S3 Mathematics Notes

Quadrilaterals

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Content:

➢ Parallelograms
➢ Rhombuses, Rectangles and Squares
➢ Mid-point Theorem and Intercept Theorem

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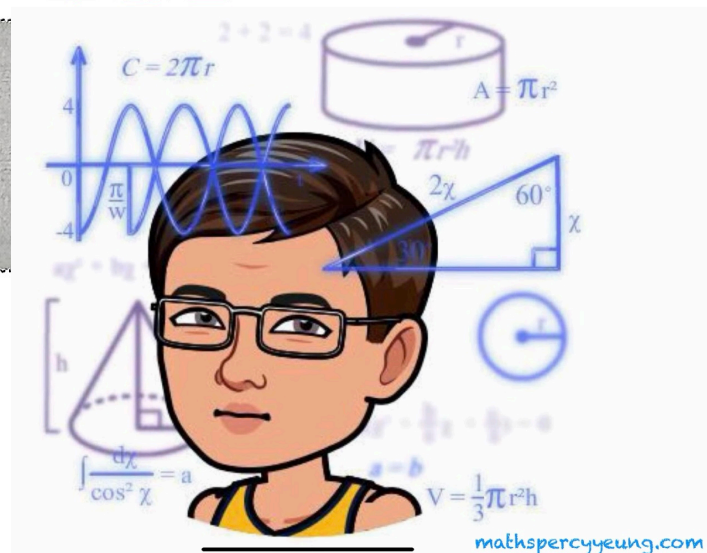
A · Parallelograms

🔑 Key Points: Properties of Parallelograms

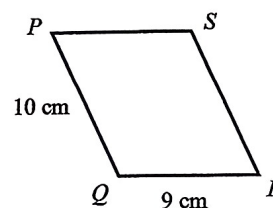|		
$AD = BC$ and $AB = DC$	$\angle A = \angle C$ and $\angle B = \angle D$	$AE = EC$ and $BE = ED$
(*opp. sides of // gram*)	(*opp. $\angle$s of // gram*)	(*diags. of // gram*)

📋 Let's Try

1. In the figure, $ABCD$ is a parallelogram. If $\angle ABC = 117°$ and $\angle CAD = 35°$, find x and y.
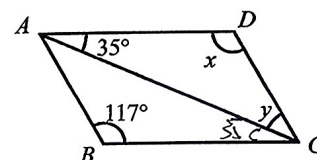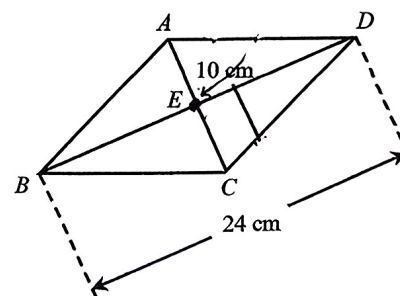
2. In the figure, $PQRS$ is a parallelogram. If $PQ = 10$ cm and $QR = 9$ cm, find the perimeter of $PQRS$.
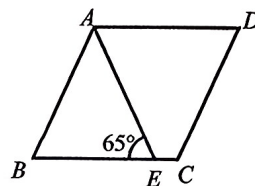
3. In the figure, $ABCD$ is a parallelogram. AC and BD intersect at E. It is given that $AC \perp BD$. If $AC = 10$ cm and $BD = 24$ cm.

   (a) Find the AE and BE.
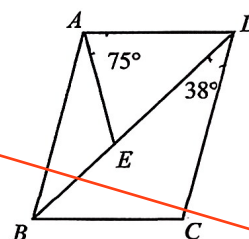
   (b) Find the AB.

Example 1

In the figure, *ABCD* is a parallelogram. *E* is a point lying on *BC* such that *AB* = *AE* and ∠*AEB* = 65°. Find ∠*ABE* and ∠*ADC*.
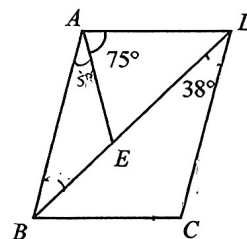

Practice Exercise

1. In the figure, *ABCD* is a parallelogram. *E* is a point lying on *AD* such that *CE* = *CD*. If ∠*BCE* = 48°, find ∠*CDE* and ∠*ABC*.

2. In the figure, *ABCD* is a parallelogram. *E* is a point lying on *BD* such that *AE* = *BE*. If ∠*BDC* = 38° and ∠*DAE* = 75°, find ∠*BCD*.

3. In the figure, *ABCD* is a parallelogram. *E* is a point lying on *AD* such that ∠*AEF* = 105°. *G* is a point lying on *EF* such that *BG* = *BF*. If ∠*ADC* = 72°, find ∠*ABG*.
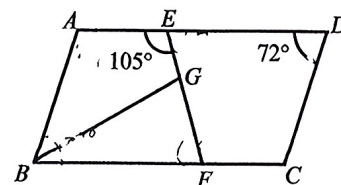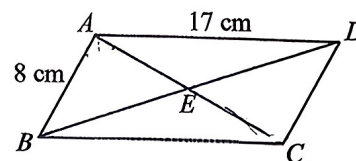

4. In the figure, *AB* = *AC*. *D*, *E* and *F* are points lying on *AB*, *AC* and *BC* respectively such that *DFCE* is a parallelogram. Cara claims that *DE* bisects ∠*ADF*. Do you agree? Explain your answer.
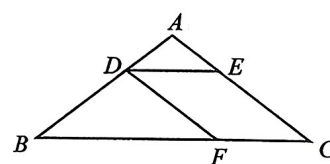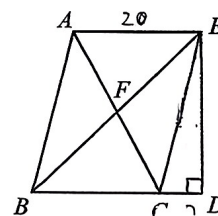
Example 2

In the figure, $ABCD$ is a parallelogram. AC and BD intersect at E. $AB = 8$ cm, $AD = 17$ cm and $\angle BAC = 90°$. Find BE correct to 3 significant figures.
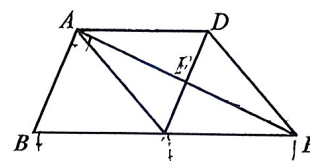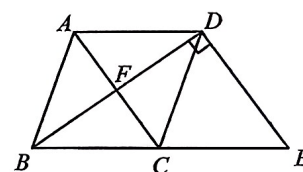

Practice Exercise

5. In the figure, C is a point lying on BD such that $BD \perp ED$ and $ABCE$ is a parallelogram. AC and BE intersect at point F. It is given that $AB = 26$, $AE = 20$ and $DE = 24$. Find BF correct to 3 significant figures.

6. In the figure, C is a point lying on BE such that $ABCD$ and $ACED$ are parallelograms. AE and CD intersect at point F. It is given that $AB \perp AE$, $AD = 17$ and $AF = 15$. Find CD.
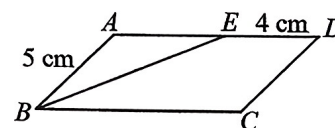

7. In the figure, C is a point lying on BE such that $ABCD$ and $ACED$ are parallelograms. AC and BD intersect at point F. It is given that $BD \perp DE$ and $BC = 15$. Find AB.
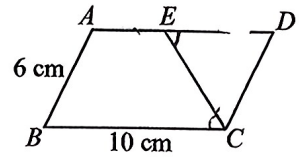

Example 3

In the figure, $ABCD$ is a parallelogram with $AB = 5$ cm and $ED = 4$ cm. E is a point lying on AD such that BE bisects $\angle ABC$. Find BC.
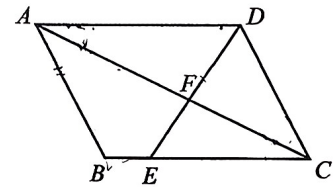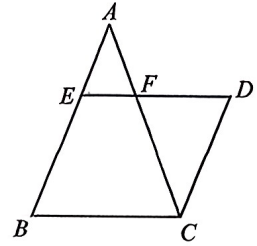
8. In the figure, $ABCD$ is a parallelogram with $AB = 6$ cm and $BC = 10$ cm. E is a point lying on AD such that EC bisects $\angle BCD$. Find AE.


9. In the figure, $ABCD$ is a parallelogram. E is a point lying on BC such that DE bisects $\angle ADC$. It is given that $AB = 7$ and $BE = 2$. Find AD.

10. In the figure, E is a point lying on AB such that $BCDE$ is a parallelogram. AC and DE intersect at point F. It is given that $AB : CD = 3 : 2$. Find $EF : DF$.

11. In the figure, $ABCD$ is a parallelogram. E is the mid-point of CD. AC and BE intersect at point F. Find the ratio of the area of $\triangle BCF$ to that of quadrilateral $AFED$.
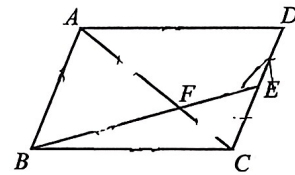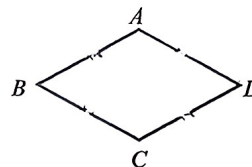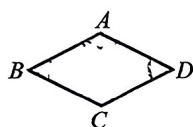
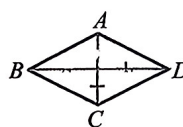A rhombus is a parallelogram with four equal sides.

[Ref.: *definition of rhombus*]

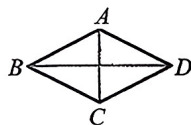(1) *ABCD* has all the properties of a parallelogram.

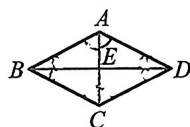$\angle ABC = \angle ADC$,
$\angle BAD = \angle BCD$

$AE = CE$,
$BE = DE$

(2) The diagonals are perpendicular to each other.

$AC \perp BD$

(3) The diagonals bisect each interior angle.

$\angle EAB = \angle EAD = \angle ECB = \angle ECD$,
$\angle EBA = \angle EBC = \angle EDA = \angle EDC$

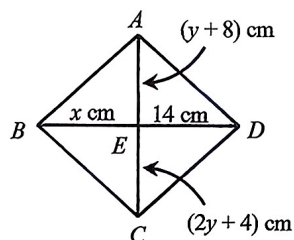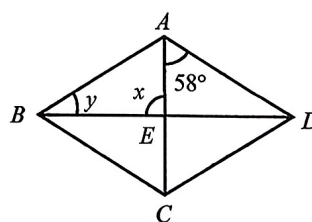[Ref.: *property of rhombus*]

Let's

In each of the following, *ABCD* is a rhombus and *AC* and *BD* intersect at *E*. Find x and y. (1 – 2)

1.

$(y + 8)$ cm
x cm 14 cm
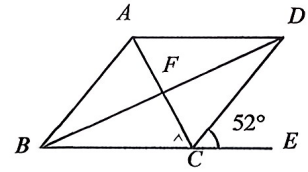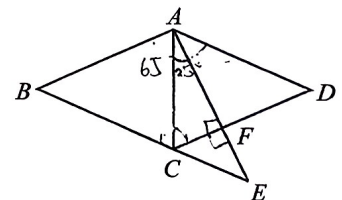$(2y + 4)$ cm

2.

y x 58°

Example 4

In the figure, *ABCD* is a rhombus. *BCE* is a straight line. *AC* and *BD* intersect at *F*. It is given that $\angle DCE = 52°$. Find $\angle ACB$ and $\angle CBF$.
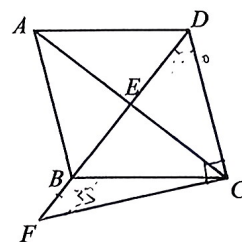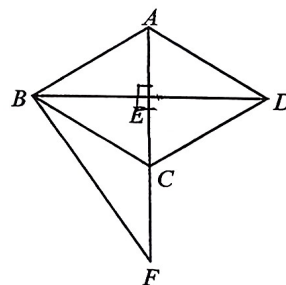

Practice Exercise

12. In the figure, *ABCD* is a rhombus. *BC* is produced to point *E*. *AE* and *CD* intersect at point *F* such that $AE \perp CD$ and $\angle CAE = 25°$. Find $\angle DAF$ and $\angle ABC$.

13. In the figure, $ABCD$ is a rhombus. AC and BD intersect at point E. DB is produced to point F such that $DC \perp FC$ and $\angle EFC = 33°$. Find $\angle ECF$ and $\angle EBC$.
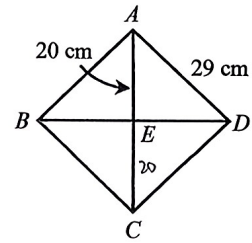
14. In the figure, $ABCD$ is a rhombus. AC and BD intersect at point E. AC is produced to point F such that $AB \perp BF$ and $\angle BFA = \angle ABE$. Barry claims that $\triangle ABF \sim \triangle AED$. Do you agree? Explain your answer.
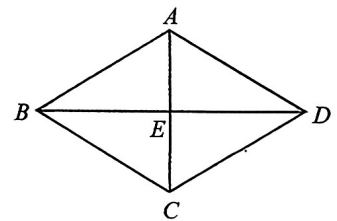
Example 5

In the figure, *ABCD* is a rhombus. *AC* and *BD* intersect at *E*. *AD* = 29 cm and *AE* = 20 cm. Find *AC* and *BE*.
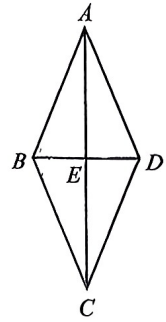

Practice Exercise

15. In the figure, *ABCD* is a rhombus of perimeter 136 cm. *AC* and *BD* intersect at point *E*. It is given that *AC* = 32 cm. Find the area of *ABCD*.

16. In the figure, *ABCD* is a rhombus of area 120 cm². *AC* and *BD* intersect at point *E*. It is given that *BD* = 10 cm. Find the perimeter of *ABCD*.

17. In the figure, *ABCD* is a rhombus of perimeter 20 cm. *AC* and *BD* intersect at point *E*. If *AC* = *BD*, find the area of *ABCD*.
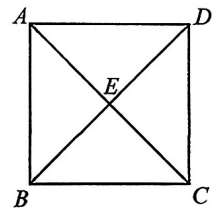
18. In the figure, *POR* is a sector with centre *O* and radius 8 cm. *PORQ* is a rhombus. Find the perimeter and the area of the shaded region. (Give your answers correct to 3 significant figures.)
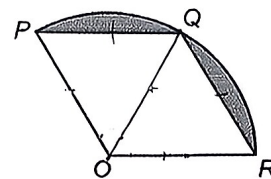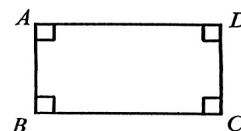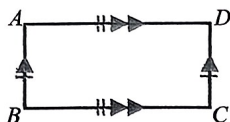
Key Points: Definition of a Rectangle

A rectangle is a quadrilateral with all interior angles equal to 90°.

[Ref.: *definition of rectangle*]
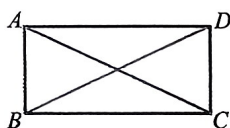
Key Points: Properties of Rectangles

(1) *ABCD* has all the properties of a parallelogram.
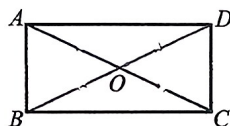

> **(i)** *AB* // *DC* and *AD* // *BC*
>
> **(ii)** *AB* = *DC* and *AD* = *BC*

(2) The diagonals are equal.

> *AC* = *BD*

(3) The diagonals bisect each other into four equal segments.

> *OA* = *OB* = *OC* = *OD*
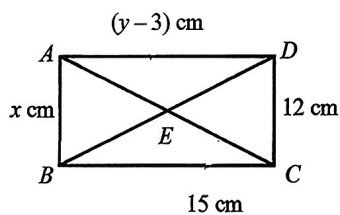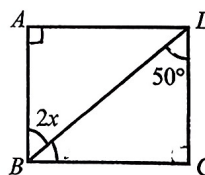
[Ref.: *property of rectangle*]

Let's Try

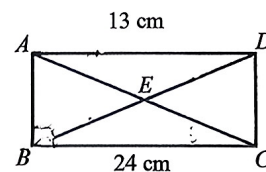In each of the following, *ABCD* is a rectangle. Find x and y. **(1 – 2)**

1.

$(y-3)$ cm

x cm
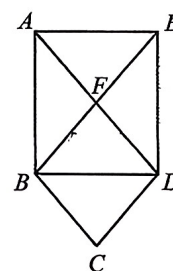
12 cm

15 cm

AC and *BD* intersect at *E*.

2.

50°

$2x$

Example 6

In the figure, $ABCD$ is a rectangle. AC and BD intersect at E. $AE = 13$ cm and $BC = 24$ cm. Find BD and AB.
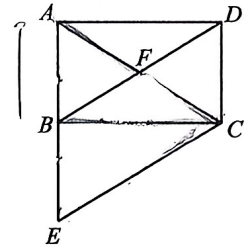

13 cm

A D

E

B 24 cm C

Practice Exercise

19. In the figure, $ABDE$ is a rectangle. AD and BE intersect at point F. $BCDF$ is a rhombus of perimeter 50 cm. It is given that $AB = 20$ cm. Find BD.
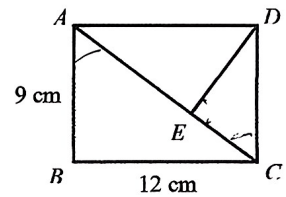

A E

F

B D

C

20. In the figure, $ABCD$ is a rectangle with $BC = 15$ cm. AC and BD intersect at point F. AB is produced to point E such that $CE = AC$. It is given that $AE = 16$ cm. Find BD.

21. In the figure, $ABCD$ is a rectangle. E is a point on AC such that $DE \perp AC$. $AB = 9$ cm and $BC = 12$ cm.

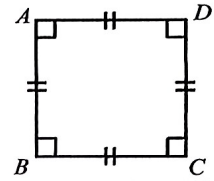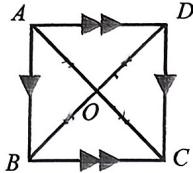(a) Prove that $\triangle ABC \sim \triangle CED$.

(b) Find the length of CE.

A square is a quadrilateral that four sides are equal and all interior angles are 90°.

[Ref.: *definition of square*]

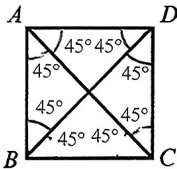(1) *ABCD* has all the properties of rhombuses and rectangles .

(i) *AB // DC* and *AD // BC*
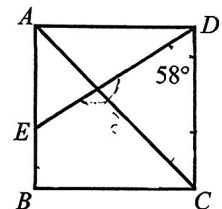
(ii) *OA = OB = OC = OD*

(iii)*AC ⊥ BD*

(2) Any angle between a side and a diagonal is 45°.
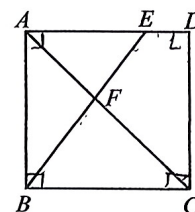

[Ref.: *property of square*]

Example 7

In the figure, *ABCD* is a square. *E* is a point lying on *AB* such that $\angle CDE = 58°$. *AC* and *DE* intersect at *F*. Find $\angle BED$ and $\angle CFD$.
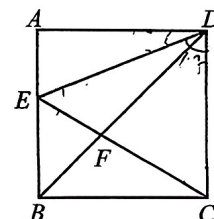
22. In the figure, $ABCD$ is a square. E is a point lying on AD such that $\angle BED = 120°$. AC and BE intersect at point F. Find $\angle BFC$.
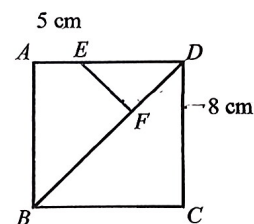
23. In the figure, $ABCD$ is a square. E is a point lying on AB such that $\angle ADE = 23°$ and $\angle DEF = 57°$. BD and CE intersect at point F. Find $\angle DFC$.
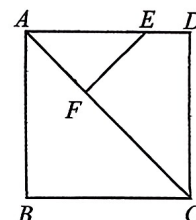
<u>**Example 8**</u>

In the figure, $ABCD$ is a square. E and F are points lying on AD and BD respectively such that $EF \perp BD$. It is given that $AE = 5$ cm and $DF = 8$ cm. Find CD correct to 3 significant figures.

<u>**Practice Exercise**</u>

24. In the figure, $ABCD$ is a square. E and F are points lying on AD and AC respectively such that $DE = 3$ and $EF \perp AC$. It is given that $AF = 6$. Find AB correct to 3 significant figures.

25. In the figure, $ABCD$ is a square. E and F are points lying on BC and AC respectively such that $EF = 5$ and $EF \perp AC$. AC and DE intersect at point G. It is given that $BE = 8$. Find DE correct to 3 significant figures.
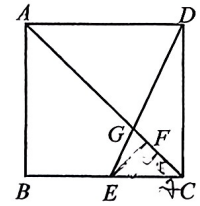
26. In the figure, $ABCD$ is a square. E is a point lying on BC. AE and BD intersect at point F. It is given that $AB : BE = 5 : 2$. Debby claims that $DF > AD$. Do you agree? Explain your answer.
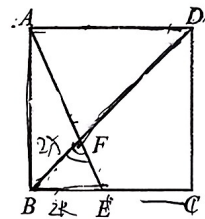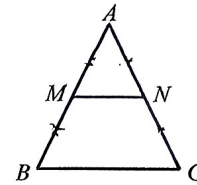
🔑 Key Points: Mid-point Theorem

If M and N are the mid-points of AB and AC respectively, then

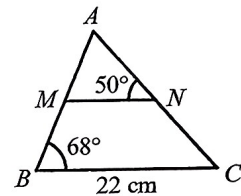(a) $MN \parallel BC$,

(b) $MN = \dfrac{1}{2}BC$.
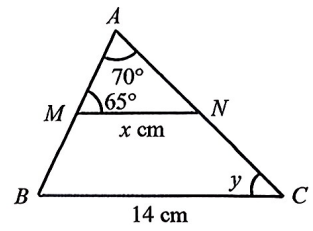
[Ref.: *mid-pt. theorem*]

Example 9

In the figure, M and N are the mid-points of AB and AC respectively.
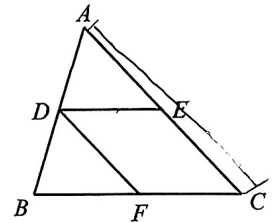
(a) Find MN.

(b) Find $\angle AMN$ and $\angle MAN$.

27. In the figure, M and N are the mid-points of AB and AC respectively. $\angle AMN = 65°$, $\angle MAN = 70°$ and $BC = 14$ cm. Find x and y.
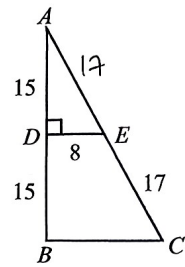

28. In the figure, D and E are the mid-points of AB and AC respectively. F is a point lying on BC such that $CF = 6$. It is given that $DE = 6$ and $AC = 14$. Find
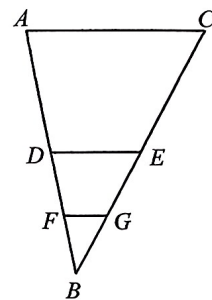
    (a) BF,

    (b) DF.

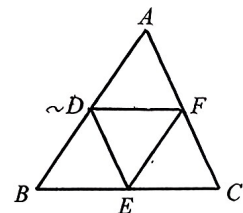**29.** In the figure, D and E are the mid-points of AB and AC respectively. Find

   (a) AE,

   (b) BC.

30. In the figure, D and E are the mid-points of AB and CB respectively. F and G are the mid-points of BD and BE respectively. Find $AC : FG$.

31. In the figure, D, E and F are the mid-points of AB, BC and AC respectively. If the perimeter of $\triangle DEF$ is 25 cm, find the perimeter of $\triangle ABC$.

32. In Figure (a), $ABCD$ is a parallelogram. The diagonals AC and BD intersect at Q. P is the mid-point of AB.
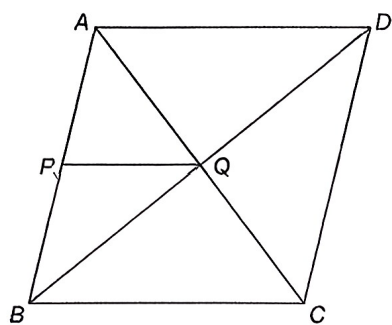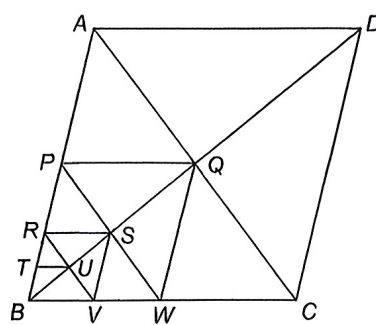

Figure (a)   *Figure (b)*

(a) Prove that $\dfrac{1}{AD}+\dfrac{1}{BC}=\dfrac{1}{PQ}$.

(b) In Figure (b), R and T are the mid-points of PB and RB respectively. $PBWQ$ and $RBVS$ are two parallelograms similar to $ABCD$. $BUSQD$, $BVWC$, PSW and RUV are straight lines.
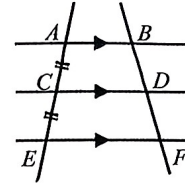
Prove that $\dfrac{1}{AD}+\dfrac{1}{BC}+\dfrac{1}{BW}+\dfrac{1}{BV}=\dfrac{1}{TU}$.
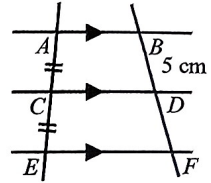
Example 10

In the figure, ACE and BDF are straight lines. $AB \parallel CD \parallel EF$, $AC = CE$ and $BD = 5$ cm.

(a) Find DF.

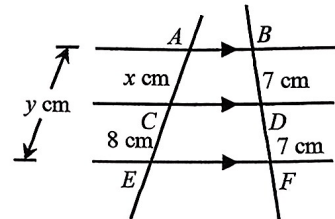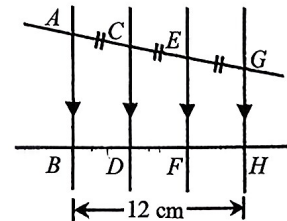**(b)** Find BF.

Practice Exercise

33. In the figure, ACE and BDF are straight lines. Find x and y.

34. In the figure, $ACEG$ and $BDFH$ are straight lines. $AC = CE = EG$ and $BH = 12$ cm. Find DH.
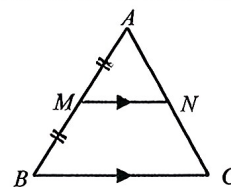
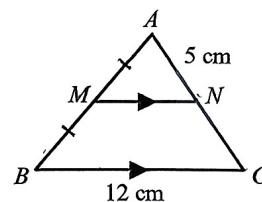If M is the mid-point of AB and $MN \mathbin{/\mkern-4mu/} BC$,

then $AN = NC$.

[Ref.: *intercept theorem*]

Example 11

In the figure, M and N are points lying on AB and AC respectively. $AM = MB$ and $MN \mathbin{/\mkern-4mu/} BC$. It is given that $AN = 5$ cm and $BC = 12$ cm.
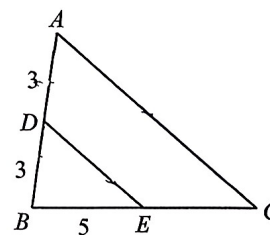
(a) Find NC.

(b) Find MN.

Practice Exercise

35. In the figure, D and E are points lying on AB and BC respectively. If $DE \mathbin{/\mkern-4mu/} AC$, find

(a) CE,

(b) AC.

36. In the figure, $ABCD$ and $AGFE$ are straight lines. $AB = 8$ cm, $BC = CD = 4$ cm, $BG = 6$ cm, $FE = 5$ cm and $BG \mathbin{/\!/} CF \mathbin{/\!/} DE$.

    (a) Find the length of AG.

    (b) Find the length of DE.
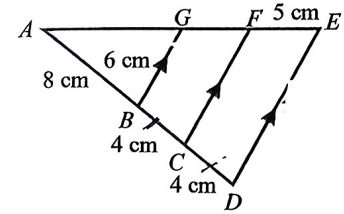
37. In the figure, E, F and G are points lying on AB, AC and AD respectively such that $EG \mathbin{/\!/} BD$ and $FG \mathbin{/\!/} CD$. Find AG, EG and CD.
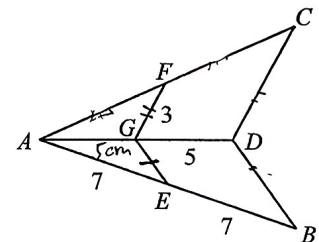
38. In the figure, D and E are points on AB and AC respectively such that $DE \parallel BC$. F is a point on BC. AF and DE intersect at G such that $AG = GF$. If the perimeter of $\triangle ADE$ is 18 cm, find the perimeter of $\triangle ABC$.
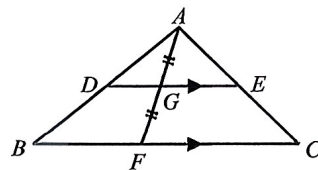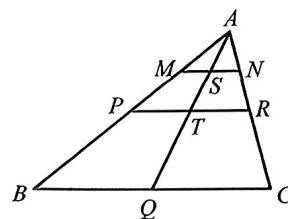

39. In the figure, $ABCD$ is a parallelogram. AC and BD intersect at point E. F and G are points lying on AB and BC respectively such that $FG \parallel AC$. It is given that $AD \parallel FE$ and $AC = 18$. Find FH.
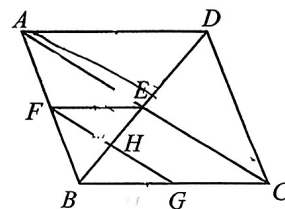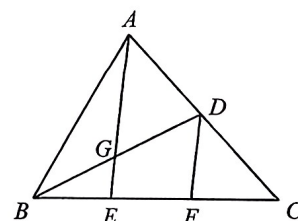
Example 12

In the figure, P and R are the mid-points of AB and AC respectively. Q is a point lying on BC. M and N are the mid-points of AP and AR. AQ cuts MN and PR at S and T respectively. Prove that $AS : SQ = 1 : 3$.
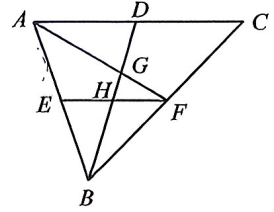
Practice Exercise

40. In the figure, D is the mid-point of AC. E and F are points lying on BC such that $AE \parallel DF$. AE and BD intersect at point G such that $BG = DG$. Prove that $AG = 3EG$.

41. In the figure, D, E and F are the mid-points of AC, AB and BC respectively. BD cuts AF and EF at points G and H respectively. Prove that $DG : BG = 1 : 2$.

42. In the figure, $ABCD$ is a parallelogram. F is a point lying on CD. AC cuts BD and BF at points E and G respectively. If EF // BC, prove that $AE : EG = 3 : 1$.