2023-2024 S5
1st TERM UT
MATH EP
M2

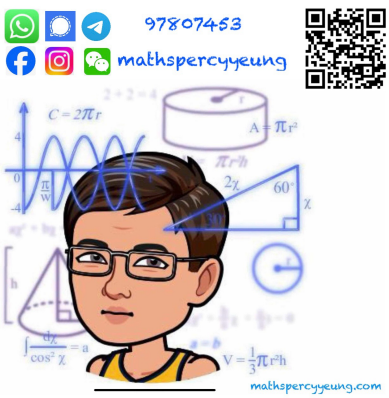
2023 – 2024
S5 First Term Uniform Test

MATHEMATICS Extended Part

Module 2 (Algebra and Calculus)

Question–Answer Book

9th November, 2023
9:30 am – 10:30 am (1 hour)
This paper must be answered in English



Section	Marks
A Total	**/32**
B Total	**/14**
TOTAL	**/46**

INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.

2. This paper consists of TWO sections, A and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question – Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers must be exact.

6. The diagrams in this paper are not necessarily drawn to scale.

FORMULAS FOR REFERENCE

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$2\sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A \sin B = \cos(A-B) - \cos(A+B)$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

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Section A (31 marks)

1. Let $f(x) = \sqrt{x+3}$ for all $x \geq -3$. Find $f'(1)$ from the first principles.

(4 marks)

Answers written in the margins will not be marked

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2. The rate of inflation of a spherical balloon is 0.5 m^3/s. When the radius of the balloon is 1.5 m, find, with respect to time, the rate of change of

(a) its radius,

(b) its surface area.

(5 marks)

Answers written in the margins will not be marked

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3. Find the equation of tangent to the curve $x \ln y + y = 2$ at the point where the curve cuts the y-axis. (5 marks)

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4. Find the following integrals.

(a) $\int 6x^2(2x^3-1)^5\,dx$

(b) $\int \frac{e^{2x}}{e^{2x}+8}\,dx$

(c) $\int \tan^4 2x\,dx$

(7 marks)

Answers written in the margins will not be marked

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5. Find the following integrals.

(a) $\int \frac{1}{\sqrt{1-4x^2}}\,dx$

(b) $\int \frac{8x}{\sqrt{1-4x^2}}\,dx$

(7 marks)

Answers written in the margins will not be marked

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6. Find $\int e^x \cos 2x \, dx$.

(4 marks)

Answers written in the margins will not be marked

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Section B (14 marks)

7. Let $f(x) = x - \dfrac{x}{x+1}$, where $x \neq -1$.

(a) Find $f'(x)$ and $f''(x)$, where $x \neq -1$. (3 marks)

(b) **(i)** Find the relative extreme points of the graph of where $y = f(x)$.

(ii) Show that the graph of $y = f(x)$ does not have any point of inflexion.

(6 marks)

(c) Find the asymptote(s) of the graph of $y = f(x)$.

(2 marks)

(d) Sketch the graph of $y = f(x)$.

(3 marks)

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