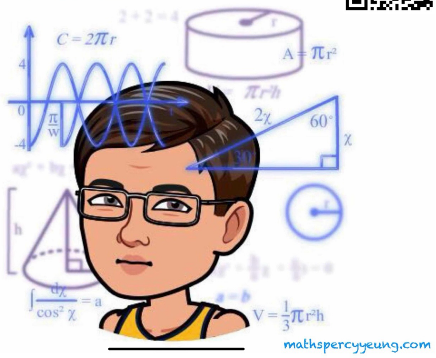
2018-19 F.4
MATH EP
M2



2018 - 2019
Form 4 2nd Term Uniform Test

MATHEMATICS Extended Part
Module 2 (Algebra and Calculus)

Question–Answer Book

27th March, 2019. (Wednesday)

9:30 – 10:30 a.m. (1 hour)

This paper must be answered in English.

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your name, class and class number in the spaces provided on this cover.
2. Answer ALL questions. Write your answers in the spaces provided in this Question-Answer Book.
3. Supplementary answer sheets will be supplied on request. Write your name, class, class number and mark the question number box on each sheet.
4. Unless otherwise specified, all working must be clearly shown.
5. Unless otherwise specified, numerical answers must be exact.

Grand Total	**/ 42**

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

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1. Prove that $1+\cos(\pi-2\theta)-2\cos^4\left(\frac{\pi}{2}+\theta\right) \equiv \frac{\sin^2 2\theta}{2}$. (4 marks)

Answers written in the margins will not be marked.

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2. **(a)** Prove that $\dfrac{\cos(A+B)+\cos A}{\sin(A+B)-\sin A}=\cot\dfrac{B}{2}$. (3 marks)

(b) Using **(a)**, find the value of $\dfrac{\sin 20^\circ+\cos 10^\circ}{\cos 20^\circ-\sin 10^\circ}$ without using a calculator. (3 marks)

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3. **(a)** Using mathematical induction, prove that

$$\cos\theta+\cos3\theta+\cos5\theta+\cdots+\cos(2n-1)\theta=\frac{\sin2n\theta}{2\sin\theta}\text{, where }\sin\theta\neq0,$$

for all positive integers n. (6 marks)

(b) It is given that $\sin\frac{\pi}{12}=\frac{\sqrt{6}-\sqrt{2}}{4}$. Using **(a)**, evaluate $\sum_{k=1}^{404}\cos\frac{(2k-1)\pi}{12}$. (3 marks)

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4. Find the following limits.

(a) $\lim_{x\to 5}\frac{x-5}{\sqrt{x+20}-5}$ (3 marks)

(b) $\lim_{x\to 0}\frac{\cos^2 2x+2x^2-1}{1+\sin^2 3x-\cos^2 3x}$ (3 marks)

(c) $\lim_{x\to\infty}\left(\sqrt{4x^2+3x}-2x\right)$ (3 marks)

(d) $\lim_{x\to\infty}\left(\frac{x^2-x-6}{x^2+7x-8}\right)^x$ (3 marks)

(e) $\lim_{x\to 0}\cot 2x\left(e^{3x}-1\right)$ (3 marks)

(f) $\lim_{x\to 0}\frac{e^{3x}-e^{2x}-e^x+1}{x^2}$ (3 marks)

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5. **(a)** Prove the identity $\cot x = \dfrac{\sin 2x}{1-\cos 2x}$. (2 marks)

(b) Using **(a)**, prove the identity $\cot y = \dfrac{\sin 8y}{1-\cos 8y} \cdot \dfrac{1-\cos 4y}{\cos 4y} \cdot \dfrac{\cos 2y}{1-\cos 2y}$. (3 marks)

END OF PAPER

Answers written in the margins will not be marked.

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