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| Class | F.5 (            ) |
| Class Number | |



F.5 FIRST TERM EXAMINATION

2022 – 2023

# MATHEMATICS Extended Part
# Module 2 (Algebra and Calculus)

## Question-Answer Book

11:00 – 13:00 (2 hours)

This paper must be answered in English

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Name, Class and Class Number in the space provided on Page 1.

2. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

3. Graph paper and supplementary answer sheets will be supplied on request. Write your Name and mark the question number box on each sheet, and fasten them with string INSIDE the book.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers must be exact.

6. The diagrams in this paper are not necessarily drawn to scale.

7. No extra time will be given to candidates for writing names or filling in the question number boxes after the 'Time is up' announcement.

8. The full mark of this paper is 90.

**FORMULAS FOR REFERENCE**

| | |
|---|---|
| $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ | $\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$ |
| $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ | $\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$ |
| $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ | $\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$ |
| $2\sin A \cos B = \sin(A+B) + \sin(A-B)$ | $\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$ |
| $2\cos A \cos B = \cos(A+B) + \cos(A-B)$ | |
| $2\sin A \sin B = \cos(A-B) - \cos(A+B)$ | |

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Answers written in the margins will not be marked

1. (a) Expand $(2+x)^4$.

(b) Find the coefficient of $x$ in the expansion of $\left(1-\dfrac{3}{x}\right)^2(2+x)^4$.

(5 marks)

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2. Let $f(x) = (x^2 - \pi^2)\cos x$. Find $f'(\pi)$ from first principles. (4 marks)

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3. Let $f(x) = x^{\cos x}$. Find $f'(\pi)$. (4 marks)

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4. Find

(a) $\displaystyle\int \frac{x}{\sqrt{x^2+1}}\,dx$ ,

(b) $\displaystyle\int \frac{x^3}{\sqrt{x^2+1}}\,dx$ ,

(c) $\displaystyle\int \sec^6 x\,dx$ ,

(d) $\displaystyle\int x\sin^2 x\,dx$ ,

(e) $\displaystyle\int e^x \sin x\,dx$ .

(14 marks)

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5. Solve $\sin x + \sin 2x = \sin 3x$ for $0 \le x < \pi$ . (4 marks)

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6. At any point $(x, y)$ on curve $C$, the slope of tangent to the curve is $\dfrac{4\sqrt{x}+2\ln x}{x}$. It is given that the point $P(1, 10)$ lies on $C$.

(a) Find the equation of the tangent to $C$ at $P$.

(b) Find the equation of $C$.

(7 marks)

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7. By considering $\frac{\mathrm{d}}{\mathrm{d}x}\left((x^4-1)\ln(x^2+1)\right)$, find $\int x^3\ln(x^2+1)\,\mathrm{d}x$. (5 marks)

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8. Let $f(x)=e^{2x}(x^2-2)$. Find the greatest value and the least value of $f(x)$ for $-3\le x\le 2$. (6 marks)

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9. Let $f(x)=\dfrac{x^2+x-6}{x+7}$, where $x \neq -7$.

(a) Find the $x$-intercept(s) and the $y$-intercept of the graph of $y=f(x)$.

(b) Show that $f'(x)=\dfrac{x^2+14x+13}{(x+7)^2}$ and $f''(x)=\dfrac{72}{(x+7)^3}$.

(c) Find all the turning points of the graph of $y=f(x)$.

(d) Find all the asymptotes of the graph of $y=f(x)$.

(e) Sketch the graph of $y=f(x)$.

(12 marks)

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10. (a) Using mathematical induction, prove that $\sum_{k=1}^{n}(-1)^k k^2 = \frac{(-1)^n n(n+1)}{2}$ for all positive integers $n$.

(b) Using (a), find $\sum_{k=1}^{n}(-1)^k (k+1)^2$ in terms of $n$.

(c) Using the fact that $\sum_{k=1}^{n}(-1)^k = \frac{(-1)^n - 1}{2}$ and the above results, show that $\sum_{k=1}^{n}(-1)^k k = \frac{1}{4}\left[(-1)^n(2n+1)-1\right]$ .

(9 marks)

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11. $P(x, y)$ is a moving point on the curve $y = 3\ln(x+1)$ such that the $x$-coordinate of $P$ is increasing at a rate of 8 units per second. The tangent to the curve at $P$ is denoted by $L$ and has an inclination angle $\theta$ .

(a) Find the rate of change of the $y$-coordinate of $P$ when $x = 3$ .

(b) Find the rate of change of $\theta$ when $x = 3$ .

(c) Let $Q$ be the point where $L$ cuts the $y$-axis. Find the rate of change of $PQ$ when $x = 3$ .

(8 marks)

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12. (a) Show that $\int \frac{1}{1+u^2}\,du = \tan^{-1} u + C$ , where $C$ is a constant.

(b) Using the substitution $u = 1 + x^{\frac{1}{3}}$ , find $\int \frac{1}{x^{\frac{2}{3}}(1+x^{\frac{1}{3}})}\,dx$ .

(c) Using the substitution $u = x^{\frac{1}{6}}$ , find $\int \frac{1}{x^{\frac{1}{2}}(1+x^{\frac{1}{3}})}\,dx$ .

(d) Using integration by parts and the above results, show that

$$\int \frac{\tan^{-1}(x^{\frac{1}{6}})}{x^{\frac{1}{2}}(1+x^{\frac{1}{3}})}\,dx = -3\ln(1+x^{\frac{1}{3}}) - 3\left(\tan^{-1}(x^{\frac{1}{6}})\right)^2 + 6x^{\frac{1}{6}}\tan^{-1}(x^{\frac{1}{6}}) + C \text{ , where } C \text{ is a constant.}$$

(12 marks)

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**END OF PAPER**

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