TT F4 Ch2 Equation of Straight Lines
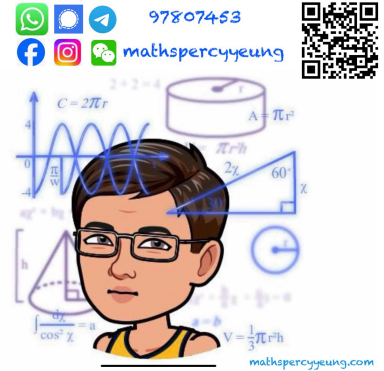
A. Review

I. Pre-lesson

1. Find the distance between the points A and B.
   $A(-1 , 9)$, $B(5 , 1)$

2. Find the slope of the straight line PQ.
   $P(-2 , -6)$, $Q(-7 , -9)$

3. If $P(3 , 1)$, $Q(q + 1 , 3)$ and $R(9 , 7)$ are collinear, find the value of q.

4. L_1 is a straight line passing through $(-5 , -3)$ and $(-1 , 5)$. L_2 is a straight line passing through $(-2 , 1)$ and $(6 , -3)$. Determine whether L_1 is perpendicular to L_2.

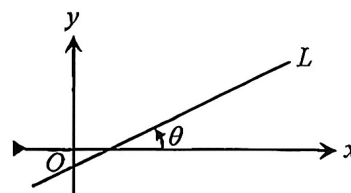## II. Teaching example and classwork

5. L_1 is a straight line with slope 2. L_2 is a straight line passing through $A(k, -5)$ and $B(-3, 3)$. Find the value of k in each of the following cases.
   (a) $L_1 \parallel L_2$
   (b) $L_1 \perp L_2$

6. In each of the following, find the coordinates of the mid-point of the line segment AB.
   (a) $A(5, 8), B(3, 10)$
   (b) $A(-7, 6), B(-9, -2)$

7. P is a point lying on the line segment joining $A(8, 7)$ and $B(-7, -3)$. If $AP : PB = 2 : 3$, find the coordinates of P.

8. Q is a point lying on the line segment joining $A(-6, -1)$ and $B(6, 5)$. If $AB = 6QB$, find the coordinates of Q.
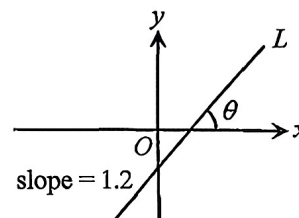
B. Section 2.1   Slope and Inclination

Inclination

For a straight line L with inclination θ (where $\theta \neq 90°$):

slope of $L = \tan \theta$

θ is the angle measured anticlockwise from the positive x-axis to L.

1. Find the inclination θ of L, correct to 3 significant figures.

slope = 1.2

II. Teaching example and classwork

2. Find the slope and the inclination of the straight line L passing through $A(3 , 3)$ and the origin.
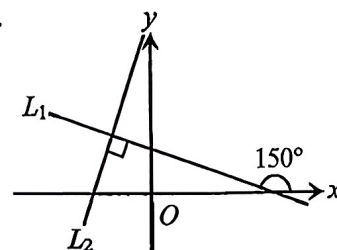
3. Find the slope and the inclination of the straight line L passing through $P(5 , 3)$ and $Q(-1 , -2)$.
    (*Give the answers correct to 3 significant figures if necessary.*)

4. In the figure, the straight lines L_1 and L_2 are perpendicular to each other.
   (a) Find the inclination of L_2.
   (b) Find the slope of L_2.
   (*Leave the radical sign '$\sqrt{}$' in the answer.*)

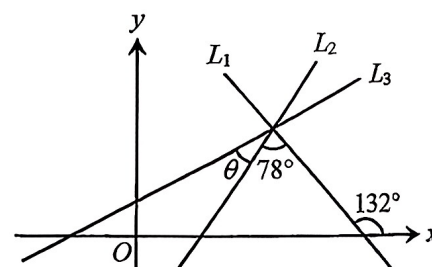|	30°	45°	60°
$\sin \theta$			
$\cos \theta$			
$\tan \theta$			

Level 2

5. In the figure, straight lines L_1, L_2 and L_3 intersect at the same point.
   Denote the acute angle between L_2 and L_3 by θ.
   (a) Find the inclination of L_2.
   (b) If the slope of L_3 is 0.5, find θ, correct to the nearest degree.

Section 2.2   Finding Equation of Straight Lines

1. Please refer to the textbook P.7 – 12.
2. Video from Identity: https://youtu.be/FLJtpS15Vso
   00:05 – quick review on basics of coordinate geometry
   02:34 – the fundamental idea and point-slope form
   11:01 – two-point form
   13:47 – slope-intercept form
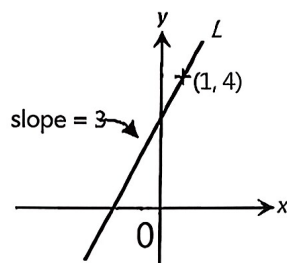   15:32 – two-intercept form (Enhancement)

I.   Pre-lesson

Find the equations of straight lines by considering different conditions.

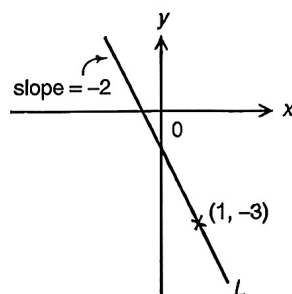|	Point-slope form	Two-point form	Slope-intercept from
Graph			
Equation of L			

II.   Teaching example and classwork

1. In each of the following, find the equation of the straight line L.

(a)

(b)

2. In each of the following, find the equation of the straight line satisfying the given conditions.
   (a)   It passes through $(3, 1)$, and its slope is 4.
   (b)   It passes through $(-5, 2)$, and its slope is -1.
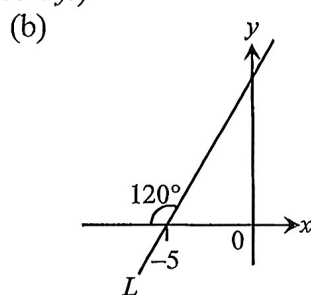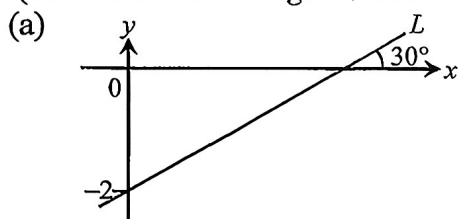
3. In each of the following, find the equation of the straight line satisfying the given conditions.

   (a) Its slope is 3, and its y-intercept is -2.

   (b) Its slope is $-\dfrac{2}{3}$, and its y-intercept is 5.

4. In each of the following, find the equation of the straight line L.
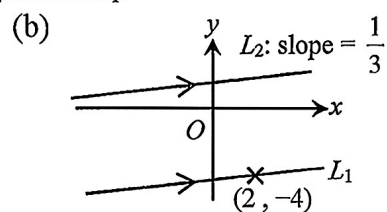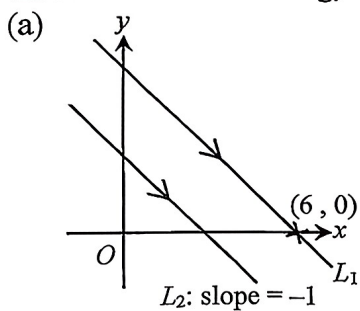
   (a)

   (b)
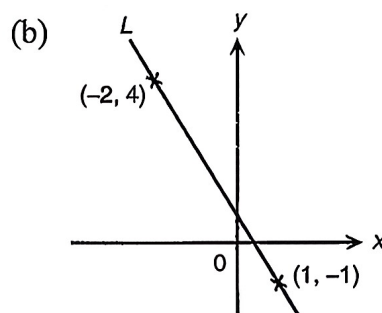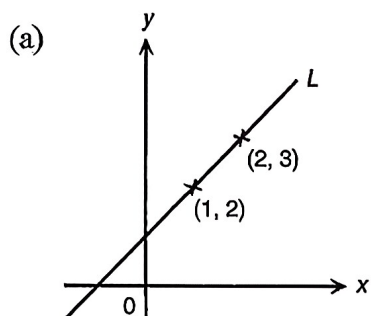
5. Find the equation of the straight line with x-intercept -2 and y-intercept -5.
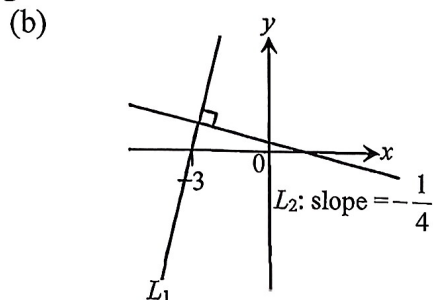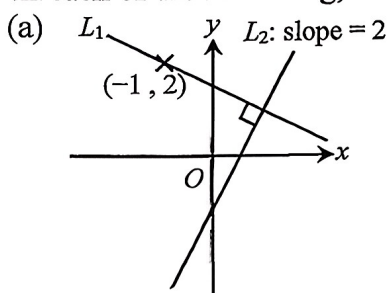
6. In each of the following, find the equation of the straight line L.
   (*Leave the radical sign '$\sqrt{}$' in the answers if necessary.*)
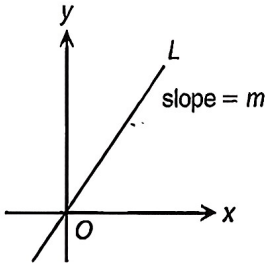   (a)

   (b)

7. In each of the following, find the equation of the straight line L_1.
   (a)

   (b)

8. In each of the following, find the equation of the straight line L_1.
   (a)

   (b)

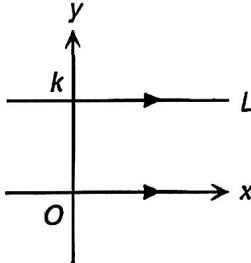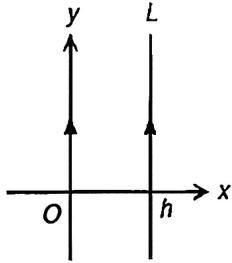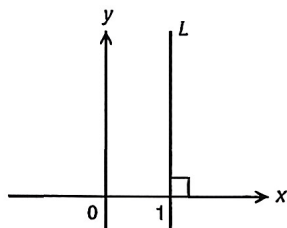| Equation of special straight lines			
	Oblique lines Passing through the Origin	**Horizontal Lines**	**Vertical Lines**
Graph			
Equation of L	$y = mx$	$y = k$	$x = h$

9. In each of the following, find the equation of the straight line L.

(a)

(b)
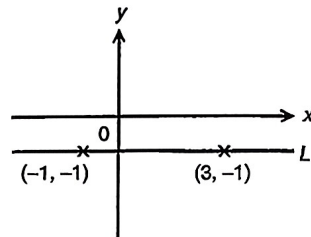
10. In each of the following, find the equation of the straight line satisfying the given conditions.

(a) It passes through the origin, and its slope is $-\dfrac{1}{6}$.

(b) It passes through the origin and $(-1, 4)$.

(c) It passes through $(2, -2)$ and parallel to the x-axis.

(d) It passes through $(1, -3)$ and perpendicular to the x-axis.

11. In the figure, the straight line L passes through $A(3, 3)$, and cuts the y-axis at B. The y-intercept of L is -3.
    (a) Find the equation of L.
    (b) Are the three points A, B and $C(2, 1)$ **collinear**? Explain your answer.
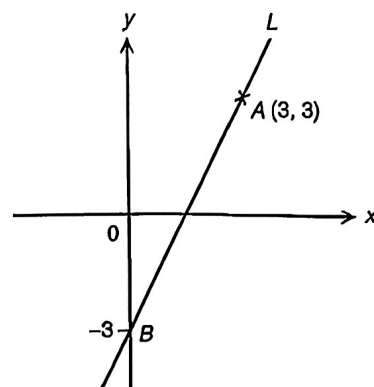
12. It is given that the straight line L passes through x-intercept -5 and has slope $\dfrac{2}{3}$.

    (a) Find the equation of L.
    (b) Does the point $Q(-2, 2)$ lie on L? Explain your answer.

Sketch the graph:

13. It is given that the straight line L passes through $(2, -1)$ with slope 3. L cuts the y-axis at P.
    (a) Find the equation of L.
    (b) Find the coordinates of P.

Sketch the graph:

14. In the figure, the straight line L passes through $(0, -6)$ and $(8, 0)$.
    (a) Find the slope of L.
    (b) ℓ is a straight line passing through $(2, -5)$. If ℓ is perpendicular to L, find the equation of ℓ.
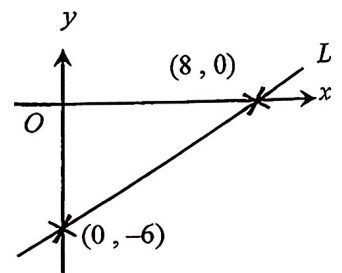
15. It is given that the straight line L_1 passes through $(-2, -3)$ and with y-intercept 1. The straight line L_2 is perpendicular to L_1 and with x-intercept 6.

(a) Find the equation of L_1.
(b) Find the equation of L_2.

Sketch the graph:

16. In the figure, the straight line $L : y = (k-1)x - 4k$ cuts the x-axis and the y-axis at A and $B(0, -6)$ respectively. OP is the median of AB in $\triangle OAB$. Find
    (a) the value of k,
    (b) the coordinates of A and P,
    (c) the equation of OP.
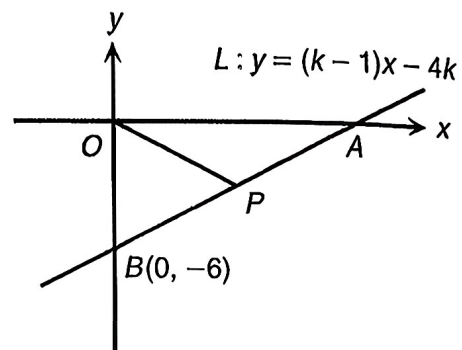
17. A straight line L cuts the y-axis at $A(0 , 10)$ and the x-axis at $B(5 , 0)$.
    (a) Find the slope of L.
    (b) ℓ is the perpendicular bisector of AB.

    (i) Find the equation of ℓ.

    (ii) If ℓ cuts the y-axis at the point P, find the equation of the horizontal line passing through P.

C. Section 4.2   General Form of Equation of a Straight Lines

1. Please refer to the textbook P.25 – 28.
2. Video from Identity: https://youtu.be/7mr7M1zFh98
   00:05 – general form
   04:42 – illustrative example

> For a straight line L in the general form $L: Ax + By + C = 0$ (where $A \neq 0$ and $B \neq 0$), we have
> $$\text{slope} = -\frac{A}{B}, \quad x\text{-intercept} = -\frac{C}{A} \quad \text{and} \quad y\text{-intercept} = -\frac{C}{B}$$

I. Pre lesion Worksheet

1. For each of the following equations of straight lines, find the slope and the y-intercept of the straight line.

   (a) $3x - y = 0$

   (b) $6x + 4y - 7 = 0$

2. Find the slopes, the x-intercepts and the y-intercepts of the following straight lines.

		Slope	x-intercept	y-intercept
(a)	$L_1: x + y - 5 = 0$			
(b)	$L_2: y = -2x - 4$			

3. In each of the following,
   (i)   find the slopes of the straight lines L_1 and L_2,
   (ii) determine whether L_1 and L_2 are parallel or perpendicular and put a '✓' in the appropriate box.

		Slope of L_1	Slope of L_2	$L_1 /\!/ L_2$	$L_1 \perp L_2$
(a)	$L_1: x - 3y + 2 = 0$ $L_2: x - 3y + 5 = 0$				
(b)	$L_1: 5x - 2y + 1 = 0$ $L_2: 4x + 10y + 3 = 0$				
(c)	$L_1: 3x + 4y + 2 = 0$ $L_2: -6x - 8y + 3 = 0$				

II. Teaching example and classwork

1. Find the slopes, the x-intercepts and the y-intercepts of the following straight lines.

   (a) $2x + 3y - 12 = 0$

   (b) $y - 3 = \dfrac{1}{2}(x - 4)$

2. If the two straight lines $L_1: x + 2y - 5 = 0$ and $L_2: 3x - Cy + 8 = 0$ are parallel, find
   (a) the slope of L_1,
   (b) the value of C.

3. If the two straight lines $L_1: 3x - y + 3 = 0$ and $L_2: Dx + 6y - 1 = 0$ are perpendicular to each other, find the value of D.

4. In the figure, the straight lines L_1: $2x + 3y - 6 = 0$ and L_2 are perpendicular to each other, and they have the same y-intercept.
    (a) Find the slope and the y-intercept of L_2.
    (b) Find the equation of L_2 in the general form.


$L_1 : 2x + 3y - 6 = 0$
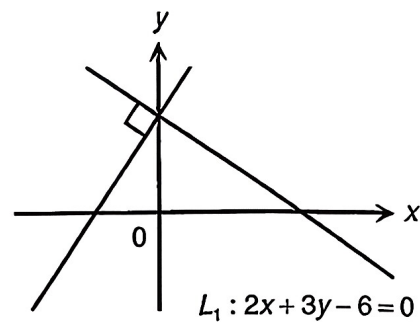
5. Find the equation of the straight line L_1 which passes through the point $B(-2, -7)$ and is parallel to the straight line $L_2 : 5x + 3y = 0$.

6. In the figure, the straight lines $L_1 : 3x + y + 6 = 0$ and $L_2 : x - 2y + k = 0$ cut the x-axis at the same point A and cut the y-axis at points B and C respectively.

   (a)   Find the coordinates of A and the value of k.

   (b)   Find the coordinates of B and C.

   (c)   Hence, find the area of $\triangle ABC$.
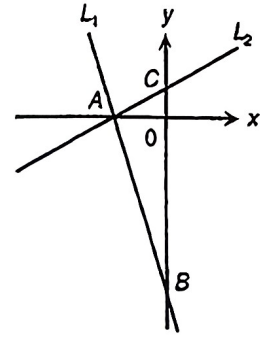
7. In the figure, $A(-2, 5)$ and $B(3, 2)$ are two points. The equation of the straight line L_2 is $4x + y - 8 = 0$. The straight line L_1 passes through A and is parallel to L_2.
   (a) Find the equation of L_1.
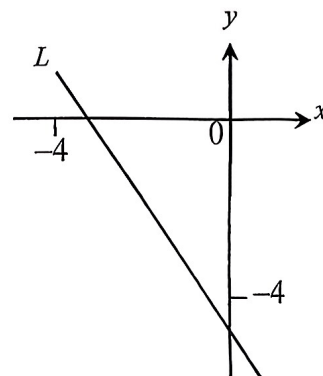   (b) If C is a point lying on L_1 such that $AC = BC$, find the coordinates of C.
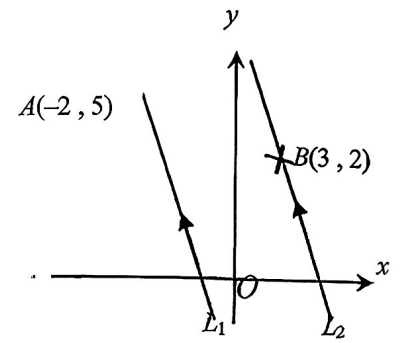
8. In the figure, the equation of the straight line L is $x + ay + b = 0$.
Determine whether each of the following is true. Explain your answers.
   (a) $a > 0$
   (b) $b > 3$
   (c) $b > -3a$
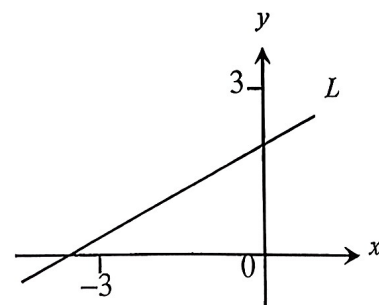
9. In the figure, the equation of the straight line L is $cx - 2y + d = 0$.
Determine whether each of the following is true. Explain your answers.
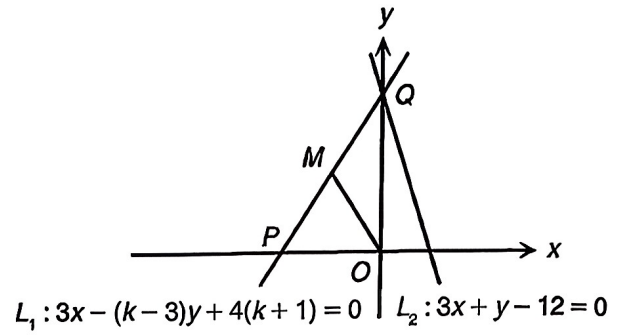   (a) $c > 0$
   (b) $d > -8$
   (c) $d > 4c$

10. In the figure, the straight lines
L_1: $3x - (k-3)y + 4(k+1) = 0$ and L_2: $3x + y - 12 = 0$
cut the y-axis at the same point Q. L_1 cuts the x-axis at P.
OM is the median of PQ in $\triangle POQ$.

(a) (i) Find the value of k.
    (ii) Hence, find the x-intercept of L_1.
(b) (i) Find the equation of OM.
    (ii) Determine whether OM is the perpendicular
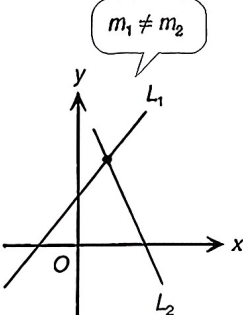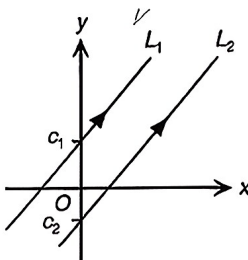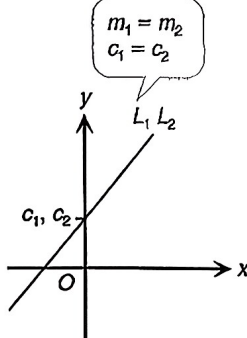   bisector of PQ. Explain your answer.

$L_1 : 3x - (k-3)y + 4(k+1) = 0$ $L_2 : 3x + y - 12 = 0$

11. The straight line L_1: $x + 2y - 8 = 0$ cuts the x-axis and the y-axis at the points P and Q respectively. The straight line L_2 is perpendicular to L_1 and intersects L_1 at P. L_2 cuts the y-axis at the point R.

   (a) Find the coordinates of P and Q.

   (b) Find the equation of L_2.

   (c) Let M be a point such that QM is a median of $\triangle PQR$. Find the area of $\triangle QRM$.

# D. Section 2.4   Possible Intersection of Two Straight Lines

I. Pre lesson Worksheet

For two non-vertical straight lines $L_1 : y = m_1 x + c_1$ **and** $L_2 : y = m_2 x + c_2$, **we have**

	Case 1	Case 2	Case 3
No. of intersections	one intersection	no intersections	an infinite number of intersections
Graphical presentation			
Condition	unequal slopes	equal slope and unequal y-intercepts	equal slope and equal y-intercept

1. In each of the following, find the coordinates of the intersection of the straight lines L_1 and L_2.

   (a)

   $L_1 : x + y - 5 = 0$
   $L_2 : x - y + 5 = 0$
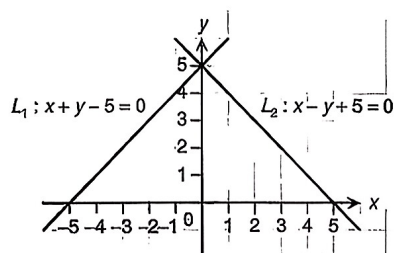
   (b) $L_1: x - y = 0$, $L_2: x + y - 2 = 0$

2. If the two straight lines $L_1: ax + by + 15 = 0$ and $L_2: x - 2y - 5 = 0$ have an infinite number of intersections, find the values of a and b.

3. In each of the following, determine the number of intersections between the two straight lines L_1 and L_2, and put a '✓' in the appropriate box and explain your answer

|		No intersections	One intersection	Infinite number of intersections	Explain your answer
(a)	$L_1: y = 1, L_2: y = -5$				
(b)	$L_1: x = 3, L_2: y = 3$				
(c)	$L_1: y = 5x - 3, L_2: y = 5x + 3$				
(d)	$L_1: y = -x + 4, L_2: y = x + 4$				
(e)	$L_1: 3x = 1, L_2: x = \dfrac{1}{3}$				

II. Teaching example and classwork

4. If the two straight lines $L_1: -3x + y + h = 0$ and $L_2: -9x + (h - 3)y - 2k = 0$ are **coincident**, find the values of h and k.

5. In the figure, the two straight lines L_1: $2x + y + 1 = 0$ and L_2 intersect at a point A. The slope and the y-intercept of L_2 are 3 and 4 respectively.
   (a) Find the equation of L_2.
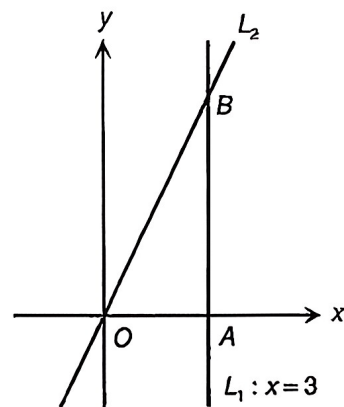   (b) Find the coordinate of A.
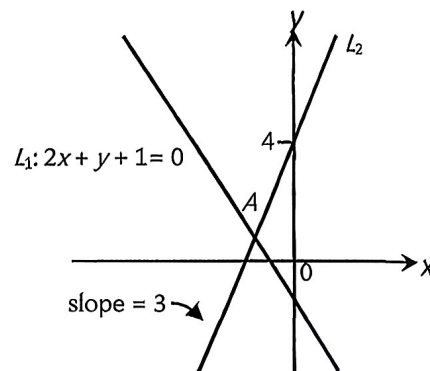
6. In the figure, the two straight lines $L_1 : 2x - 3y - 19 = 0$ and L_2 intersect at A. The x-intercept and the y-intercept of L_2 are $-\dfrac{4}{3}$ and -2 respectively.
   (a) Find the equation of L_2.
   (b) Find the coordinates of A.
   (c) If L_3 is the straight line passing through O and A, find the equation of L_3.
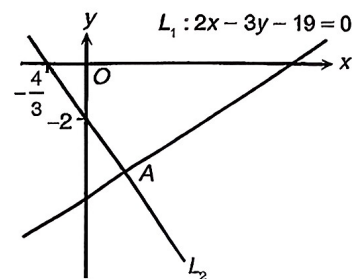
8. t is given that the two straight lines L_1: $4x + y + 1 = 0$ and L_2: $Ax + 2y - 3 = 0$ do not intersect.
   (a) Find the value of A.
   (b) If L is the straight line passing through the origin and perpendicular to L_2, find the equation of L.

9. In the figure, the straight line L_1: $x = 3$ cuts the x-axis and L_2 at A and B respectively. The area of the triangle OAB is 9 sq. units.
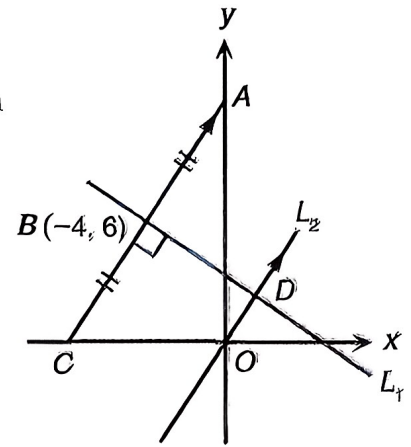   (a) Find the coordinates of A and B.
   (b) Find the equation of L_2.

10. In the figure, the straight line $L: y = -\dfrac{2}{3}x + 4$ cuts the x-axis

and y-axis at A and D respectively. C is a point on L and $B(2, 0)$ is a point on the x-axis.

(a) Find the coordinates of A.

(b) If the area of $\triangle ABC$ is 4 square units, find
   (i) the coordinates of C,
   (ii) $AC: CD$,
   (iii) the equation of BC.
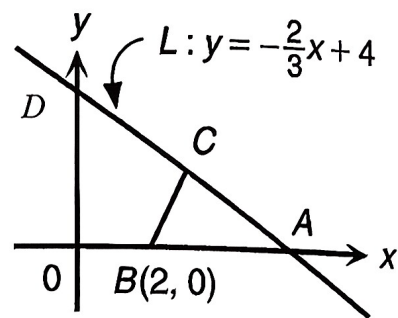
11. In the figure, the straight line L_1 is the perpendicular bisector of the line segment AC, and L_1 cuts AC at $B(-4, 6)$. The straight line L_2 passes through O and is parallel to AC. L_2 intersects L_1 at a point D. If A lies on the y-axis and C lies on the x-axis, find

    (a)   the coordinates of A and C,

    (b)   the equation of BD,

    (c)   the equation of OD,

    (d)   the coordinates of D.

12. $A(1, 6)$, $B(6, 13)$ and $C(6, -11)$ are three points on the rectangular coordinate plane and M is the mid-point of BC. A straight line L with slope 3 passing through B cuts line segment AM at K.

    (a)   Find the equations of BK and AM.

    (b)   Find the coordinates of K.

    (c)   Hence, find area of $\triangle BAK$: area of $\triangle BKM$.
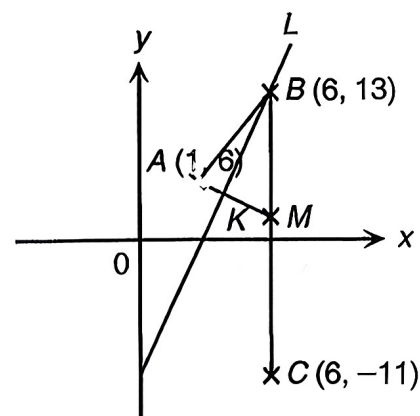
E. Past Paper (MC)

1	2	3	4	5	6	7	8

9	10	11	12	13	14	15	16

1. In the figure, the slopes of the straight lines l_1, l_2, l_3 and l_4 are m_1, m_2, m_3 and m_4 respectively. Which of the following is true?
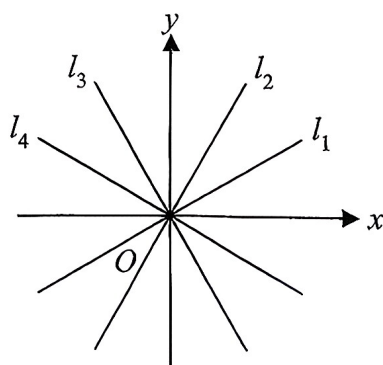
- A. $m_1 > m_2 > m_3 > m_4$
- B. $m_2 > m_1 > m_3 > m_4$
- C. $m_1 > m_2 > m_4 > m_3$
- D. $m_2 > m_1 > m_4 > m_3$
- E. $m_4 > m_3 > m_2 > m_1$

[HKCEE 90 #29]

2. If a, b and c are all positive, which of the following may represent the graph of $ax + by + c = 0$?

[HKCEE 96 #29]

3. If $a > 0$, $b > 0$ and $c < 0$, which of the following may represent the graph of the straight line $ax + by + c = 0$?
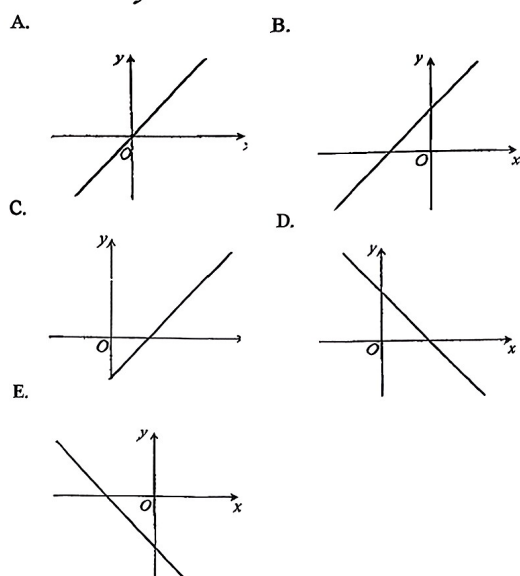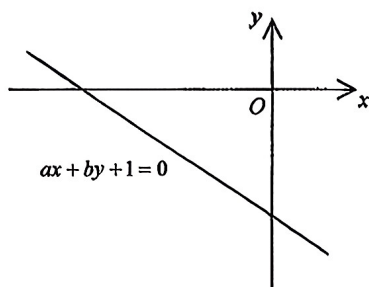

[HKCEE 04 #29]

4. If $k < 0$, which of the following may represent the graph of the straight line $x - y = k$?
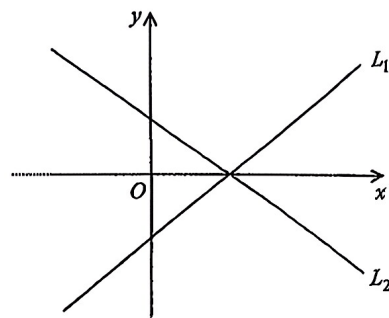

[HKCEE 06 #28]

29
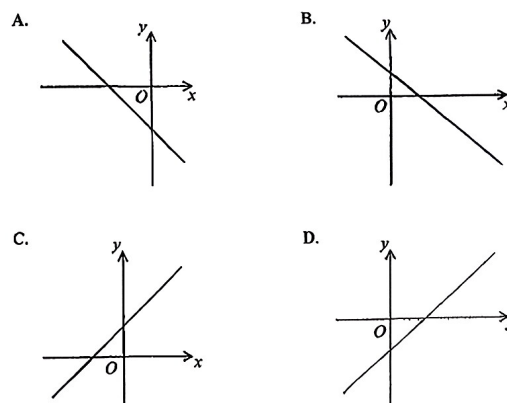
5. The figure shows the graph of the straight line $ax+by+1=0$. Which of the following is true?

A. $a>0$ and $b>0$
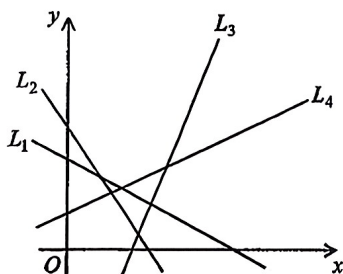B. $a>0$ and $b<0$
C. $a<0$ and $b>0$
D. $a<0$ and $b<0$

6. In the figure, L_1, L_2, L_3 and L_4 are straight lines. If m_1, m_2, m_3 and m_4 are the slopes of L_1, L_2, L_3 and L_4 respectively, which of the following must be true?

A. $m_1<m_2<m_3<m_4$
B. $m_1<m_2<m_4<m_3$
C. $m_2<m_1<m_3<m_4$
D. $m_2<m_1<m_4<m_3$

7. In the figure, the straight line $L_1: y=ax+b$ and the straight line $L_2: y=cx+d$ intersect at a point on the positive x-axis. Which of the following must be true?

A. $ab>0$
B. $cd>0$
C. $ac=bd$
D. $ad=bc$

8. In the figure, the two straight lines intersect at a point on the negative y-axis. Which of the following must be true?
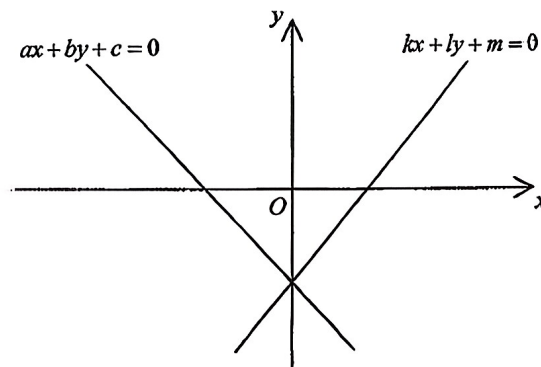
I. $ac>0$
II. $km>0$
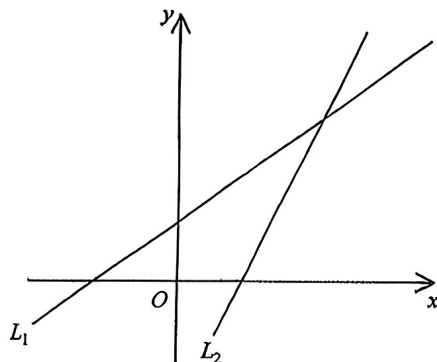III. $am=ck$
IV. $bm=cl$

A. I and III only
B. I and IV only
C. II and III only
D. II and IV only

9. In the figure, the equations of the straight lines L_1 and L_2 are $ax+y=b$ and $cx+y=d$ respectively. Which of the following are true?
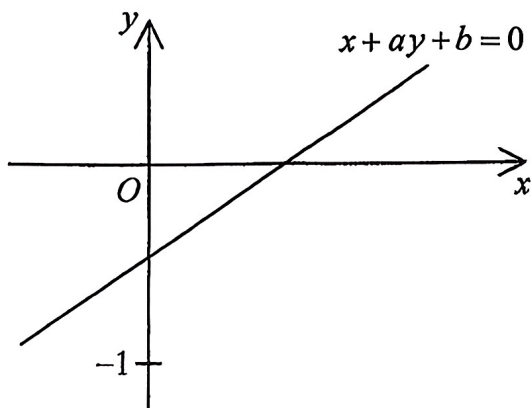
I. $a<0$
II. $a<c$
III. $b>d$
IV. $ad>bc$

    A.   I, II and III only
    B.   I, II and IV only
    C.   I, III and IV only
    D.   II, III and IV only

[HKDSE 12 #25]

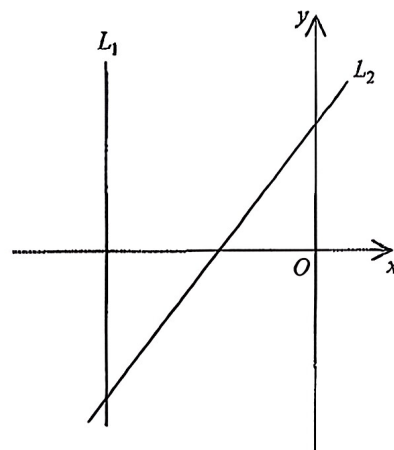10. The figure shows the graph of the straight line $x+ay+b=0$. Which of the following are true?

I. $a<0$
II. $b<0$
III. $a<b$

    A.   I and II only
    B.   I and III only
    C.   II and III only
    D.   I, II and III

[HKDSE 13 #14]

11. In the figure, the equations of the straight lines L_1 and L_2 are $ax=1$ and $bx+cy=1$ respectively. Which of the following are true?
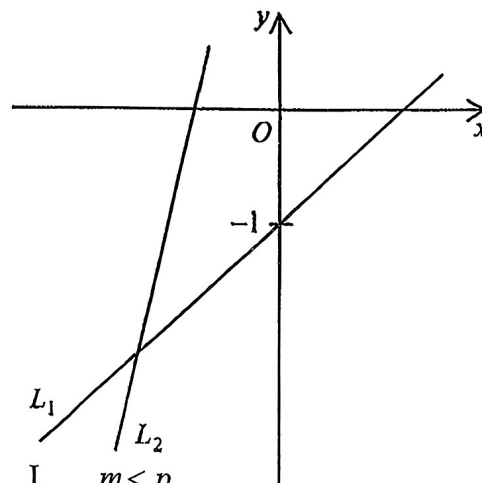
I. $a<0$
II. $a<b$
III. $c>0$

    A.   I and II only
    B.   I and III only
    C.   II and III only
    D.   I , II and III

[HKDSE 15 #25]

12. In the figure, the equations of the straight lines L_1 and L_2 are $x+my=n$ and $x+py=q$ respectively. Which of the following are true?
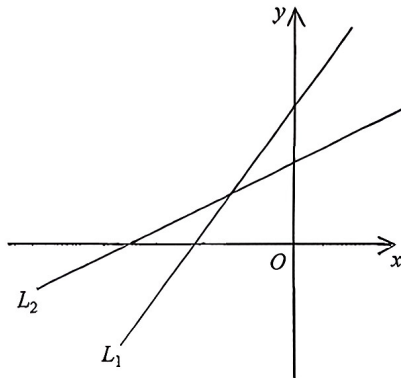
I. $m<p$
II. $n>q$
III. $n+m<p+q$

    A.   I and II only
    B.   I and III only
    C.   II and III only
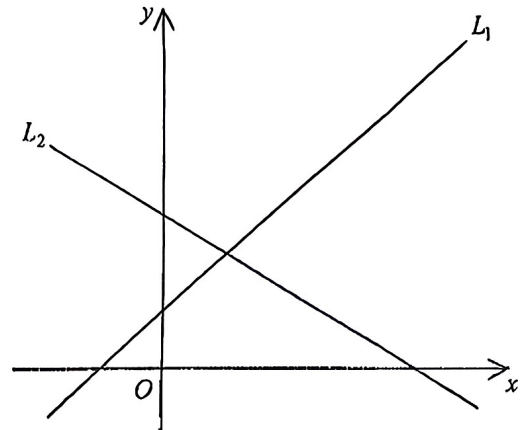   D.   I, II and III

[HKDSE 17 #23]

13. In the figure, the equations of the straight lines L_1 and L_2 are $3x + ay = b$ and $cx + y = d$ respectively. Which of the following is/are true?

I. $ac < 3$
II. $ad < b$
III. $bc < 3d$
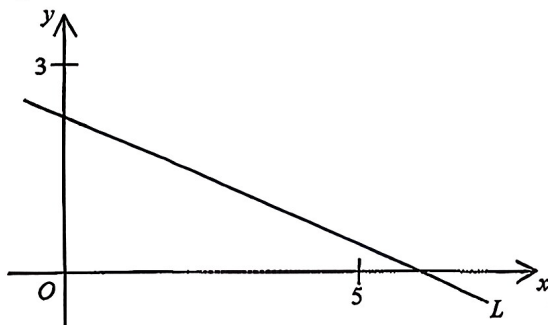
A. II only
B. III only
C. I and II only
D. I and III only

14. In the figure, the equation of the straight line L is $ax + by + 15 = 0$ .Which of the following are true?

I. $a > b$
II. $a > -3$
III. $b > -5$

A. I and II only
B. I and III only
C. II and III only
D. I , II and III

15. In the figure, the equations of the straight lines L_1 and L_2 are $x + ay + b = 0$ and $bx + y + c = 0$ respectively. Which of the following are true?

I. $c < 0$
II. $ab < 1$
III. $ac < b$

A. I and II only
B. I and III only
C. II and III only
D. I, II and III

16. The figure shows the graphs of the straight line $mx + ny = 3$. Which of the following are true?
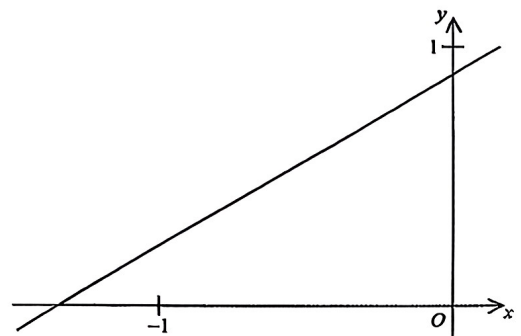
I. $m < 0$
II. $n > 3$
III. $m + n = 0$

A. I and II only
B. I and III only
C. II and III only
D. I , II and III