**2023-2024 S4**
**1st TERM EXAM**
**MATH EP**
**M2**

2023 – 2024
S4 First Term Examination

# MATHEMATICS Extended Part

# Module 2 (Algebra and Calculus)

# Question–Answer Book

15th January, 2024
8:15 am – 9:30 am (1 hour 15 minutes)
**This paper must be answered in English**

## INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.

2. This paper consists of TWO sections, A and B.

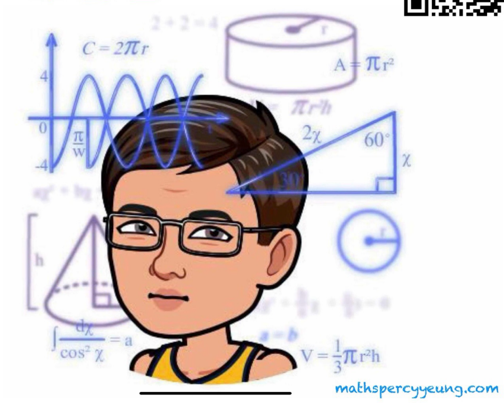
3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers must be exact.

6. The diagrams in this paper are not necessarily drawn to scale.






| Section | Marks |
|---|---|
| **A Total** | **/38** |
| **B Total** | **/12** |
| **TOTAL** | **/50** |

## FORMULAS FOR REFERENCE

| | |
|---|---|
| $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ | $\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$ |
| $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$ | $\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$ |
| $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ | $\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$ |
| $2\sin A \cos B = \sin(A+B) + \sin(A-B)$ | $\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$ |
| $2\cos A \cos B = \cos(A+B) + \cos(A-B)$ | |
| $2\sin A \sin B = \cos(A-B) - \cos(A+B)$ | |

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**Section A** (38 marks)

1. (a) Rationalize the denominator of $\dfrac{1}{\sqrt{x}+\sqrt{x-1}}$.

   (b) Using the result of (a), evaluate
   $$\frac{5}{\sqrt{300}+\sqrt{297}}+\frac{5}{\sqrt{297}+\sqrt{294}}+\frac{5}{\sqrt{294}+\sqrt{291}}+\ldots\frac{5}{\sqrt{6}+\sqrt{3}}.$$

(4 marks)

Answers written in the margins will not be marked

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2. In the expansion of $\left(x^2+\frac{a}{x}\right)^8$ , where $a \neq 0$, the coefficient of $x^r$ is denoted by $\lambda_r$. Find the value of $a$ if $\lambda_7 = 4\lambda_{10}$. (5 marks)



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3. Find the term in $x^2$ in the expansion of $(1+x)^7\left(1-\frac{2}{x}\right)^3$. (4 marks)



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4. Evaluate

(a) $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9}$,

(b) $\lim_{x \to \infty} \frac{x^3+5x-1}{3x^3-7x^2+6}$,

(c) $\lim_{x \to 0} \frac{\sin^3 4x}{x \sin^2 5x}$.

(8 marks)



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5. (a) Using mathematical induction, prove that $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$ for all positive integers $n$.

(b) Hence, show that $1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \cdots + (n-1)n(n+1) = \frac{(n-1)n(n+1)(n+2)}{4}$ for all positive integers $n$ greater than 1.

(8 marks)

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6. Simplify $[\sec(\pi+\theta)+1]\left[\csc\left(\frac{\pi}{2}+\theta\right)+1\right]$. (3 marks)

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7. (a) Prove that $x+1$ is a factor of $4x^3+2x^2-3x-1$.

(b) Prove that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

(c) Using the results of (a) and (b), prove that $\cos\frac{3\pi}{5} = \frac{1-\sqrt{5}}{4}$.

(6 marks)

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**Section B** (12 marks)

8. Let $m = \dfrac{\sin\beta}{\sin(2\alpha+\beta)}$.

(a) Prove that $1+m = \dfrac{2\sin(\alpha+\beta)\cos\alpha}{\sin(2\alpha+\beta)}$.

(2 marks)

(b) Prove that $(1+m)\tan\alpha = (1-m)\tan(\alpha+\beta)$.

(3 marks)

(c) (i) If $\sin(2x+\dfrac{\pi}{4}) = -\dfrac{\sqrt{2}}{10}$, by using the result of (b), prove that $2\tan^2 x - 5\tan x - 3 = 0$.

(ii) Solve $\sin(2x+\dfrac{\pi}{4}) = -\dfrac{\sqrt{2}}{10}$ where $\dfrac{\pi}{2} < x < \dfrac{3\pi}{2}$.

(Give your answers correct to 3 significant figures.)

(7 marks)

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**END OF PAPER**

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