2023-2024 S6
1st TERM UT
MATH CP
PAPER 1
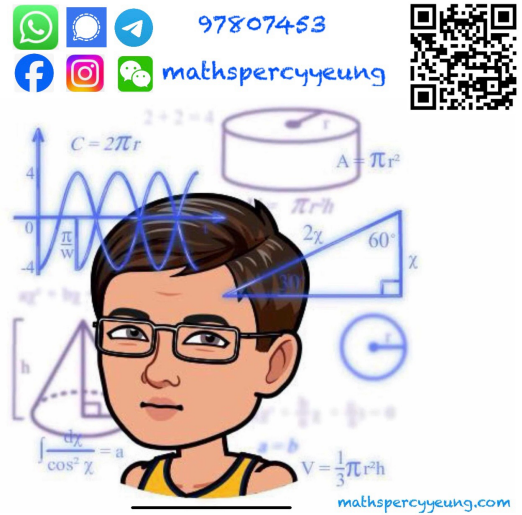
2023 – 2024
S6 First Term Uniform Test

MATHEMATICS Compulsory Part

PAPER 1

Question–Answer Book

13th November, 2023
8:15 am – 9:45 am (1 hour 30 minutes)
This paper must be answered in English

INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.

2. This paper consists of THREE sections, A(1), A(2) and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question – Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

6. The diagrams in this paper are not necessarily drawn to scale.

97807453
mathspercyyeung
mathspercyyeung.com

$C = 2\pi r$
$A = \pi r^2$
$\pi r^2 h$
$V = \frac{1}{3}\pi r^2 h$

Sections	Marks
A (1 – 4)	
A (5 – 10)	
A Total	**/48**
B Total	**/22**
TOTAL	**/70**

Section A(1) (22 marks)

1. Simplify $\dfrac{(xy^{-1})^3}{x^{-2}y^5}$ and express your answer with positive indices. (3 marks)

2. Make x the subject of the formula $y = \dfrac{3x}{5-2x}$. (3 marks)

3. Factorize the following.
    (a) $p^2 + pq - 6q^2$
    (b) $p^2 + pq - 6q^2 - 2p + 4q$

    (3 marks)

4. In a furniture shop, the marked price of a sofa is 30% higher than the cost price. The sofa is sold at a discount of 25%, and the selling price is $1755.
   (a) Find the cost price of the sofa.
   (b) Find the percentage profit or loss made by selling the sofa.

   (4 marks)

5. Consider the compound inequality
   $$\frac{5-2x}{3} > 4(x+2) \quad \text{or} \quad 3x+11 \le 0 \quad \text{………..} \ (*)$$
   (a) Solve (*).
   (b) Write down the greatest integer satisfying (*).

   (4 marks)

6. Consider an arithmetic sequence with general term $T(n)$. Given that the 8th term is 15 and the sum of the 5th term and the 9th term is 26, find
   (a) the first term and the common difference of the sequence,
   (b) the least possible value of k such that $T(k)$ is greater than 100.

(5 marks)

Section A(2) (26 marks)

7. The cost (C) of holding a charity concert is partly constant and partly varies directly as the number of guests (n). It is given that when there are 40 guests, the cost is $3900; when there are 70 guests, the cost is $6450.
    (a) Express C in terms of n.   (3 marks)
    (b) Simon claims that the cost will be more than doubled if the number of guests is doubled. Is he correct? Explain your answer.   (2 marks)

8. The following box-and-whisker diagram shows the distribution of the number of correct questions of 12 students in a Mathematics competition. It is known that the range of the distribution is twice the inter-quartile range of the distribution.

Numbers of correct questions
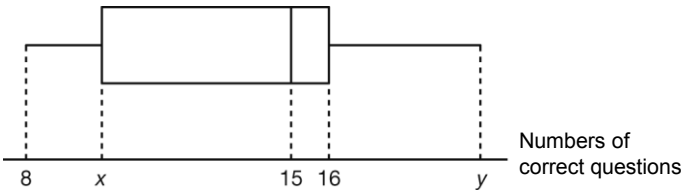
(a) Find all possible pairs of integral values of x and y. (3 marks)

(b) After review, the marker found that the numbers of correct questions of 4 students were recorded wrongly. The table shows the original and amended numbers of correct questions of those 4 students.

Student	A	B	C	D
Original numbers of correct questions	8	8	18	20
Amended numbers of correct questions	7	9	21	26

It is given that the mean of the original numbers of correct questions in the distribution is 14. Find the mean, median and range of the amended numbers of correct questions.

(4 marks)

9. Eunice needs to drive to work through either road X or road Y. The probabilities of having traffic congestion in roads X and Y are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. The chance for Eunice to drive through road X is 5 times that to drive though road Y.

   (a) Find the probabilities that, in a certain day,
       (i) Eunice drives through road X,
       (ii) Eunice gets into traffic congestion.

   (4 marks)

   (b) If Eunice gets into traffic congestion, the probability that she will be late for work is $\frac{4}{5}$. Otherwise, the probability that she will be late for work is $\frac{1}{8}$. Find the probability that she will **NOT** be late for work.

   (2 marks)

10. Given that the equation of a circle C is $x^2 + y^2 - 10x - 22y + 46 = 0$. Denote the centre of C by Q. The equation of a straight line L is $12x + 5y + 54 = 0$, where L and C do not intersect. Let P be a point on L which is closest to Q.

   (a) Find the coordinates of Q and the radius of the circle C. (2 marks)

   (b) Find the coordinates of P. (3 marks)

   (c) Let R be a point on C such that it is farthest from P.

      (i) Describe the geometric relationship between P, Q and R.

      (ii) Find the ratio of the area of $\triangle OPQ$ to the area of $\triangle OQR$, where O is the origin.

      (3 marks)

Section B (22 marks)

11. An examination paper consists of sections A, B and C. It is required to answer 5 out of 7 questions in section A, 4 out of 6 questions in section B, 3 out of 5 questions in section C. In how many ways can a candidate attempt the paper if

    (a) the candidate should answer all the sections? (2 marks)

    (b) the candidate can choose 2 out of 3 sections to answer? (2 marks)

12. Three towns X, Y and Z form a triangle XYZ, where $XY = 12\,\text{km}$ and $XZ = 8\,\text{km}$. The compass bearing of Z from X is N22°W and Y is due west of X.

    (a) (i) Find the area of $\triangle XYZ$.

    (ii) Find the length of YZ.

    (Give your answers correct to 3 significant figures.)

    (4 marks)

    (b) Let Q be the point of a position in the triangle XYZ. It is given that the compass bearings of Q from X and Y are N32°W and N54°E respectively. Two cars drive to Q from X and Y at the speeds of 20 km/h and 25 km/h respectively. Which car (car from X or car from Y) will arrive at the position Q earlier? Explain your answer.

    (4 marks)

13. At the beginning of a month, Mr. Lee borrowed P from a bank at an interest rate of $r\%$ p.a., compounded monthly. From the beginning of the second month, he repays x at the beginning of each month until the loan is fully repaid. The last repayment may be less than x.

(a) Find the balances owed in the following periods in terms of P, $r\%$ and x.

   (i) At the end of the first month

   (ii) At the end of the second month

(3 marks)

(b) Prove that the balance owed at the end of the nth month is

$$\$\left\{ P\left(1+\frac{r\%}{12}\right)^{n} - x\left(1+\frac{r\%}{12}\right)\left[\frac{(1+\frac{r\%}{12})^{n-1}-1}{\frac{r\%}{12}}\right]\right\}.$$

(3 marks)

(c) Let $P = 100\ 000$ and $r = 12$.

   (i) If Mr. Lee repays the loan in 28 instalments, find the least value of x. (Give your answer correct to 2 decimal places.)

   (ii) If Mr. Lee repays $5\ 000$ each month, when will he fully repay the loan?

(4 marks)

END OF PAPER