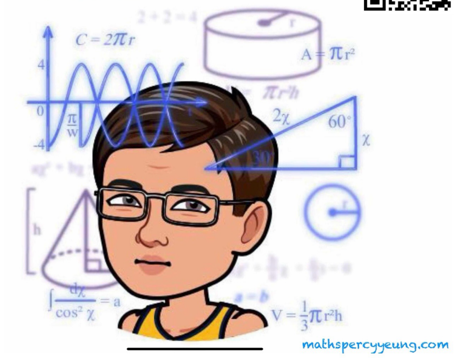
2024-2025 S4
1st TERM EXAM
MATH EP
M2



2024 – 2025
S4 First Term Examination

MATHEMATICS Extended Part

Module 2 (Algebra and Calculus)

Question–Answer Book

13th January, 2025
8:15 am – 9:30 am (1 hour 15 minutes)
This paper must be answered in English

INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.

2. This paper consists of TWO sections, A and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers must be exact.

6. The diagrams in this paper are not necessarily drawn to scale.

Section	Marks
A	/ 31
B	/ 19
TOTAL	/ 50
	%

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

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Section A (31 marks)

1. The coefficient of x^2 in the expansion of $(1-3x)^n$ is 135, where n is a positive integer.

(a) Find the value of n.

(b) Find the coefficient of x^3 in the expansion of $(1-3x)^n$.

(4 marks)

Answers written in the margins will not be marked.

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2. In the expansion of $(1+ax)^2(1+2x)^n$, the coefficients of x and x^2 are 16 and 109 respectively. Find the values of n.

(5 marks)

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3. Using mathematical induction, prove that $\sum_{k=1}^{n}(5k^4+k^2)=\frac{n^2(n+1)^2(2n+1)}{2}$ for all positive integers n.

(5 marks)

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4. Determine whether $f(x)=\dfrac{\sin\left(\dfrac{3\pi}{2}+x\right)}{2x^2+3}$ is an odd function, an even function or neither of them.

(3 marks)

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5. Prove that $\dfrac{\sin\theta+1}{\csc\theta-\cot\theta}+\dfrac{\sin\theta-1}{\csc\theta+\cot\theta}=2(1+\cot\theta)$.

(3 marks)

6. Simplify $\sin(180°-\theta)\sec(270°-\theta)+\cos(360°-\theta)\csc(270°-\theta)$.

(3 marks)

Answers written in the margins will not be marked.

7. Evaluate the following limits.

(a) $\lim_{x \to 5} \frac{x^2 - 25}{5x - 25}$ **(b)** $\lim_{x \to 0} \frac{e^{4x} - 1}{\sin \frac{x}{2}}$ **(c)** $\lim_{x \to \infty} \frac{1}{3x - \sqrt{9x^2 + x}}$

(8 marks)

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Section B (19 marks)

8. **(a)** Prove that $\cos 3x = 4\cos^3 x - 3\cos x$.

(3 marks)

(b) Prove that $\dfrac{\cos 3\left(x-\dfrac{\pi}{4}\right)}{\cos\left(x-\dfrac{\pi}{4}\right)} = \dfrac{\sin 3x - \cos 3x}{\sin x + \cos x}$.

(2 marks)

(c) Solve the equation $\dfrac{\sin 3x - \cos 3x}{\sin x + \cos x} = -2$, where $\dfrac{\pi}{2} < x < \dfrac{3\pi}{4}$.

(4 marks)

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9. **(a)** **(i)** Show that $a^n+b^n=(a+b)\left(a^{n-1}+b^{n-1}\right)-ab\left(a^{n-2}+b^{n-2}\right)$, where a and b are real numbers.

(ii) Let $x+\dfrac{1}{x}=2\cos\theta$, where $\theta \neq k\pi$ and x is a non-zero real number. Suppose $x^k+\dfrac{1}{x^k}=2\cos k\theta$ and $x^{k+1}+\dfrac{1}{x^{k+1}}=2\cos(k+1)\theta$ for a positive integer k . Using **(i)**, show that $x^{k+2}+\dfrac{1}{x^{k+2}}=2\cos(k+2)\theta$.

(4 marks)

(b) It is given that $x^n+\dfrac{1}{x^n}=2\cos n\theta$. Prove, by mathematical induction, that

$$\left(x+x^3+x^5+\ldots+x^{2n-1}\right)+\left(\frac{1}{x}+\frac{1}{x^3}+\frac{1}{x^5}+\ldots+\frac{1}{x^{2n-1}}\right)=\frac{\sin 2n\theta}{\sin\theta}$$

for all positive integers n , where $\theta \neq k\pi$.

(6 marks)

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END OF PAPER

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