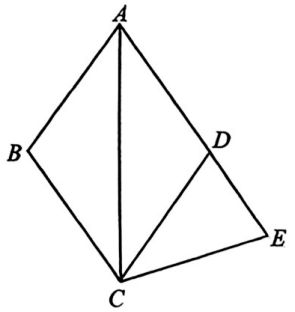
Example 7

In the figure, $ABCD$ is a rectangle. $BEFD$ is a parallelogram. ADF and BCE are straight lines. CD and BF intersect at point G. If $BG = GD$ and $\angle DGF = 60°$, prove that $\triangle BEF$ is an isosceles triangle.

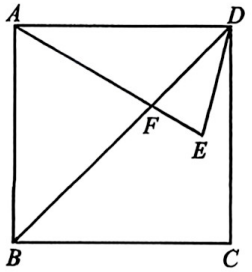


Practice Exercise

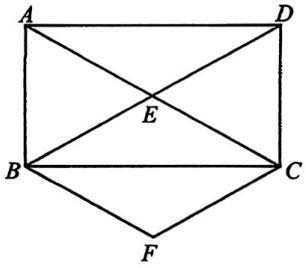
24. In the figure, $ABCD$ is a rhombus. AD is produced to point E such that $AC = AE$. It is given that $\angle ACB = \angle DCE$. Prove that $\triangle CDE$ is an isosceles triangle.



25. In the figure, $ABCD$ is a square. AE and BD intersect at point F. It is given that $\angle CDE = 15°$ and $DE = DF$. Prove that $\triangle ADE$ is an isosceles triangle.

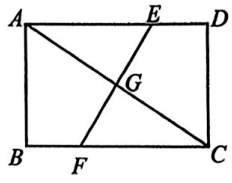


26. In the figure, $ABCD$ is a rectangle. AC and BD intersect at point E. It is given that $BFCE$ is a rhombus and $\angle BFC = 120°$. Prove that $\triangle ABE$ is an equilateral triangle.



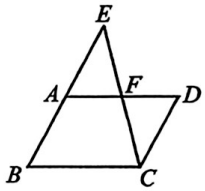
Example 8

In the figure, $ABCD$ is a rectangle. E and F are points lying on AD and BC respectively. AC and EF intersect at point G. It is given that $ED = BF$. Simon claims that G is the mid-point of EF. Do you agree? Explain your answer.

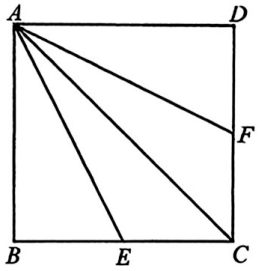


Practice Exercise

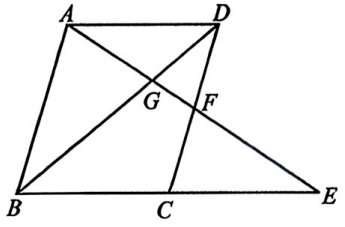
27. In the figure, $ABCD$ is a parallelogram. BA is produced to E. CE and AD intersect at point F such that $CF = EF$. Kelly claims that A is the mid-point of BE. Do you agree? Explain your answer.



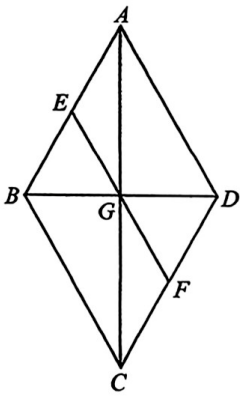
28. In the figure, $ABCD$ is a square. E is the mid-point of BC. F is a point lying on CD such that AC bisects $\angle EAF$. Barry claims that F is the mid-point of CD. Do you agree? Explain your answer.



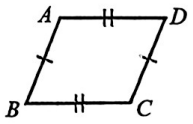
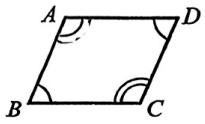
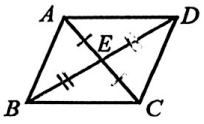
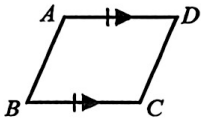
29. In the figure, $ABCD$ is a parallelogram. BC is produced to point E. AE cuts CD and BD at points F and G respectively. If F is the mid-point of CD, prove that $3AG = 2EF$.



30. In the figure, $ABCD$ is a rhombus. AC and BD intersect at point G. E and F are points lying on AB and CD respectively such that $AE = BE$. Use two different methods to prove that F is the mid-point of CD.



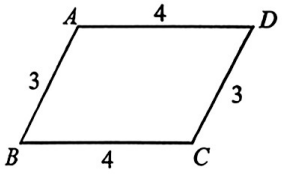
Key Points: Conditions for Parallelograms

Graphs				
If	$AD = BC$ and $AB = DC$	$\angle A = \angle C$ and $\angle B = \angle D$	$AE = EC$ and $BE = ED$	$AD \parallel BC$ and $AD = BC$
Then	$ABCD$ is a parallelogram			
Reason	(*opp. sides equal*)	(*opp. ∠s equal*)	(*diags. bisect each other*)	(*2 sides // and equal*)

Let's Try

In each of the following, determine whether $ABCD$ is necessarily a parallelogram. Put a tick '✓' in the appropriate box. If it is, give a reason. **(1 – 6)**

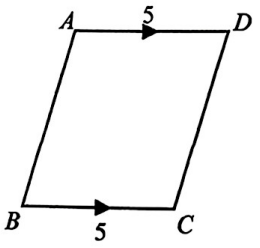
1.



☐ Yes, reason: _____

☐ No

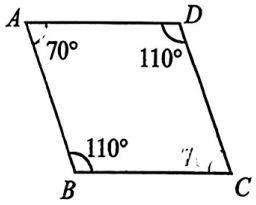
2.



☐ Yes, reason: _____

☐ No

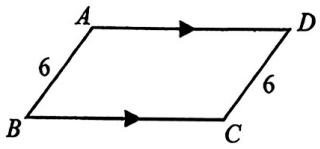
3.



☐ Yes, reason: _____

☐ No

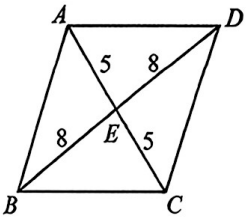
4.



☐ Yes, reason: _____

☐ No

5.

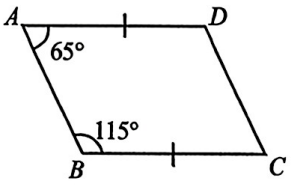


AEC and *BED* are straight lines.

☐ Yes, reason: _____

☐ No

6.



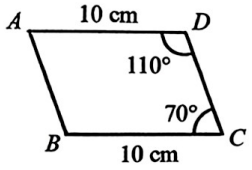
☐ Yes, reason:

☐ No

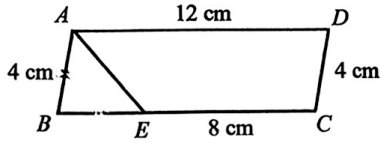
Example 9

In each of the following, prove that $ABCD$ is a parallogram.

(a)



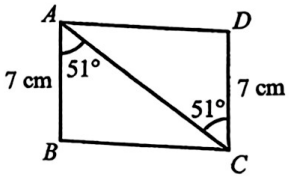
(b)



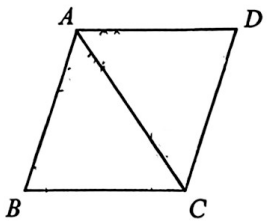
BEC is a straight line.

Practice Exercise

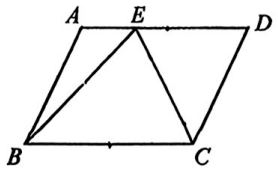
31. Prove that $ABCD$ is a parallogram.



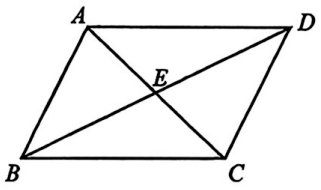
32. In the figure, $\triangle ABC \cong \triangle CDA$. Prove that $ABCD$ is a parallelogram.



33. In the figure, E is a point lying on AD such that CE bisects $\angle BED$ and $BE = BC$. If $AD = BC$, prove that $ABCD$ is a parallelogram.

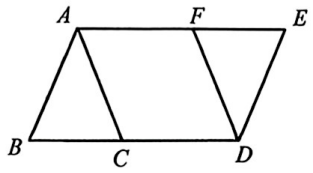


34. In the figure, AC and BD intersect at point E such that $\triangle ABE \cong \triangle CDE$ and $\triangle ADE \cong \triangle CBE$. Use two different methods to prove that $ABCD$ is a parallelogram.



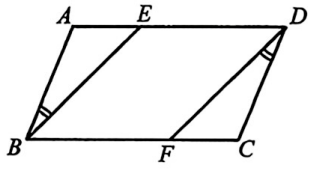
Example 10

In the figure, $ACDF$ is a parallelogram. AFE and BCD are straight lines. $\triangle ABC \cong \triangle DEF$. Prove that $ABDE$ is a parallelogram.

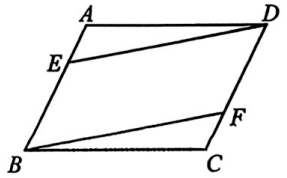


Practice Exercise

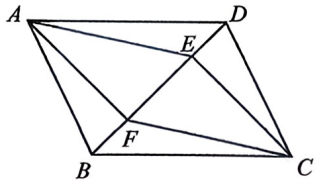
35. In the figure, $ABCD$ is a parallelogram. E and F are points lying on AD and BC respectively such that $\angle ABE = \angle CDF$. Prove that $EBFD$ is a parallelogram.



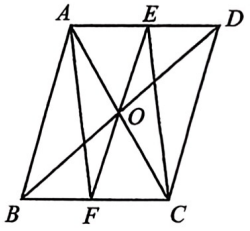
36. In the figure, $ABCD$ is a parallelogram. E and F are points lying on AB and CD respectively such that $\angle AED = \angle CFB$. Prove that $BFDE$ is a parallelogram.



37. In the figure, $ABCD$ is a parallelogram. E and F are points lying on BD such that $AF = CE$ and $BF = DE$. Prove that $AFCE$ is a parallelogram.

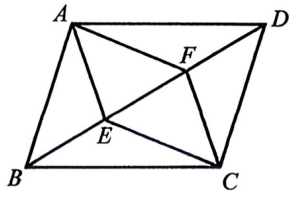


38. In the figure, E and F are points lying on AD and BC respectively such that $AFCE$ is a parallelogram. AC, BD and EF intersect at point O. Use two different methods to prove that $ABCD$ is a parallelogram.



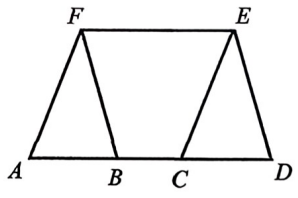
Example 11

In the figure, $ABCD$ is a parallelogram. E and F are points lying on BD such that $BE = EF = FD$. Prove that $AECF$ is a parallelogram.

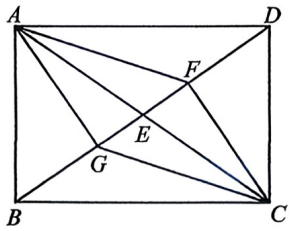


Practice Exercise

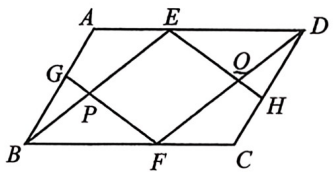
39. In the figure, $ABCD$ is a straight line. $ACEF$ is a parallelogram and $AB = CD$. Prove that $BDEF$ is a parallelogram.



40. In the figure, $ABCD$ is a rectangle. AC and BD intersect at point E. F and G are points lying on BD such that $\angle AFD = \angle CGB$. Is $AGCF$ a parallelogram? Explain your answer.

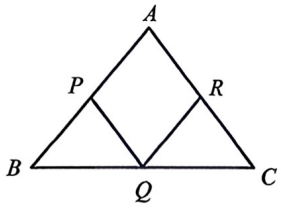


41. In the figure, $ABCD$ is a parallelogram. E, F, G and H are points lying on AD, BC, AB and CD respectively. BE and FG intersect at point P while DF and EH intersect at point Q. It is given that $BG = DH$ and $BF = DE$. Prove that $EPFQ$ is a parallelogram.



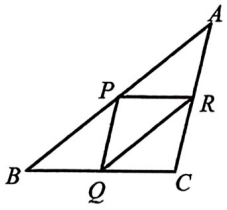
Example 12

In the figure, P, Q and R are the mid-points of AB, BC and CA respectively. Prove that $APQR$ is a paralleogram.

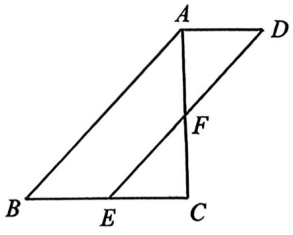


Practice Exercise

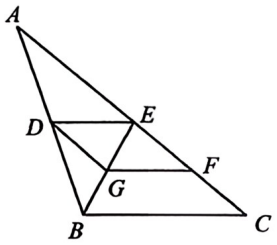
42. In the figure, P, Q and R are the mid-points of AB, BC and CA respectively. Prove that $BPRQ$ is a paralleogram.



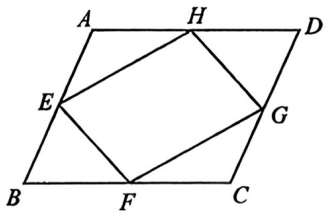
43. In the figure, E is the mid-point of BC. AC and DE intersect at point F such that F is the mid-point of AC and DE. Prove $ABED$ is a parallelogram.



44. In the figure, D and E are the mid-points of AB and AC respectively. F and G are the mid-points of CE and BE respectively. Prove $DGFE$ is a parallelogram.



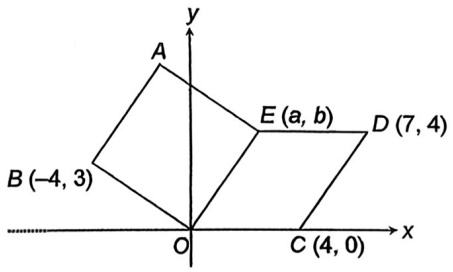
45. In the figure, $ABCD$ is a quadrilateral. E, F, G and H are the mid-points of AB, BC, CD and AD respectively. Is $EFGH$ a parallelogram? Explain your answer.



46. The figure shows a square $ABOE$ and a quadrilateral $EOCD$ on a rectangular coordinate plane.

(a) Find the values of a and b.

(b) Is $EOCD$ a parallelogram? Explain your answer.



47. In the figure, PSQ and $QUVR$ are straight lines. RS bisects $\angle PRQ$ and cuts PV at T. $PQ \perp PR$, $PV \perp QR$ and $SU \perp QR$.

(a) Prove that $\triangle PSR \cong \triangle USR$.

(b) Prove that $PS = PT$.

(c) Prove that $PSUT$ is a rhombus.