Name	
Class	F.5 ( )
Class Number	

F.5 SECOND TERM TEST

2022 − 2023

MATHEMATICS Extended Part
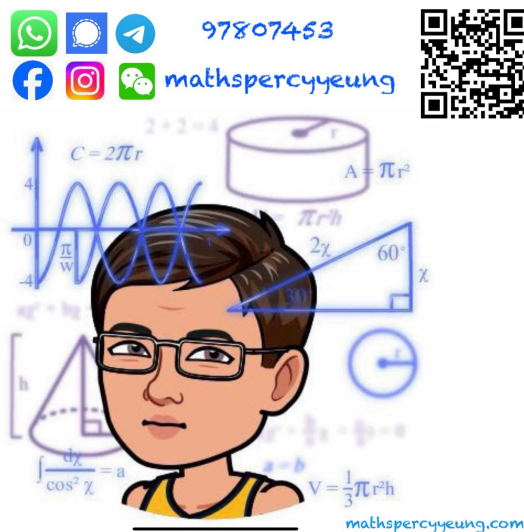# Module 2 (Algebra and Calculus)

Question-Answer Book

Time allowed: 75 minutes

This paper must be answered in English

INSTRUCTIONS

1.   After the announcement of the start of the examination, you should first write your Name, Class and Class Number in the space provided on Page 1.

2.   Attempt ALL questions in this paper.   Write your answers in the spaces provided in this Question-Answer Book.   Do not write in the margins.   Answers written in the margins will not be marked.

3.   Graph paper and supplementary answer sheets will be supplied on request.   Write your Name and mark the question number box on each sheet, and fasten them with string INSIDE the book.

4.   Unless otherwise specified, all working must be clearly shown.

5.   Unless otherwise specified, numerical answers must be exact.

6.   The diagrams in this paper are not necessarily drawn to scale.

7.   No extra time will be given to candidates for writing names or filling in the question number boxes after the 'Time is up' announcement.

8.   The full mark of this paper is 55.

FORMULAS FOR REFERENCE

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$2\sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A \sin B = \cos(A-B) - \cos(A+B)$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

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1. Let $f(x) = e^{3x}$. Find $f'(x)$ from first principles. (3 marks)

2. Find $\displaystyle\int \frac{x}{\sqrt{x^2 + 2023}}\,dx$. (2 marks)

3. At any point (x, y) on the curve Γ, the slope of the tangent to Γ is $(\pi + x)\cos x$. It is given that $\frac{\pi}{2}$ is one of the x-intercepts of Γ. Using integration by parts, find the equation of Γ.

(5 marks)

4. Evaluate

(a) $\displaystyle\int_0^1 \frac{x^2+1}{x+1}\,dx$,

(b) $\displaystyle\int_0^3 x\sqrt{x+1}\,dx$,

(c) $\displaystyle\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} 24\cos^2 3x\,dx$.

(10 marks)

Answers written in the margins will not be marked.

5. Denote the origin by O. $P(x, y)$ is a moving point on the curve $y = 3^{2x}$, where $x > 0$. Q and R are moving points on the x-axis and y-axis respectively such that $OQPR$ remains a rectangle.   If OR is increasing at a constant rate $2\ln 3$ units per second, find the rate of change of the area of $OQPR$ when $x = 1$.

(4 marks)

6. It is given that $4x - y + h = 0$ is a tangent to the curve $xy^2 + \ln y = k$ at P, where h and k are constants.
   If the y-coordinate of P is 1, find h and k.

   (5 marks)

7. Define $f(x) = \dfrac{e^{-x}}{x^2 + x + 1}$.

(a) Find the range of values of x for which $f(x)$ is increasing.

(b) Find the least value of $f(x)$ for $-3 \le x \le 0$.

(6 marks)

Answers written in the margins will not be marked.

8. Define $f(x) = \dfrac{(2x+3)(x+1)}{x^3}$ for all $x \neq 0$. Denote the graph of $y = f(x)$ by H.

(a) Write down all asymptote(s) of H.

(b) Find the point(s) of inflexion of H.

(6 marks)

9. (a) Using $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$, or otherwise, prove that $\tan\dfrac{\pi}{12}=2-\sqrt{3}$. (2 marks)

(b) Let $0<x<\dfrac{\pi}{2}$. Show that $\displaystyle\int\frac{1}{(\cos x+\sin x)\cos x}\,dx=\ln(1+\tan x)+\text{constant}$. (3 marks)

(c) Using the substitution $u=\dfrac{\pi}{4}-x$, show that $\displaystyle\int_{\frac{\pi}{12}}^{\frac{\pi}{6}}\frac{x}{(\cos x+\sin x)\cos x}\,dx=\frac{\pi}{8}\int_{\frac{\pi}{12}}^{\frac{\pi}{6}}\frac{1}{(\cos x+\sin x)\cos x}\,dx$. (4 marks)

(d) Using integration by substitution and the above results, evaluate $\displaystyle\int_{\frac{\pi}{3}}^{\frac{5\pi}{12}}\frac{x+\tan x}{(\cos x+\sin x)\sin x}\,dx$. (5 marks)

Answers written in the margins will not be marked.

END OF PAPER

Answers written in the margins will not be marked.