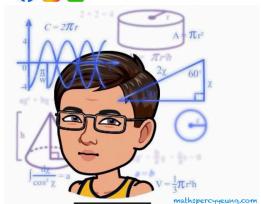# TT F4 Ch1 Number System-Notes

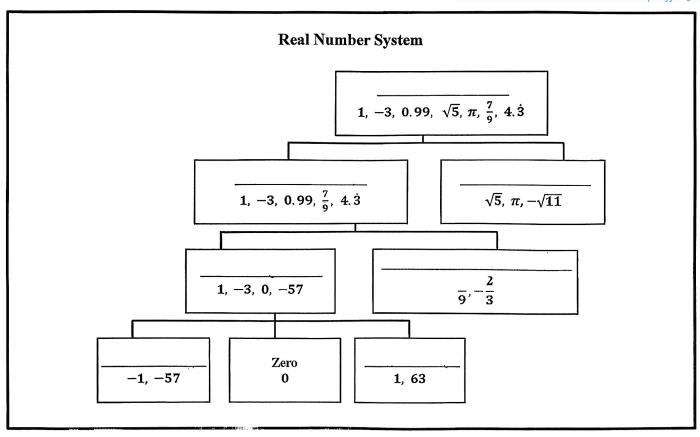**I.** <u>Section 1.1</u>   <u>Real Number System</u>

A. Pre-lesson:

1. Refer to P.1 – 6, complete the following diagram.

Real Number System

$$1, -3, 0.99, \sqrt{5}, \pi, \frac{7}{9}, 4.\dot{3}$$

$$1, -3, 0.99, \frac{7}{9}, 4.\dot{3}$$

$$\sqrt{5}, \pi, -\sqrt{11}$$

$$1, -3, 0, -57$$

$$\frac{7}{9}, -\frac{2}{3}$$

$$-1, -57$$

Zero
$$0$$

$$1, 63$$

2. Refer to P.1 – 6, complete the following table by putting "✓" in the correct spaces.

		$\frac{1}{12}$	$1.\dot{2}$	-3	$\frac{\pi}{5}$	$\sqrt{5}$	$\sqrt{9}$	$\tan 45°$	$\tan 60°$	$\sin 30°$	$\sin 50°$
(a)	**Natural number**										
(b)	**Integer**										
(c)	**Rational**										
(d)	**Irrational number**										
(e)	**Terminating decimals**										
(f)	**Recurring decimals**										

3. Refer to P.4, convert $0.\overset{\circ}{3}$ into a fraction.

B. Teaching Example and Classwork:

Level 2

1. Convert each of the following recurring decimals into a fraction.

(a) $0.5\overset{\bullet\bullet}{18}$

(b) $0.\overset{\bullet}{5}1\overset{\bullet}{8}$

2. Determine whether the result of each of the following is a rational number or an irrational number.

(a) $\dfrac{\sqrt{9}}{2} + \dfrac{1}{3}$

(b) $4\pi + 2\pi$

(c) $\sqrt{2 + \dfrac{1}{4}}$

(d) $(1 - \sqrt{6})^2$

Level 3

3. Determine whether each of the following statements must be true. If the statement may not be true, give an example to justify your answer.

(a) The difference of two irrational numbers is an irrational number.

(b) The square of an irrational number is an irrational number.

II. Section 1.2 Complex Number System

A. Pre-lesson:

1. Refer to P.7 - 9, complete the following summary.

> **Imaginary Numbers and Complex Numbers**
> **(a)** The square roots of a negative number is called an imaginary numbers, such as $\sqrt{-1}$, $\sqrt{-2}$ and $\sqrt{-4}$.
> **(b)** $i = \underline{\quad\quad}$ is called the imaginary unit.
> **(c)** $\sqrt{-N} = \sqrt{N}i$, where N is a positive real number.
> **(d)** A complex number can be written in the form $a + bi$. The real numbers a and b are called the real part and the imaginary part respectively.
>
> $$a + bi$$
>
> $\underline{\quad real \ part \quad}$ $\underline{\quad imaginary \quad}$

2. Identify the real part and the imaginary part of each of the following complex numbers.

		Real part	Imaginary part
(a)	$2 - 8i$		
(b)	$6i - \sqrt{7}$		
(c)	$-2i$		
(d)	$\sqrt{-4} - 5$		

3. Classify the following numbers, complete the following table by putting "✓" in the correct spaces.

		Integers	Rational numbers	Real numbers	Complex numbers
(a)	$3.\dot{2}$				
(b)	-7				
(c)	7π				
(d)	$\sqrt{-6} + \dfrac{1}{2}$				

B. Teaching Example and Classwork:

Level 2

1. The imaginary part of the complex number $(5 - 2x) + (3x - 2)i$ is 1, where x is a real number.
   (a) Find the value of x.
   (b) Find the real part of the complex number.

2. The imaginary part of the complex number $(x - 5) + 4xi$ is 4, where x is a real number.
   (a) Find the value of x.
   (b) Find the real part of the complex number.

3. The real part of the complex number $14 - 3x - (7x - 1)i$ is 20, where x is a real number.
   (a) Find the value of x.
   (b) Find the imaginary part of the complex number.

4. Let $z = (2 - 3x) + (2x + 1)i$, where x is a real number. Find the value of x such that
   (a) z is a real number,
   (b) z is an imaginary number.

III. Section 1.3   Operations of Complex Numbers

A. Pre-lesson:

1. Video from Identity: https://youtu.be/4iVQMSxC4RY

    00:05 – addition and subtraction of complex numbers
    02:06 – multiplication and division of complex numbers
    09:04 – complex numbers on calculator

2. Pattern of i^n.

Refer to P. 14-15, simplify the following expressions.

(a) i^{13}	(b) $3i^5 \times 2i^7$	(c) $\dfrac{12i^6}{6i^4}$

3. Operations of complex numbers (Addition, Substraction and Multiplication)
    Refer to P. 17-18, simplify the following expressions.

   (a) $(3 + 2i) + (8 - 5i)$

   (b) $(3 - 2i) + (7 + i) - (4 - 6i)$

   (c) $(3 + i)(2 + 5i)$

   (d) $(4 - i)^2$

B. Teaching Example and Classwork:

Level 1

1. Simplify the following expressions and express the answer in the form $a + bi$.

   (a) $(2 + 5i) + (1 - 3i) - (5 - 4i)$

   (b) $(2 + 5i)(3 - 2i)$

   (c) $(-4i + 3)^2$

   (d) $(4 - 3i)(4 + 3i)$

2. Division of Complex Numbers

   Simplify the following expressions and express the answers in the form $a + bi$.

Case 1 Denominator is a non-zero real number	Case 2 Denominator is an imaginary number	Case 3 Denominator is a complex number
(a) $\dfrac{12-3i}{3}$	(b) $\dfrac{1+i}{2i}$	(b) $\dfrac{25}{4-3i}$

3. Simplify the following expressions and express the answers in the form $a + bi$.

   (a) $\dfrac{7+21i}{14}$

   (b) $\dfrac{-2+6i}{3i}$

   (c) $\dfrac{1}{3+2i}$

   (d) $\dfrac{2-3i}{2+3i}$

4. Find the real part of $\dfrac{2i^5 - 3i^8 + i^7}{3+i}$.

5. Find the imaginary part of $\dfrac{3i^{11} - 4i^5 - 6i^6}{4+i}$.

6. Simplify the following expressions and express the answers in the form $a + bi$.

(a) $\dfrac{(2+2i)(-3+5i)}{-1+i}$

(b) $\left(\dfrac{1+2i}{2-i}\right)^2$

> **Equality of Complex Numbers**
>
> If $a + bi = c + di$, then $a = c$ and $b = d$.

7. If $(y - xi) + (2x + yi) = 1 + 4i$, find the values of the real numbers x and y.

8. If $(-y + 2xi) + (4x + yi) = 9 + 3i$, find the values of the real numbers x and y.

9. If $(2a+3)+4bi=0$, find the values of the real numbers a and b.

10. If $(2+3i)(2+ai)=4+3b$, find the values of the real numbers a and b.

11. If $\dfrac{4+ai}{2-i} = b - 4i$, find the values of the real numbers a and b.

12. Let $z_1 = (k - 1 + i)(1 + i)$ and $z_2 = \dfrac{-k + 5i}{1+i}$, where k is a real number.

    (a) Express z_1 and z_2 in the form $a + bi$.

    (b) If the imaginary parts of z_1 and z_2 are equal, find the value of k.

13. If the result of $(k + 2i)(3 - 5i)$ is a real number, find the value of the real number k.

14. If the result of $\dfrac{2 - 5i}{a - i}$ is an imaginary number, find the value of the real number a.

1. (a) Express $\dfrac{1}{1+2i}$ in the form of $a+bi$, where a and b are real numbers.   (2 marks)

    (b) The roots of the quadratic equation $x^2 + px + q = 0$ are $\dfrac{10}{1+2i}$ and $\dfrac{10}{1-2i}$. Find

         (i) p and q ,

         (ii) the range of values of r such that the quadratic equation $x^2 + px + q = r$ has real roots.

                                                                                              (5 marks)
                                                                                    [HKDSE PP #17]

2. The real part of $i + 2i^2 + 3i^3 + 4i^4$ is

         A. 2 .

         B. –2 .

         C. 6 .

         D. –6 .

                                                                                    [HKDSE 13 #36]

3. If β is a real number, then $\dfrac{\beta^2 + 4}{\beta + 2i} =$

         A. $\beta - 2i$.

         B. $\beta + 2i$.

         C. $2 - \beta i$.

         D. $2 + \beta i$.

                                                                                    [HKDSE 14 #36]

4. Let $z = (a+5)i^6 + (a-3)i^7$, where a is a real number. If z is a real number, then $a =$

    A. -5 .
    B. -3 .
    C. 3 .
    D. 5 .

5. Let $u = \dfrac{7}{a+i}$ and $v = \dfrac{7}{a-i}$, where a is a real number. Which of the following must be true?

    I. uv is a rational number.
    II. The real part of u is equal to the real part of v .
    III. The imaginary part of $\dfrac{1}{u}$ is equal to the imaginary part of $\dfrac{1}{v}$.

    A. I only
    B. II only
    C. I and III only
    D. II and III only

6. If k and $\dfrac{5}{2-i} + ki$ are real numbers, then $k =$

    A. -2 .
    B. -1 .
    C. 1 .
    D. 2 .

7. The real part of $\dfrac{2i^{12} + 3i^{13} + 4i^{14} + 5i^{15} + 6i^{16}}{1-i}$ is

    A. -3 .
    B. -1 .
    C. 1 .
    D. 3 .

8. If a is a real number, then the real part of $\dfrac{4+i^5}{a+i} - i^6$ is

    A. $\dfrac{4a+1}{a^2-1}$.

    B. $\dfrac{4a+1}{a^2+1}$.

    C. $\dfrac{a^2+4a+2}{a^2-1}$.

    D. $\dfrac{a^2+4a+2}{a^2+1}$.

9. Define $z_1 = \dfrac{2+ki}{1+i}$ and $z_2 = \dfrac{k+5i}{2-i}$, where k is a real number. If the imaginary part of z_1 is equal to the imaginary part of z_2 , then $z_1 - z_2 =$

    A. -20 .

    B. 0 .

    C. 3

    D. 10 .

10. Let α be real number. Define $u = w + \dfrac{1}{w}$ and $v = w - \dfrac{1}{w}$, where $w = \dfrac{\alpha+i}{\alpha-i}$. Which of the following must be true?

    I. u is a real number.

    II.   The real part of v is equal to 0 .

    III. The imaginary part of w is equal to the imaginary part of $2w$.

    A. I and II only

    B. I and III only

    C. II and III only

    D. I, II and III

12. Let $z = 4 + 5i^{10} - ki^{15} + 6i^{21} + 2ki^{28}$, where k is a   real number. If the real part and the imaginary part of z are equal, then the real part of z is

   A.   7

   B.   13 .

   C.   17 .

   D.   25 .

13. If k is a real number, then the real part of $\dfrac{i}{k-i} + \dfrac{2}{k+i}$ is

   A. $\dfrac{2k+1}{k^2-1}$

   B. $\dfrac{2k-1}{k^2+1}$

   C. $\dfrac{k+2}{k^2-1}$

   D. $\dfrac{k-2}{k^2+1}$

14. Let $z = (a-5)i + \dfrac{(a+2)i}{2+i}$. If a and z are real numbers, then $a - z =$

   A.   2 .

   B.   3 .

   C.   4 .

   D.   5 .