S3 First Term Examination (2018-2019)

Mathematics

(2 hours)

Date: 3rd January 2019

Time: 10:00 a.m. - 12:00 nn

Instructions to students:

1. This paper consists of THREE parts, Conventional Questions, Multiple-choice Questions and Bonus Question. There are Section A(1), Section A(2) and Section B in Conventional Questions. Section A(1) carries 33 marks, Section A(2) carries 27 marks, Section B carries 20 marks, Multiple-choice Questions carry 20 marks and Bonus Question carries 4 marks.

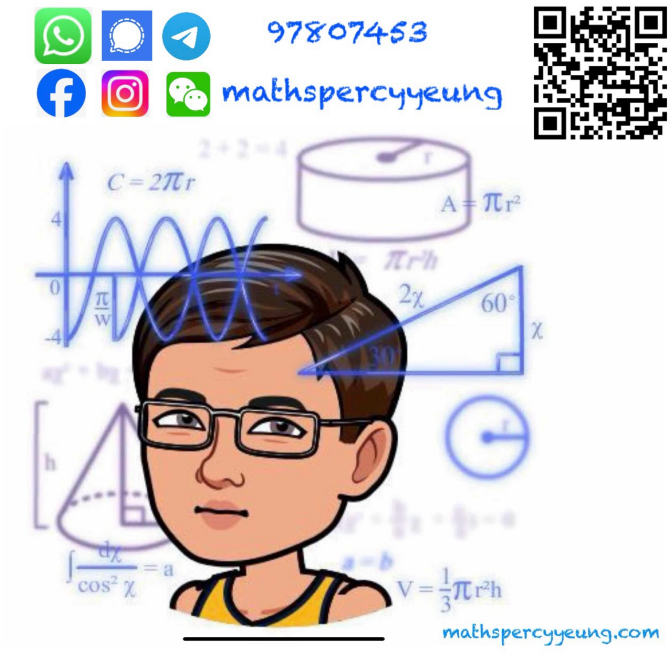
2. The maximum score of this paper is 100.

3. Attempt ALL questions in Conventional Questions and Multiple-choice Questions. Write your answers in the spaces provided in this Question / Answer Book.

4. Unless otherwise specified, show all workings clearly.

5. Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.

6. The diagrams in this paper are not necessarily drawn to scale.



Conventional Questions
Section A(1) (33 marks)

1. Simplify $x^2(\frac{x^{-2}}{y^3})^{-2}$ and express your answer with positive indices. (3 marks)

2. Make x the subject of the formula $a = b + \frac{c}{x}$. (3 marks)

3. Factorize

   (a) $x^2 + 2xy - 15y^2$,

   (b) Hence factorize $x^2 + 2xy - 15y^2 - (x - 3y)^2$.

   (3 marks)

4. (a) Solve the inequality $\frac{x+2}{2} > -3(x-4)$ and **represent the solutions on a number line.**

(b) If x is an integer, find the smallest possible value of x.

(4 marks)

5. In Figure 1, AC is a median of ΔABD. $AC = CD = AD = 8$ cm.

Find $\angle ABC$. (3 marks)

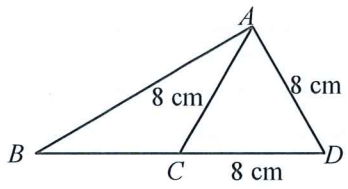


Figure 1

6. (a) Evaluate the expression $\frac{(8\times10^9)\times(6\times10^5)}{12\times10^6}$ without using a calculator and express the answer in scientific notation.

(b) Convert $FACE_{16}$ to a denary number.

(4 marks)

7. It is given that 7 times x is not less than 84. If x is an integer, find the smallest possible value of $3x + 4$. (3 marks)

8. Figure 2 shows an inverted right circular cone with base radius 5 cm and height of 12 cm. Find the curved surface area of the circular cone. (4 marks)

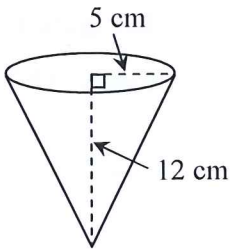


Figure 2

9. Figure 3 shows a hemisphere of diameter 10 cm.
Find the volume of the hemisphere in terms of π. (3 marks)

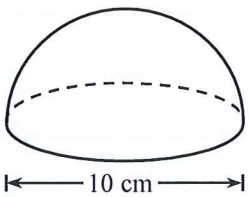


Figure 3

10. Figure 4 shows a right triangular prism *ABCDEF*.

(a) Write down the projection of *AD* on plane *ABFE*.

(b) Name the angle between *AD* and plane *BCDF*.

(c) Name the angle between planes *ABD* and *ABFE*.

(3 marks)

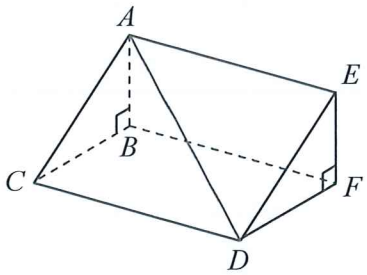


Figure 4

Section A(2) (27 marks)

11. By using Figure 5, draw the orthographic views for the following solid on grid paper, each grid represents dimension 1 cm x 1 cm. (3 marks)

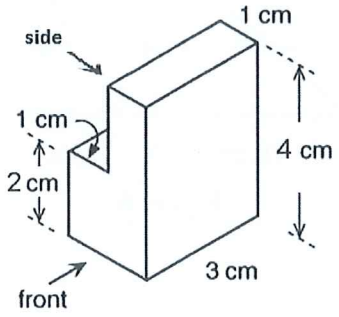
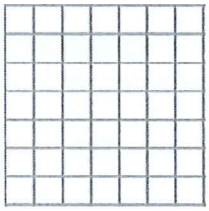
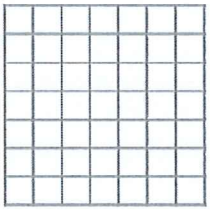


Figure 5

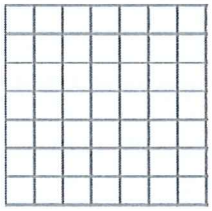
Front View



Top View



Side View



12. The volume of a sphere is 36π cm^3. Find the radius and the surface area of the sphere. (Express your answers in terms of π if necessary.) (4 marks)

13. If $5^{x+1}+5^{x-1}-26=0$, find the value of x. (4 marks)

14. In Figure 6, D is a point on BC. AD is the angle bisector of $\angle BAC$.
$AC = 12$ cm, $BD = 10$ cm and $DC = 8$ cm.

(a) Prove that $\Delta ABC \sim \Delta DAC$.

(b) Find AD.

(5 marks)

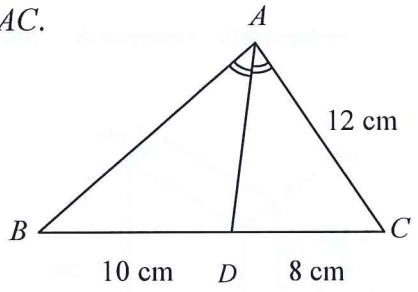


Figure 6

15. Consider two similar cuboids P and Q. The ratio of the total surface area of cuboid P to that of cuboid Q is 9 : 25.

    (a) Find the ratio of the volume of cuboid P to that of cuboid Q.

    (b) Cuboids P and Q are melted together and then recast to cuboid R. It is known that the volume of cuboid R is 380 cm^3. Find the volume of cuboid P.

(5 marks)

16. Figure 7 shows a right frustum $PQRSTUVW$. The upper base and the lower base are rectangles of dimensions 12 cm × 9 cm and 16 cm × 12 cm respectively. The height of the frustum is 20 cm. Find the volume of the frustum. (6 marks)

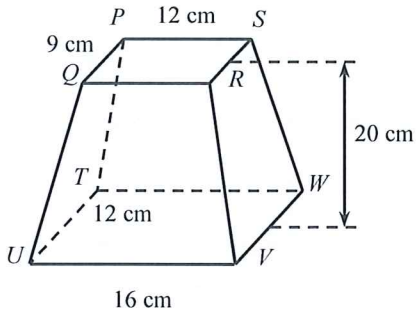


Figure 7

Section B (20 marks)

17. In Figure 8, $ADEC$ and BFC are straight lines. BD is the altitude of ΔABC on AC. EF is the perpendicular bisector of AC.

(a) Find BD.

(b) (i) Prove that $\Delta CEF \sim \Delta CBA$.

(ii) Find EF.

(c) Sam claims that $\angle ABD = \angle ACB$.
Do you agree with him?
Explain your answer.

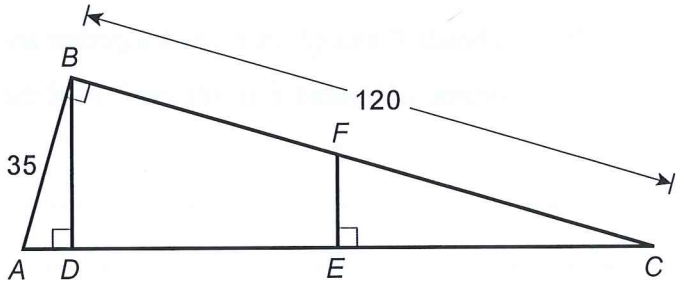


Figure 8

(12 marks)

18. Figure 9 shows a solid which is formed by combining a right circular cone and a right cylinder. The base radii of the cone and the cylinder are both 6 cm. The height of the cylinder is 7 cm and the slant height of the cone is l cm. It is given that the total surface area of the solid is at least 180π cm^2.

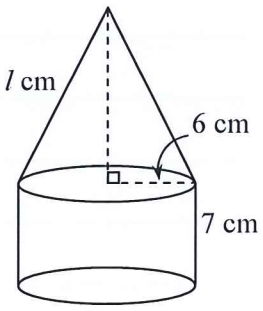


Figure 9

(a) Find the range of values of l.

(b) Using the result of (a), find the minimum volume of the solid. (Leave the answer in terms of π.)

(8 marks)

Multiple-choice Questions (20 marks)

Write down the correct answers into the boxes.

19.	20.	21.	22.	23.	24.	25.	26.	27.	28.

19. If $a > 0 > b$ and k is a negative odd integer, which of the following must be true?

I. $\frac{a^2}{k} > \frac{b^2}{k}$

II. $a + k > b + k$

III. $a^k > b^k$

A. I and II only

B. II and III only

C. I and III only

D. I, II and III

20. In each of the following, the lengths of three line segments are shown. Which set of line segments cannot form a triangle?

I. 25, 11, 12

II. 33, 13, 21

III. 35, 16, 18

A. II only

B. I and III only

C. II and III only

D. I, II and III

21. In Figure 10, the net is folded into a cuboid. Which of the following pairs of letters shown on the faces are opposite to each other?

A. P and Q

B. Q and R

C. U and T

D. R and T

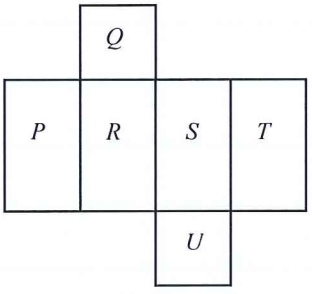


Figure 10

22. $4^{2m} \cdot 32^{n} =$

A. 2^{4m+5n}.

B. 16^{2m+2n}.

C. 32^{2m+n}.

D. 128^{2m+n}.

23. Figure 11 shows a solid made up of 6 identical cubes.

Which of the following must be true?

I. The solid has 2 planes of reflection.

II. The solid has 1 axis of 2-fold rotational symmetry.

III. The solid has 1 axis of 4-fold rotational symmetry.

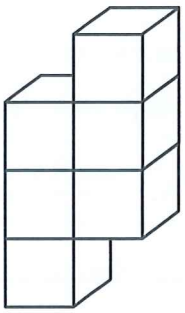


Figure 11

A. I only

B. II only

C. I and III only

D. II and III only

24. $2^{10}+5\times2^{7}+7\times2^{4}-2^{4} =$

A. 11011100000_2.

B. 11001110000_2.

C. 11011100001_2.

D. 11010110111_2.

25. In Figure 12, $ABCD$ is a tetrahedron with $AB \perp BD$, $AB \perp BC$ and $AC = AD$. M and N are mid-points of AB and CD respectively.

Which of the following is the angle between planes ABC and ADB?

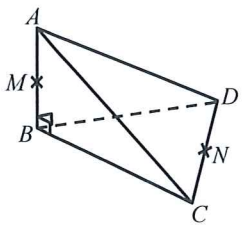


Figure 12

A. $\angle ANB$

B. $\angle CMD$

C. $\angle CBD$

D. $\angle CAD$

26. In Figure 13, AEB, ADC and BFC are straight lines. $CDEF$ is a parallelogram and $AE : EB = 4 : 7$. If the area of $CDEF$ is 84 cm^2, then the area of $AEFC$ is

A. 97.5 cm^2.

B. 108 cm^2.

C. 157.5 cm^2.

D. 181.5 cm^2.

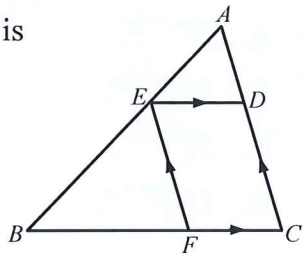


Figure 13

27. In Figure 14, the sector AOB (O is the centre of the sector) is folded to form a paper cup in the shape of a circular cone by joining OA and OB together. Find the capacity of the cup formed.

A. $\frac{9\pi\sqrt{15}}{8}$ cm^3

B. 9π cm^3

C. $\frac{27\pi\sqrt{15}}{8}$ cm^3

D. 72π cm^3

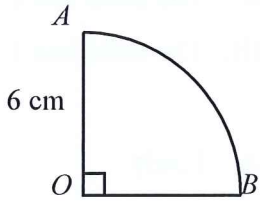


Figure 14

28. In Figure 15, AED is a straight line and E is the centroid of ΔABC. It is given that $BE = CE$. Which of the following must be true?

I. The in-centre of ΔABC lies on AD.

II. The circumcentre of ΔABC lies on AD.

III. ΔABC is an isosceles triangle.

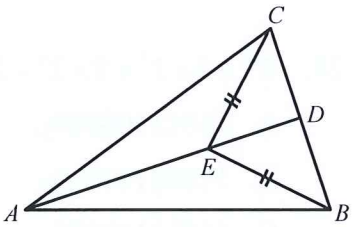


Figure 15

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

Bonus Question (4 marks)

29. ΔOPQ is an obtuse-angled triangle. Denote the in-centre and the circumcentre of ΔOPQ by I and J respectively. It is given that P, I and J are collinear. Prove that $OP = PQ$. (4 marks)

— END OF PAPER —