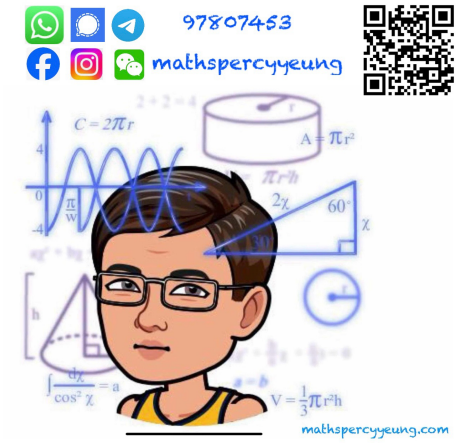
2023-2024 S3
1st TERM EXAM
MATH

2023 – 2024
S3 First Term Examination

MATHEMATICS

Question–Answer Book

3rd January, 2024
8:15 am – 9:45 am (1 hour 30 minutes)
This paper must be answered in English



INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.

2. Answer ALL questions in Section A. You should use an HB pencil to mark all the answers on the Answer Sheet, so that wrong marks can be completely erased with a clean rubber. You must mark the answers clearly; otherwise you will lose marks if the answers cannot be captured. You should mark only ONE answer for each question. If you mark more than one answer, you will receive NO MARKS for that question.

3. Attempt ALL questions in Sections B and C. Write your answers in the spaces provided in this Question – Answer Book.

4. Unless otherwise specified, all working must be clearly shown and numerical answers should be either exact or correct to 3 significant figures.

5. The diagrams in this paper are not necessarily drawn to scale.

Sections	Marks
A Total	**/30**
B (31 – 33)	
B (34 – 39)	
B Total	**/40**
C Total	**/30**
TOTAL	**/100**

Section A (30 marks)

Choose the best answer for each question.

1. Suppose that a and b are non-zero numbers, and n is a positive integer. Which of the following must be correct?

   A. $(ab)^{-n} = \dfrac{b}{a^n}$

   B. $(ab)^0 = b$

   C. $\left(\dfrac{a}{b}\right)^{-n} = \dfrac{b^n}{a^n}$

   D. $a^0 b^{-n} = \dfrac{1}{ab^n}$

2. $\left(\dfrac{k}{h^{-2}}\right)^4 =$

   A. $\dfrac{k^4}{h^2}$.

   B. h^2k^4.

   C. h^8k^4.

   D. $\dfrac{1}{h^8k^4}$.

3. In which of the following numbers the place value of 0 is the largest?

   A. 10111111_2

   B. 1011_{10}

   C. 101111_2

   D. 11101_{10}

4. Convert $2^{10}+2^8+2^2+1$ to a binary number.

   A. 10100000011_2

   B. 10100000101_2

   C. 11000000011_2

   D. 11000000101_2

5. 1 day =

   A. 1.44×10^6 s.

   B. 8.64×10^4 s.

   C. 1.44×10^4 min.

   D. 8.64×10^3 min.

6. If $3^x = \dfrac{1}{27}$, then

   A. $x = -3$.

   B. $x = 2$.

   C. $x = 3$.

   D. $x = 9$.

7. Which of the following can be factorized?

   I. a^2+5a+6

   II. a^2-5a+6

   III. a^2+5a-6

   IV. a^2-5a-6

   A. I and II only

   B. II and III only

   C. III and IV only

   D. I, II, III and IV

8. Factorize $6x^2 - 7x - 3$.

   A. $(3x - 1)(2x + 3)$

   B. $(3x + 1)(2x - 3)$

   C. $(6x - 1)(x + 3)$

   D. $(6x + 1)(x - 3)$

9. Factorize a^4-8a^2+16.

   A. $\left(a^2+4\right)(a+2)(a-2)$

   B. $(a-2)^2(a+2)^2$

   C. $\left(a^2+4\right)^2$

   D. $(a-2)^4$

10. Which of the following is/are (an) identity/identities?

    I. $(a-2)(a+2) = a^2-2$

    II. $(5a-1)^2 = 25a^2-10a+1$

    III. $(2+3a)^2 = 4+6a+9a^2$

    A. II only

    B. I and III only

    C. II and III only

    D. I, II and III

11. A sum of \$30 000 is deposited at an interest rate of 6% p.a. for 4 years, compounded half-yearly. Find the interest correct to the nearest dollar.

A. \$1781
B. \$3765
C. \$7874
D. \$8003

12. Larry deposits \$30 000 into a bank account at a simple interest rate of r% p.a. If he will get an amount of \$36 000 after 4 years, find the value of r.

A. 2
B. 3
C. 4
D. 5

13. A number is first decreased by 20% and then increased by 15%. If the result is 414, find the original number.

A. 400
B. 410
C. 422
D. 450

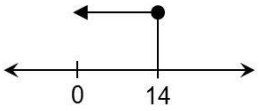
14. The number of a kind of bacteria increases at a constant rate of 4% per minute. If there are 1 000 000 bacteria now, the number of bacteria after one hour will be

A. 1.051×10^7 (corr. to 4 sig. fig.).
B. 1.052×10^7 (corr. to 4 sig. fig.).
C. 1.051×10^8 (corr. to 4 sig. fig.).
D. 1.052×10^8 (corr. to 4 sig. fig.).

15. A is 10% greater than B and C is 5% less than B. A is greater than C by

A. 13% *(corr. to the nearest 1%)*.
B. 14% *(corr. to the nearest 1%)*.
C. 15% *(corr. to the nearest 1%)*.
D. 16% *(corr. to the nearest 1%)*.

16. Which of the following diagrams represents $x \leq 14$?

A.


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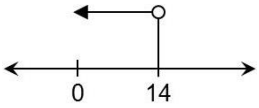
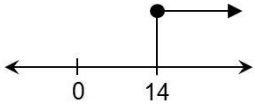


C.


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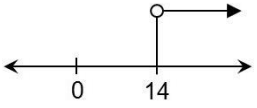
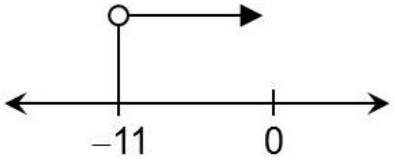


17. How many negative integers are the solutions of the inequality represented by the following diagram?



A. 9
B. 10
C. 11
D. 12

18. If $x > y$, then $4x + 3$ (I) $4y + 3$ and $1 - 2x$ (II) $1 - 2y$.

	(I)	(II)
A.	>	>
B.	<	<
C.	>	<
D.	<	>

19. Solve the inequality $4x + 7 > 6x - 11$.

A. $x > 2$
B. $x < 2$
C. $x > 9$
D. $x < 9$

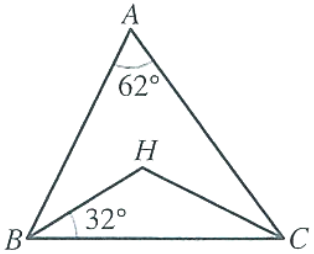
20. If $p > q > 0$, which of the following must be true?

I. $\frac{p}{q} > 1$
II. $pq < q^2$
III. $-p > -q$

A. I only
B. III only
C. I and II only
D. II and III only

21. In the figure, H is the incentre of ΔABC. $\angle BCA =$

A. $27°$.
B. $32°$.
C. $54°$.
D. $62°$.

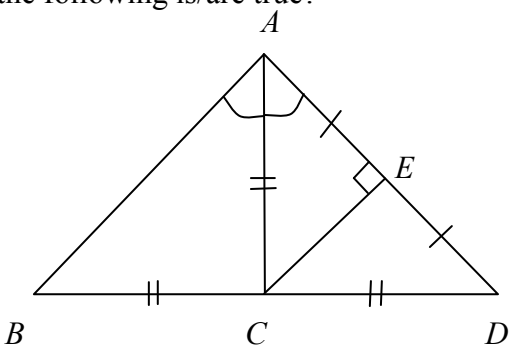


22. Which of the following centres of a triangle must lie inside the triangle?

I. Incentre
II. Centroid
III. Orthocentre

A. I only
B. II only
C. I and II only
D. I, II and III

23. In the figure, BCD and AED are straight lines. $AC = BC = CD$, $AE = DE$, $\angle AEC = 90°$, $\angle BAC = \angle CAD$. Which of the following is/are true?



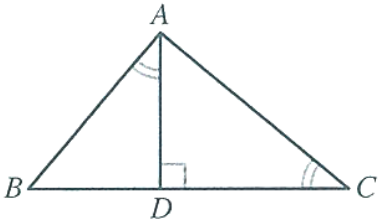
I. CE is an altitude of ΔACD.
II. AC is a perpendicular bisector of ΔABD.
III. CE is an angle bisector of ΔACD.

A. I only
B. I and III only
C. II and III only
D. I, II and III

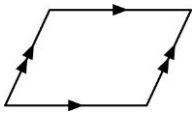
24. In the figure, AD is an altitude of ΔABC. $\angle BAD = \angle ACD$. Which of the following must be true?

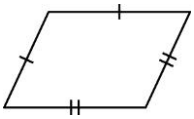
I. $\Delta ABD \sim \Delta CAD$
II. AD is the angle bisector of $\angle BAC$.
III. AD is the perpendicular bisector of BC.

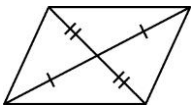
A. I only
B. III only
C. I and II only
D. I, II and III

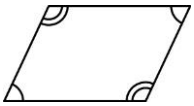


25. Which of the following may NOT be a parallelogram?

A. 

B. 

C. 

D. 

26. Which of the following must be true?

I. The diagonals of a rhombus are equal.

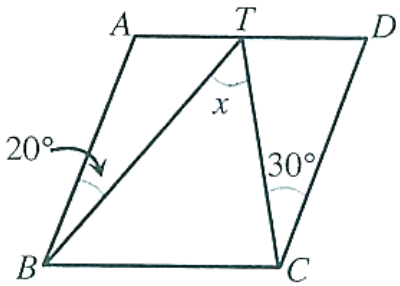
II. The diagonals of a rectangle bisect each other.

III. The diagonals of a square bisect each interior angle.

A. I only
B. III only
C. I and II only
D. II and III only

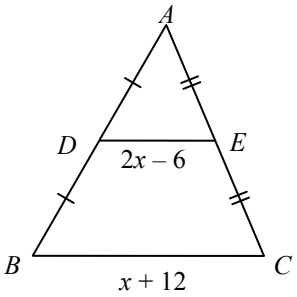
27. In the figure, $ABCD$ is a parallelogram. ATD is a straight line. Find the value of x.

A. 30°
B. 40°
C. 50°
D. 60°

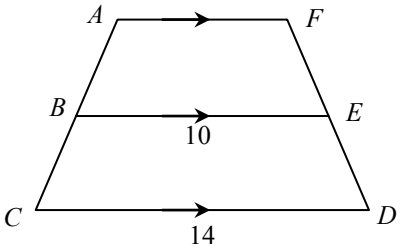


28. In the figure, ADB and AEC are straight lines. If $DE = 2x - 6$ and $BC = x + 12$, then $x =$

A. 4.
B. 5.
C. 6.
D. 8.

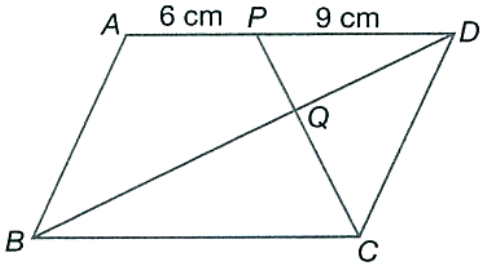


29. In the figure, $AF//BE//CD$. ABC and FED are straight lines. B is the mid-point of AC. If $BE = 10$ and $CD = 14$, find the length of AF.



A. 6
B. 7
C. 8
D. 9

30. In the figure, $ABCD$ is a parallelogram. APD is a straight line. $AP = 6\text{ cm}$ and $PD = 9\text{ cm}$. BD and CP intersect at Q. Find $PQ : QC$.



A. 3 : 2
B. 2 : 3
C. 3 : 5
D. 5 : 3

Section B (40 marks)

31. (a) Factorize $x^2 + 2xy + y^2$.

    (b) Hence, or otherwise, factorize $x^2 + 2xy + y^2 - 3x - 3y$.

(4 marks)

32. Make x the subject of the formula $\frac{2x + y}{y} = x - 2$. (3 marks)

33. A bicycle costs \$600. The marked price of the bicycle is 40% above the cost. It is sold at a 25% discount on the marked price.

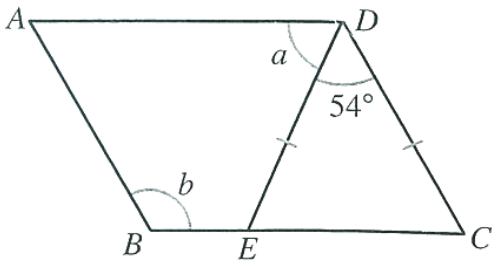
    (a) Find the selling price of the bicycle.

    (b) Find the profit percentage or loss percentage.

(4 marks)

34. Simplify $a^{23}\left(\frac{a^5}{b^{-3}}\right)^{-2}$ and express your answers with positive indices. (3 marks)

35. In the figure, $ABCD$ is a parallelogram. $\angle EDC = 54°$ and $DE = DC$. Find a and b. (5 marks)



36. Solve the inequality $\frac{8-3x}{5}-\frac{11-2x}{3}<0$ and represent the solutions graphically. (4 marks)

37. Stanley plans to deposit \$20 000 in one of the banks below for 2 years.

Bank *A*: simple interest rate of 6.5% p.a.
Bank *B*: interest rate of 6% p.a. compounded half-yearly

Which bank should he choose in order to receive more interest? Explain your answer.

(6 marks)

38. It is given that the speed of light is 3×10^8 m/s. Take 1 year = 365 days.

(a) Find the distance travelled by light in a year.

(Give the answer in scientific notation and correct to 3 significant figures.)

(b) A star X, which was 4.73×10^{17} m away from the Earth, exploded and vanished in 1988.

(i) How long should the light be travelling from star X to the Earth?

(ii) Hence, determine whether we can still observe star X in the sky from the Earth in 2024.

(5 marks)

39. (a) Factorize $16x^2 + 18x - 9$.

(b) Hence, factorize $16(6y^2 - 3)^2 + 18(6y^2 - 3) - 9$.

(6 marks)

Section C (30 marks)

40. In the fiscal year 22/23, the basic allowance is $120 000 and the progressive rates are shown in the table below.

Net chargeable income	Tax rates
First $40 000	2%
Next $40 000	7%
Next $40 000	12%
Remainder	17%

Winnie and Nancy are single and both of them are living alone. In the fiscal year 22/23, Winnie's net chargeable income is $45 000 and Nancy's is 20% more than Winnie's.

(a) How much salaries tax does each of them need to pay for the fiscal year 22/23? (5 marks)

(b) Is Nancy's tax payable 20% more than Winnie's? Explain briefly. (3 marks)

(c) What percentage of Winnie's total income does she use to pay her salaries tax for the fiscal year 22/23? (2 marks)

(Give your answers correct to 3 significant figures.)

41. The charges of two plans provided by a Streaming Platform are given below:

	Basic charge	**Additional charge per new channel**
Plan *A*	\$130/month	\$18/month
Plan *B*	\$90/month	\$30/month

It is known that both plans A and B include 10 free channels. Suppose Raymond wants to have x more channels.

(a) (i) Write down the monthly charge of plan A and that of plan B in terms of x.

(ii) Find the greatest number of additional channels Raymond has so that plan B is cheaper.

(5 marks)

(b) Recently, the Streaming Platform has offered a new plan C which also includes the 10 free channels same as those included in plans A and B. The charges of plan C are given below.

	Basic charge	**Additional charge per new channel**
Plan *C*	\$1440/year	\$19/month

Determine which plan Raymond should join if Raymond wants to have

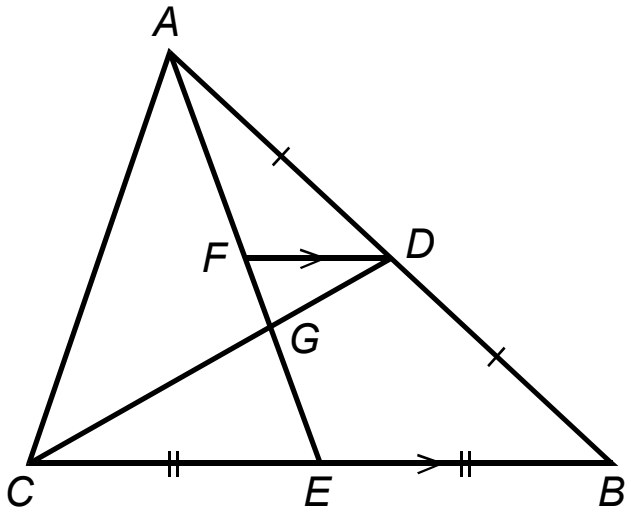
(i) 12 channels in total,

(ii) 18 channels in total.

Explain your answers.

(5 marks)

42. In the figure, D and E are the mid-points of AB and BC respectively. F is a point on AE such that FD // CB. $AFGE$ and CGD are straight lines. It is given that $AE = 20$ cm.



(a) Find EF. (2 marks)

(b) Prove that $\Delta GFD \sim \Delta GEC$. (3 marks)

(c) Find FG. (5 marks)

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