S3 First Term Examination (2016-2017)

Mathematics

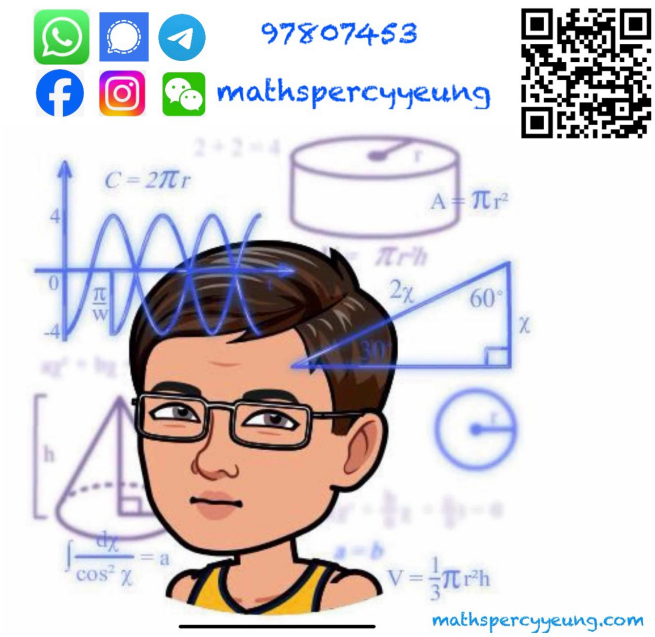
(2 hours)

Date: 11th January 2017 Name: _____

Time: 8:30 a.m. - 10:30 a.m. Class: _____ No.:_____

Instructions to students:

1. This paper consists of THREE parts, Conventional Questions, Mulitple-Choices Questions and Bonus Question. There are Section A(1), Section A(2) and Section B in Conventional Questions.
2. The maximum score of this paper is 100.
3. Attempt ALL questions. Write your answers in the spaces provided in this Question / Answer Book.
4. Unless otherwise specified, show all workings clearly.
5. Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.
6. The diagrams in this paper are not necessarily drawn to scale.



Section A(1) (35 marks)

1. Make a the subject of the formula $5a = 2(a - 3b) + c$.

(3 marks)

2. Simplify $\dfrac{(a^4b^{-3})^2}{a^2b^{-5}}$ and express your answer with positive indices.

(3 marks)

3. Factorize

   (a) $3c^2 + 10c - 8$,

   (b) $3cd + 12d + 3c^2 + 10c - 8$.

(3 marks)

4. (a) Solve the inequality $\frac{x+3}{4}-\frac{2}{3}<\frac{x}{5}$ and represent the solutions on a number line.

(b) If x is an integer, find the largest value of x.

(4 marks)

5. Winnie deposited \$35 000 in a bank. The interest rate was 5% p.a. compounded yearly. After 2 years, find

(a) the amount that Winnie would receive,

(b) the interest that Winnie would receive.

(4 marks)

6. If the price of a postcard is increased by 60% and then decreased by 80%, find the percentage change in the price of the postcard.

(3 marks)

7. In Figure 1, three lampposts A, B and C are at the same horizontal ground level. A is 80 m due south of B. C is 150 m due east of B. Find the true bearing of
(a) C from A,
(b) A from C.
(Give the answers correct to the nearest degree.)

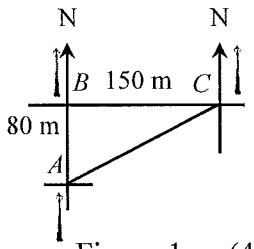


Figure 1 (4 marks)

8. Given that $\cos(90° - \theta) = \frac{2}{7}$, find the value of $\frac{1-\cos(90° - \theta)}{\sin^2(90° - \theta)} + \sin\theta$.

(4 marks)

9. Davis uses the scale shown in Figure 2 to measure the weight of a watermelon. The measured weight is 5 kg. Find the relative error of the measurement.

(3 marks)

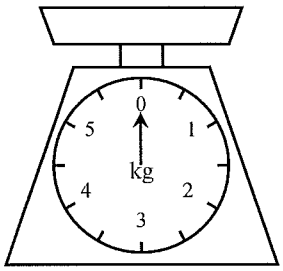


Figure 2

10. In Figure 3, Ben walks up from check point A to check point C through check point B. The inclination of AB and BC are 19° and 30° respectively. Check point A is at the sea level. Check point B and check point C are 40 m above the sea level and 100 m above the sea level respectively. Find the total distance that Ben travels.

(4 marks)

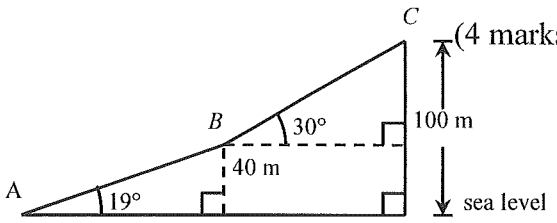


Figure 3

Section A(2) (25 marks)

11. (a) Factorize $a^5 - a^2$.

(b) Using the result of (a), factorize $x^5 + x + 1$.

(4 marks)

12. The measured volume of a bottle of fruit juice is 1.5 L. The scale interval of the measuring tool is 0.05 L.
    (a) Find the range of the actual volume of the bottle of fruit juice.
    (b) If 6 bottles of fruit juice are measured together, is it possible that the measured result is 9.13 L? Explain your result.

(5 marks)

13. In Figure 4, a lamppost AB stands on a horizontal ground. Two persons stand at P and Q on the ground and the angles of elevation of the top A of the lamppost from the two persons are 33° and 67° respectively. If the horizontal distance between P and Q is 12 m, find the height of the lamppost AB.

(5 marks)

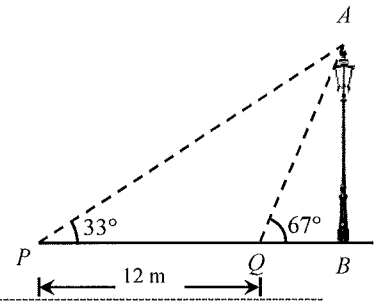


Figure 4

14. Anthony wants to deposit \$350 000 in a bank for 5 years. There are two plans for him to choose from.

Plan *A*: Compound interest at an interest rate of 5% p.a., compounded quarterly.

Plan *B*: Simple interest at an interest rate of 7% p.a. for the first year. In every subsequent year, compound interest at an interest rate of 4.5% p.a., compounded monthly.

(a) Find the amount that Anthony will receive after 5 years, correct to the nearest hundred dollars, if he chooses

(i) plan *A*, (ii) plan *B*.

(b) Which plan should Anthony choose to earn more interest?

(6 marks)

15. In Figure 5, ADC and BEC are straight lines. BD is the angle bisector of $\angle ABC$. AE meets BD at F. AE and AB are altitudes of ΔABC. $\angle ACB = 32°$.

(a) Find $\angle FBE$.

(b) Find $\angle AFD$.

(5 marks)

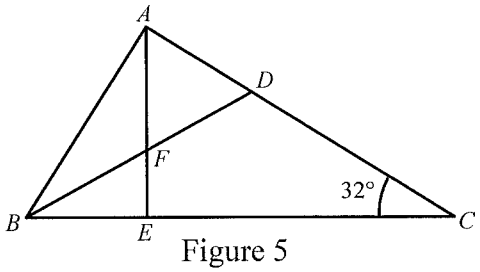


Figure 5

Section B (20 marks)

16. At the beginning, car *A* is 10 km due west of car *B*. Afterwards, car *A* travels in the direction N30°E at a constant speed 70 km/h and car *B* travels in the direction N55°W at a constant speed 110 km/h. Find the time required (in minutes) for the two cars to meet each other. (*Give the answer correct to the nearest minute*.)

(6 marks)

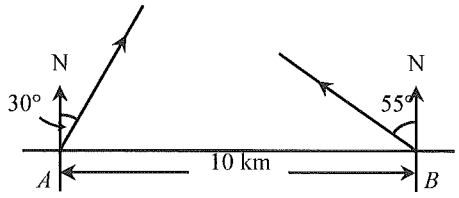


Figure 6

17. In Figure 7, $\triangle ABC$ is a right-angled triangle and BDC is a straight line.

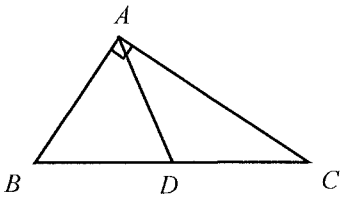


Figure 7

(a) If $AD = CD$, prove that AD is a median of $\triangle ABC$.

(b) Given that AD is a median of $\triangle ABC$, someone claims that $\triangle ACD$ is an isosceles triangle. Do you agree? Explain your answer.

(6 marks)

18. The monthly salary of Eric is \$40 000, and he is entitled to an annual tax allowance of \$108 000. To prepare for his annual tax payment, Eric would, three months before the payment is due, begin depositing a fixed amount of money into a bank account at an interest rate of 3.6% p.a., compounded monthly. How much does Eric have to deposit every month so that he saves enough for the tax payment?

(8 marks)

Multiple-Choice Questions (20 marks)

Write down the correct answers into the boxes.

19.	20.	21.	22.	23.	24.	25.	26.	27.	28.

19. $49s^2 - 56st + 16t^2 - 21s + 12t =$

A. $(7s - 4t)(7s - 4t - 3)$.

B. $(7s - 4t)(7s - 4t + 3)$.

C. $(7s - 4t)(7s + 4t - 3)$.

D. $(7s + 4t)(7s - 4t + 3)$.

20. In Figure 8, $\angle BAC = \angle ADC = 90°$. If $AB = l$, then $AC =$

A. $l \sin \theta$.

B. $l \tan \theta$.

C. $l \sin \theta \cos \theta$.

D. $\dfrac{l \sin \theta}{\tan \theta}$.

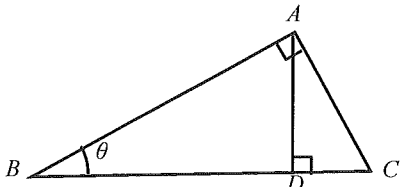


Figure 8

21. If $\sin(\theta + 25°) = \cos 14°$, then $\theta =$

A. 39°. B. 51°.

C. 65°. D. 76°.

22. Given that A and B are acute angles such that $A + B = 90°$, which of the following is / are correct?

I. $\sin A \cos B = \cos A \sin B$

II. $\tan A \tan B = 1$

III. $\sin^2 A + \cos^2 B = 1$

A. I only B. II only

C. I and III only D. II and III only

23. In Figure 9, the bearing of P from O is N56°W and the bearing of Q from O is N36°E. If P and Q are equidistant from O, then the bearing of P from Q is

A. N74°E.

B. N80°E.

C. S74°W.

D. S80°W.

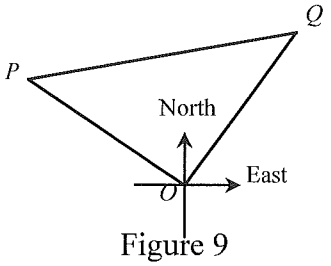


Figure 9

24. The time that Rachel takes to swim 100 m is measured as 2 minutes 30 seconds. If the relative error of the measurement is $\dfrac{1}{60}$, the actual time taken must lie between

A. 2 minutes 29.5 seconds and 2 minutes 30.5 seconds.

B. 2 minutes 27.5 seconds and 2 minutes 32.5 seconds.

C. 2 minutes 29.25 seconds and 2 minutes 30.25 seconds.

D. 2 minutes 27.25 seconds and 2 minutes 32.25 seconds.

25. The population of a city is estimated as 500 000. If the percentage error is 3%, which of the following cannot be the actual population of the city?

A. 486 550
B. 497 320
C. 510 240
D. 515 320

26. In Figure 10, AED is a straight line and E is the centroid of $\triangle ABC$.

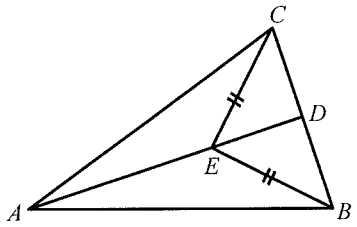


Figure 10

It is given that $BE = CE$. Which of the following must be true?

I. The in-centre of $\triangle ABC$ lies on AD.
II. The circumcentre of $\triangle ABC$ lies on AD.
III. $\triangle ABC$ is an isosceles triangle.

A. I and II only
B. I and III only
C. II and III only
D. I, II and III

27. In Figure 11, $\triangle ABC$ is triangle with $a > c > b$.

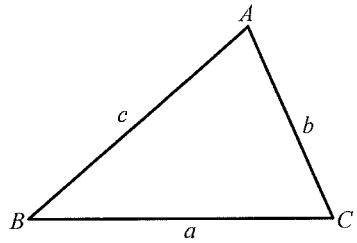


Figure 11

Which of the following must be true?

I. $a + b > c$
II. $b + c > a$
III. $b^2 + c^2 > a^2$

A. I only
B. I and II only
C. I and III only
D. I, II and III

28. The salaries tax payable of Peter is \$24 040. Find his net chargeable income. (For the salary tax rate, please refer to the Appendix.)

A. \$15 640
B. \$92 000
C. \$116 040
D. \$212 000

Bonus Question (5 marks)

29. If $AB:BE = 1:8$, $AC:CF = 1:4$, prove that AF is the median of ΔAED.

(5 marks)

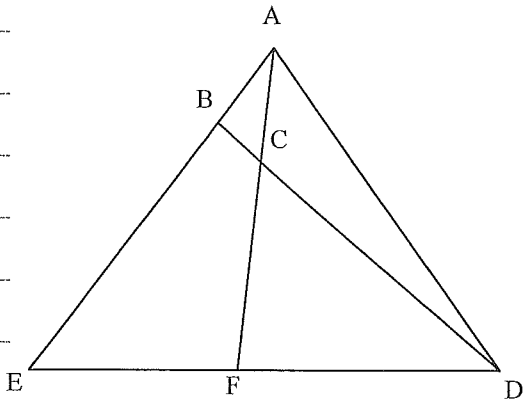


Figure 12

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Appendix

For the salary tax problem(s), candidates can take the following table as reference:

Net chargeable income	Salaries tax rates
First $40000	2%
Next $40000	7%
Next $40000	12%
Remainder	17%