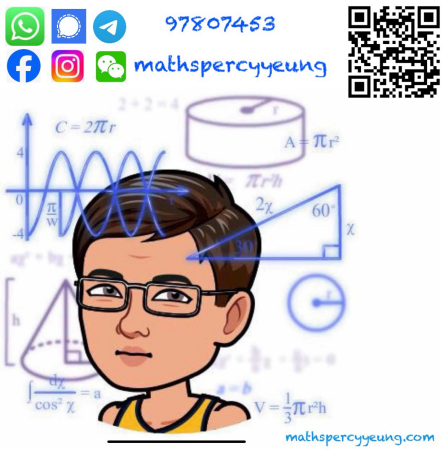
2023-2024 S6
1st TERM UT
MATH EP
M2



2023 – 2024
S6 First Term Uniform Test

MATHEMATICS Extended Part

Module 2 (Algebra and Calculus)

Question–Answer Book

13th November, 2023
10:15 am – 11:15 am (1 hour)
This paper must be answered in English

INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.

2. This paper consists of TWO sections, A and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers must be exact.

6. The diagrams in this paper are not necessarily drawn to scale.

Section	Marks
A Total	**/17**
B Total	**/23**
TOTAL	**/40**
UT	**%**

FORMULAS FOR REFERENCE

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$2\sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A \sin B = \cos(A-B) - \cos(A+B)$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

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Section A (17 marks)

1. It is given that $|\mathbf{a}| = 3$, $|\mathbf{b}| = 4$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{2\pi}{3}$.

(a) Find the value of $\mathbf{a} \cdot \mathbf{b}$.

(b) If $\mathbf{c}$ is a vector such that $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$, find the value of $|\mathbf{c}|$.

(5 marks)

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2. The figure shows a quadrilateral $OABC$. D is a point on BC such that $BD : DC = 1 : k$, where k is positive. Let $\overrightarrow{OA} = 3\mathbf{i} - 6\mathbf{j}$, $\overrightarrow{AB} = 6\mathbf{i} + 5\mathbf{j}$ and $\overrightarrow{OC} = 5\mathbf{i} + 7\mathbf{j}$.

(a) Express $\overrightarrow{OD}$ in terms of k.

(b) If $OD \,//\, AB$, find the value of k.

(5 marks)

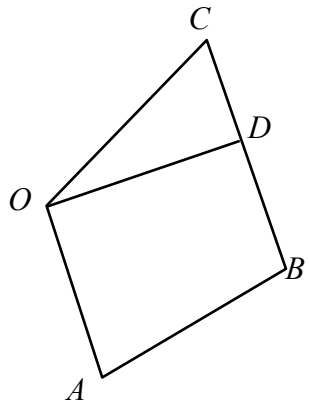

Figure 1

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3. $A\,(-2, 4, 7)$ and $B\,(-1, 3, 8)$ are points lying on the straight line L_1. $P\,(0, 2, 3)$ is a point lying on the straight line L_2 and $L_1 \,//\, L_2$. L_2 passes through a point C such that $AB = PC$ and the x-coordinate of C is positive.

(a) Find the value of $\cos \angle PBA$.

(b) Find the coordinates of C.

(c) It is known that A, B, C and P lie on the same plane. Are A, B, C and P concyclic? Use the results of **(a)** and **(b)** to explain your answer.

(7 marks)

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Section B (23 marks)

4. The position vectors of the points A, B, C and D are $t\mathbf{i}+4\mathbf{j}+s\mathbf{k}$, $8\mathbf{i}+2t\mathbf{j}+2s\mathbf{k}$, $8\mathbf{i}-4\mathbf{j}+14\mathbf{k}$ and $-8\mathbf{i}+6\mathbf{j}-2\mathbf{k}$ respectively, where $s,t\in\mathbf{R}$. Suppose that $\overrightarrow{AB}$ is parallel to $2\mathbf{i}-2\mathbf{j}+\mathbf{k}$. Denote the plane which contains A, B and C by Π.

(a) **(i)** Find s and t.

(ii) Find the area of $\triangle ABC$.

(5 marks)

(b) Let E be the projection of D on Π.

(i) Find $\overrightarrow{DE}$.

(ii) Find the volume of the tetrahedron $ABCD$.

(5 marks)

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5. In Figure 2, D is the mid-point of OB. C is a point on OA such that $OC : CA = m : (1-m)$, where $0 < m < 1$. AD and BC intersect at E such that $BE : EC = 1 : s$ and $DE : EA = 1 : t$, where s and t are positive.
Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

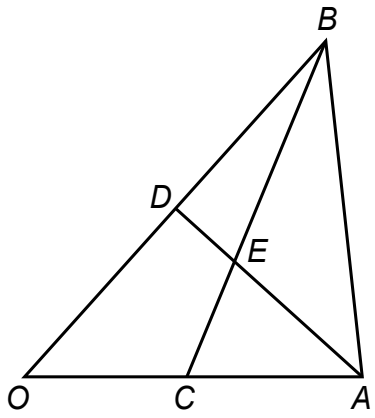

Figure 2

(a) **(i)** By considering ΔOBC, express $\overrightarrow{OE}$ in terms of m, s, $\mathbf{a}$ and $\mathbf{b}$.

(ii) By considering ΔOAD, express $\overrightarrow{OE}$ in terms of t, $\mathbf{a}$ and $\mathbf{b}$.

(iii) Express m and s in terms of t.

(iv) James claims that if $m = s$, E is the centroid of ΔOAB. Do you agree? Explain your answer.

(8 marks)

(b) It is given that $OA = AB$, $4OA = 3OB$ and F is the mid-point of AB.

(i) By considering $AD \perp OB$, prove that $\mathbf{a} \cdot \mathbf{b} = \frac{1}{2}|\mathbf{b}|^2$.

(ii) Is it possible for E to be the circumcentre of ΔOAB? Explain your answer.

(5 marks)

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