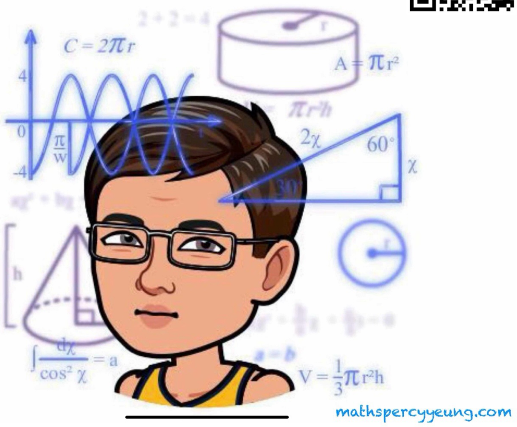


S.4 Mathematics

Ch 4 Functions and Graphs

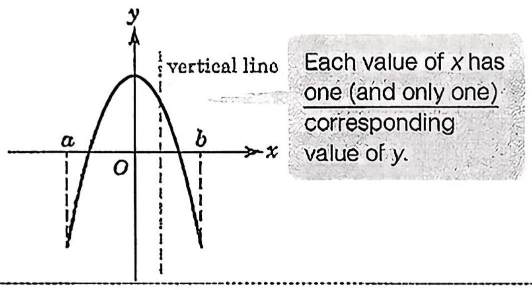
Name:________________ Class:____________

A. Section 4.1 Introduction to Functions

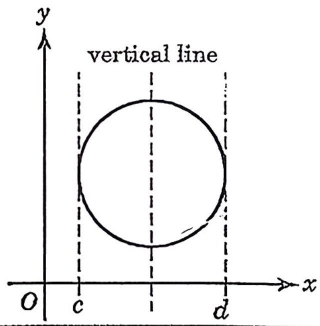
1. Please refer to the textbook P.4.2 – 4.8.
2. Video:
   https://www.youtube.com/watch?v=xmnjgQGjNwI&list=PL1Pv0-rhcQx6tW3L1NpXLKSQepnohVDVh
3. Pre-lesson

1. Definition of a function: For two variables x and y, if each value of x has one and only one corresponding value of y.

   (a) Each vertical line intersects the graph at only one point.
   $\therefore$ y is a function of x.



   (b) A vertical line intersects the graph at two points.
   $\therefore$ y is not a function of x.



This value of x has more than one corresponding value of y.

2. Domain of a function

   Domain of a function is the collection of all possible values of x.

   e.g.1 $y=\sqrt{x+1}$

   The domain is *all real numbers greater than or equal to* -1.

   e.g.2 $y=\dfrac{1}{x+1}$

   The domain is *all real numbers except* -1.

   e.g.3 $y=\dfrac{2}{\sqrt{x+1}}$

   The domain is *all real numbers greater than* -1.

Exercise:

In each of the following, y is a function of x. Find the domain of each function:

Level 1

(a) $y=\sqrt{x-9}$

(b) $y=\sqrt{10-x}$

(c) $y=\dfrac{7}{x+8}$

Level 2

In each of the following, y is a function of x. Find the domain of each function:

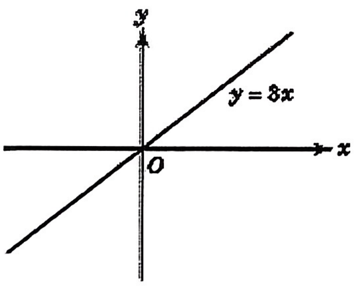
(a) $y=\dfrac{8\sqrt{5-x}}{3}$

(b) $y=x^2-8x+3$

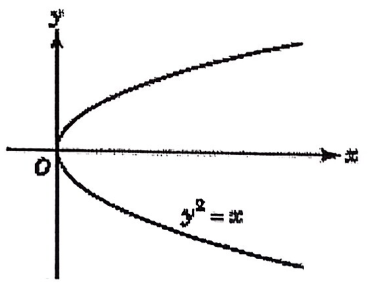
(c) $y=\dfrac{x+3}{x^2-2x-8}$

Each of the following graphs shows a relation between x and y. Determine whether y is a function of x.

(a)



(b)



B. Section 4.2 Notation of a Function

1. Please refer to the textbook P.4.15 – 4.23.
4. Video: https://www.youtube.com/watch?v=xmnjgQGjNwI&feature=emb_logo

   01:45 – Algebraic Method and Graphical Method

   07:57 – Notation of Functions

   10:26 – Value of Functions

I. Pre-lesson

1. Suppose a variable y depends on a variable x. If each value of x determines exactly one value of y, then the dependent variable y is a function of the independent variable x.

   e.g. Consider $y = 3x - 2$ where x can be any real numbers.

   When $x = -1$, $y = 3(-1) - 2 = -5$

   When $x = 0$, $y = 3(0) - 2 = -2$

   When $x = 2$, $y = 3(2) - 2 = 4$

   Each value of x gives exactly one value of y.

   $\therefore$ y is a function of x.

2. We can also use the notation '$f(x)$', '$g(x)$' and '$H(x)$', etc. to denote a function of x.

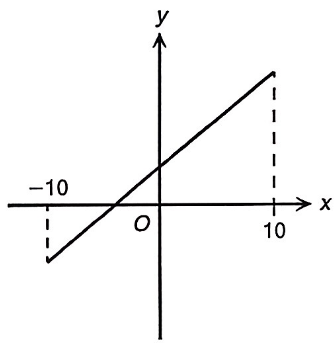
   e.g. The function $y = 4x + 3$ can be written as $f(x) = 4x + 3$.

   The value of the function at $x = \mathbf{1}$ is $f(\mathbf{1})$.

   When $x = \mathbf{1}$, $f(\mathbf{1}) = 4(1) + 3 = 7$.

   The value of the function at $x = \mathbf{-2}$ is $f(\mathbf{-2})$.

   When $x = \mathbf{-2}$, $f(\mathbf{-2}) = 4(-2) + 3 = -5$



3. Beside x, other letters such as t and s can also be used to represent independent variables.

   e.g. $G(t) = t^2 + 2t + 3$, $h(s) = 2s - 6$

$\boxed{\text{Level 1}}$

1. If $f(x) = 2x + 3$, find the values of the function when

   (a) $x = 2$, (b) $x = 0$, (c) $x = -1$.

2. If $F(x) = 2x - 4$, find

   (a) $F(r)$, (b) $F\left(\frac{r}{2}\right)$,

   (c) $F(-r)$, (d) $F(3r)$.

Teaching example and classwork

1. If $f(x) = x^2 + 2x + 2$ and $g(x) = 3x - 4$, find the values of

   (a) $f(0) + g(1)$, (b) $2f(-3) \bullet g(0)$, (c) $\dfrac{f(1)}{g(-2)}$.

2. If $H(x) = ax - 5$ and $H(5) = 10$, find the value of a.

3. If $g(x) = (x + 2)(x - k)$ and $g(-4) = 10$

(a) Find the value of k.

(b) Hence, find the value(s) of x such that $g(x) = 0$

4. It is given that $f(t) = kt^3 - 4$ and $f(3) = 50$. Find

(a) the value of k,

(b) (i) $3f(1) + f(0)$, (ii) $[f(-2)]^2$.

5. It is given that $f(x) = x^2 + ax$, where $a \neq 0$. If $f(a) = 2a$, find

(a) the value of a,

(b) $f(-1)$.

6. It is given that $f(x) = 2x^2 - 4$.

(a) Find $f(2x)$ and $f(x - 1)$.

(b) Hence, solve for x if $f(2x) = f(x - 1)$.

7. It is given that $f(x) = 4(x + k)$, $g(x) = kx$ and $f(3) = 2g(1)$.

(a) Find the value of k.

(b) Hence, find the values of x such that

(i) $f(x) - g(x) = 6$,

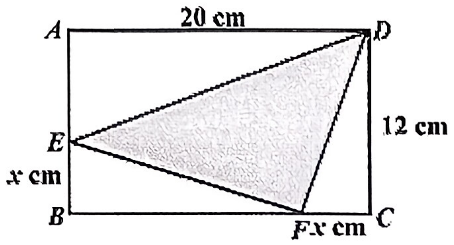
(ii) $f(x) \cdot g(x) = 120$.

$\boxed{\text{Level 2}}$

1. Let $f(x) = x^2 + ax + b$, where a and b are constants. It is given that $f(-1) = 2$ and $f(1) = 10$. Find the values of a and b.

2. If $f(x+1) = x^2 + 3x - 5$, find $f(x)$.

3. The figure shows a rectangle $ABCE$ with dimensions 20 cm$\times$12 cm. E and F are points of AB and BC respectively such that BE=FC= x cm and four triangles are formed. Let S(x) be the functions representing the area (in cm^2) of ΔDEF .



(a) Express S(x) in terms of x.

(b) Find the domain of the function S(x)

(c) (i) Find the area of ΔDEF when $AE+BF=22$ cm .

(ii) Find the value(s) of x when the area of ΔDEF is 104 cm^2 .

Public Exam Questions

[HKCEE 2003(Q1)]

1. If $f(x)=2x^2+kx-1$ such that $f(-2)=f(\frac{1}{2})$, then $k=$

   A. $-\frac{17}{3}$

   B. –5.

   C. 3.

   D. $\frac{31}{5}$.

[HKCEE 2004(Q3)]

2. If $f(x)=x^2-x+1$, then $f(x+1)-f(x)=$

   A. 0.

   B. 2.

   C. $2x$.

   D. $4x$.

[HKCEE 2005(Q3)]

3. If $f(x)=2x^2-3x+4$, then $f(1)-f(-1)=$

   A. –6.

   B. –2.

   C. 2.

   D. 6.

[HKCEE 2006(Q5)]

4. If $f(x)=\frac{x}{1+x}$, then $f(3)f(\frac{1}{3})=$

   A. $\frac{3}{16}$.

   B. $\frac{1}{2}$.

   C. $\frac{3}{4}$.

   D. 1.

[HKCEE 2007(Q8)]

5. Let $f(x)=x^2-ax+2a$, where a is a constant. If $f(-3)=29$, then $a=$

   A. –38 .

   B. –20.

   C. –4.

   D. 4.

[HKCEE 2008(Q6)]

6. Let $f(x)=x^2+kx+7$, where k is a constant. If $f(4)-f(3)=21$, then $k=$

A. 0.
B. 4.
C. 14.
D. 28.

[HKCEE 2009(Q6)]

7. Let $f(x)=x^2-9x+c$,where c is a constant. If $f(-1)=8$, then $c=$

A. -2.
B. 0.
C. 16.
D. 18.

[HKCEE 2010(Q6)]

8. If $f(x)=x^2-3x+17$, then $3f(2)-1=$

A. 27.
B. 34.
C. 44.
D. 70.

[HKCEE 2011(Q8)]

9. Let $f(x)=x^2+2x+k$, where k is a constant. Find $f(5)-f(3)$.

A. 20
B. k+8
C. k+35
D. 2k+50

[DSE 2017(Q6)]

10. Let k be a constant. If $f(x)=2x^2-5x+k$, then $f(2)-f(-2)=$

A. -20.
B. 0.
C. 16.
D. $2k$.

[DSE 2018(Q7)]

11. If $f(x)=3x^2-2x+1$, then $f(2m-1)=$

A. $6m^2-4m+2$.
B. $6m^2-4m+6$.
C. $12m^2-16m+2$.
D. $12m^2-16m+6$.

[DSE 2019(Q8)]

12. Let c be a constant. If $f(x)=x^3+cx^2+c$, then $f(c)+f(-c)=$

A. 0.
B. $2c$.
C. $2c^3+2c$.
D. $-2c^3+2c$.

[DSE 2020(Q5)]

13. Let $f(x)=3x^2-x-2$. If β is a constant, then $f(1+\beta)-f(1-\beta)=$

A. 2β.

B. 10β.

C. $6\beta^2-2$.

D. $6\beta^2-2\beta$.

[DSE 2021(Q7)]

14. Let $f(x)=(x+h)(x-3)+k$, where h and k are constants. If $f(0)=f(8)=1$, find k.

A. -14

B. -5

C. 20

D. 31

[DSE 2022(Q8)]

15. Let $f(x)=x^2-x+1$. If k is a constant, which of the following must be true?

A. $f(k)=f(-k)$

B. $f(k)=f(1-k)$

C. $f(k+1)=f(k)+f(1)$

D. $f(k-1)=f(k)-f(1)$

[DSE 2023(Q8)]

16. Let $g(x)=13-5x^2$. If α is a constant, find $g(1-3\alpha)$.

A. $8-45\alpha^2$

B. $8+45\alpha^2$

C. $8-30\alpha+45\alpha^2$

D. $8+30\alpha-45\alpha^2$

[DSE 2024(Q9)]

17. Let $g(x)=(x+1)(x+a)$, where a is a constant. If $g(1)=g(2)$, then $g(a)=$

A. -4.

B. 0.

C. 12.

D. 24.