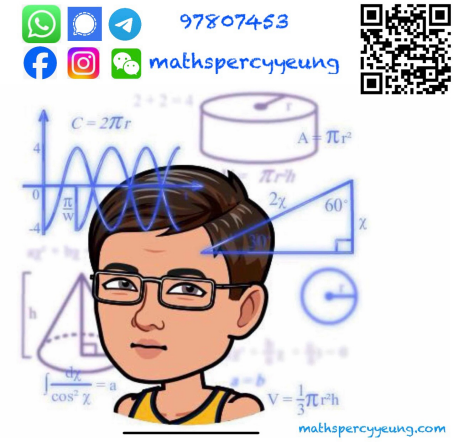
2024-2025 S6
1st TERM UT
MATH EP
M2



2024 – 2025
S6 First Term Uniform Test

MATHEMATICS Extended Part

Module 2 (Algebra and Calculus)

Question–Answer Book

31st October, 2024
9:45 am – 10:45 am (1 hour)
This paper must be answered in English

INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.

2. This paper consists of TWO sections, A and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers must be exact.

6. The diagrams in this paper are not necessarily drawn to scale.

Section	Marks
A Total	**/16**
B Total	**/24**
TOTAL	**/40**
UT	**%**

FORMULAS FOR REFERENCE

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$2\sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A \sin B = \cos(A-B) - \cos(A+B)$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

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Section A (16 marks)

1. It is given that **a** and **b** are two vectors, where $|\mathbf{a}| = 4$, $|\mathbf{b}| = 1$ and the angle between them is $\frac{2\pi}{3}$.

(a) Find the value of $\mathbf{a} \bullet \mathbf{b}$.

(b) If $\mathbf{u} = \mathbf{a} + 2\mathbf{b}$ and $\mathbf{v} = \mathbf{a} - 3\mathbf{b}$, determine whether **u** is orthogonal to **v**.

(5 marks)

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2. A square matrix X is said to be a symmetric matrix if $X^T = X$ and a square matrix Y is said to be a skew-symmetric matrix if $Y^T = -Y$. Let $A = \begin{pmatrix} 2 & 0 \\ 1 & -2 \end{pmatrix}$. Find the skew-symmetric matrix B such that $A - B$ is a symmetric matrix. (4 marks)

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3. Let $A = \begin{pmatrix} 1 & 3 \\ 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 2 & -9 \\ 0 & 5 \end{pmatrix}$ and $Y = A^{-1}BA$.

(a) Find A^{-1}. (3 marks)

(b) Show that Y is a diagonal matrix, and hence, find Y^5. (4 marks)

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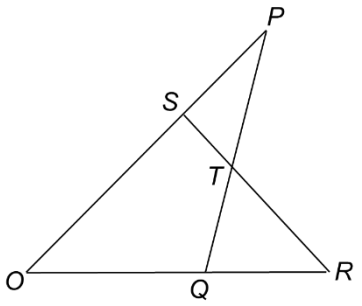
Section B (24 marks)

4. In the figure, OPQ is a triangle. S is a point on OP such that $OS : SP = 2 : 1$. R is a point on OQ produced such that $OQ : QR = 3 : 2$. PQ and RS intersect at T. It is given that $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$.



(a) Express $\overrightarrow{OR}$ and $\overrightarrow{SR}$ in terms of $\mathbf{p}$ and $\mathbf{q}$. (2 marks)

(b) It is given that $\overrightarrow{ST} = 2\lambda\mathbf{p} - 5\lambda\mathbf{q}$ and $\overrightarrow{PT} = \mu\mathbf{p} - \mu\mathbf{q}$.

(i) Find the values of λ and μ.

(ii) Find $\dfrac{PT}{TQ}$ and $\dfrac{ST}{TR}$.

(7 marks)

(c) U is a point on RS produced such that OU // QP. Express $\overrightarrow{OU}$ in terms of $\mathbf{p}$ and $\mathbf{q}$. (3 marks)

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5. Let $\overrightarrow{OP} = \mathbf{i} - 2\mathbf{k}$, $\overrightarrow{OQ} = -3\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{OR} = -5\mathbf{i} + t\mathbf{j} - 4\mathbf{k}$, where O is the origin and t is a constant. It is given that $\left|\overrightarrow{PR}\right| = \left|\overrightarrow{QR}\right|$.

(a) Find t. (3 marks)

(b) Find $\overrightarrow{PQ} \times \overrightarrow{PR}$. (2 marks)

(c) Find the volume of the tetrahedron $OPQR$. (2 marks)

(d) Denote the plane which contains P, Q and R by Π. It is given that X, Y and Z are points lying on Π such that $\overrightarrow{OX} = x\mathbf{i}$, $\overrightarrow{OY} = y\mathbf{j}$ and $\overrightarrow{OZ} = z\mathbf{k}$.

(i) Prove that $xyz \neq 0$.

(ii) Find the area of ΔXYZ.

(5 marks)

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