SWC 2024-2025 F4 M2 Final

2024-2025 2nd Term Examination
Form 4 Subject: Mathematics Extended Part
(Module 2)

Date :	24-6-2025
Time :	8:15-10:00
Time allowed :	105 min.
No. of Pages :	20
Stationery provided :	2 rough papers

Question - Answer Paper

Mark : _____/70

1. This Paper consists of TWO sections, A and B.
2. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
3. Unless otherwise specified, all working must be clearly shown.
4. The diagrams in this paper are not necessarily drawn to scale.
5. Unless otherwise specified, numerical answers must be exact.

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$

$2 \cos A \cos B = \cos(A + B) + \cos(A - B)$

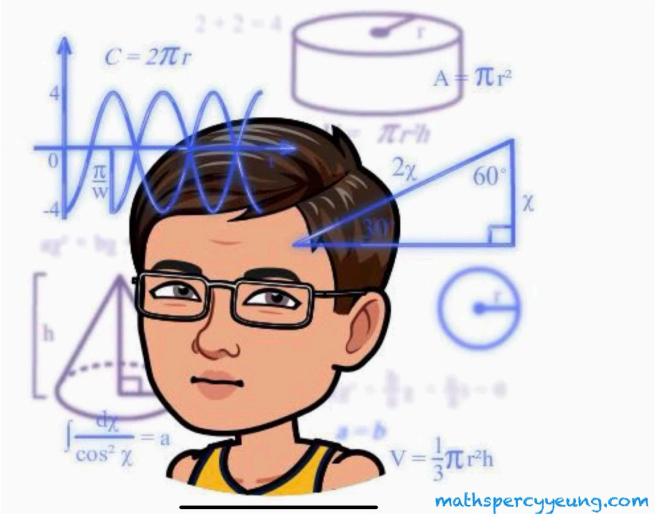
$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$

$\sin A + \sin B = 2 \sin \dfrac{A+B}{2} \cos \dfrac{A-B}{2}$

$\sin A - \sin B = 2 \cos \dfrac{A+B}{2} \sin \dfrac{A-B}{2}$

$\cos A + \cos B = 2 \cos \dfrac{A+B}{2} \cos \dfrac{A-B}{2}$

$\cos A - \cos B = -2 \sin \dfrac{A+B}{2} \sin \dfrac{A-B}{2}$



Section A (35 marks)

1. Let $f(x) = \sin x^2$. Find $f'(x)$ from first principles.

(4 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

2. Let a be a constant. If the coefficient of x^{59} in the expansion of $\left(x^4+\frac{a}{x}\right)^{16}$ is -8, find a and the coefficient of x^{39} in the expansion.

(6 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

3. **(a)** Using the identity $\cos 2A = 2\cos^2 A - 1$, prove that

(i) $\cos^6 x = \frac{1}{8}(1 + 3\cos 2x + 3\cos^2 2x + \cos^3 2x)$,

(ii) $\cos 6x = 4\cos^3 2x - 3\cos 2x$.

(b) Hence, find $\cos\theta$ if $32\cos^6\theta - 9\cos 6\theta = 9$, where $\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$.

(5 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

4. Let $y = \ln(e^{2x} - 16e^{x} + 100)$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(b) Find the point(s) of inflexion of the graph of $y = \ln(e^{2x} - 16e^{x} + 100)$.

(6 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

5. **(a)** Using mathematical induction, prove that $\sum_{k=1}^{n}\left(2^k+\frac{1}{2^{k-1}}\right)=\frac{2^{2n}-1}{2^{n-1}}$ for all positive integers n.

(b) Using **(a)**, find a pair of rational numbers a and b such that

$$\sum_{k=51}^{100}\left(2^k+\frac{1}{2^{k-1}}\right)=\frac{(2^{150}+a)(2^{50}+b)}{2^{99}}$$

(7 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

6. Find the equations of the tangents drawn from the external point (8, 0) to the curve $C: x^2 + y^2 = 16$.

(7 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Section B (35 marks)

7. Let $f(x)=\dfrac{4x^2+7x+45}{(x+5)^2}-\dfrac{21}{8}$ for all real numbers $x \neq -5$.

Denote the graph of $y = f(x)$ by H.

(a) Find the asymptote(s) of H. (3 marks)

(b) Find the maximum / minimum point(s) of H. (3 marks)

(c) Someone claims that two of the tangents to H are parallel to the straight line $11x+25y=2$. Do you agree? (2 marks)

(d) Find the x-intercept(s) and the y-intercept of H. (2 marks)

(e) Sketch H. (3 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

8. Consider the curve $C: y = \sqrt{e^x}$, where $x > 0$. Let P be a moving point on C, and Q be a point on the y-axis such that PQ is perpendicular to the tangent to C at P. The horizontal line passing through P cuts the y-axis at the point R.

(a) Denote the x-coordinate of P by r. Prove that the y-coordinate of Q is $\dfrac{2r + e^r}{e^{\frac{r}{2}}}$. (3 marks)

(b) Find the greatest area of $\triangle PQR$. (4 marks)

(c) Let O be the origin. It is given that the length of OP increases at a constant rate of $\sqrt{e}$ units per hour. Find the rate of change of the area of $\triangle PQR$ when $r = 1$. (4 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

9. Let $f(x)=\dfrac{k}{x^2-2x+2}$, where k is a positive constant. Denote the graph of $y=f(x)$ by Γ. O is the origin. P is a moving point on Γ and Q is the maximum point of Γ. P starts from Q and moves to the right of Q. M and N are the feet of perpendiculars from P to the x-axis and the y-axis respectively. Denote the x-coordinate of M by a. It is given that the extreme value of $f(x)$ is 2.

(a) Show that $a \geq 1$ (3 marks)

(b) Find k (1 marks)

(c) Find the maximum area of *OMPN*. (4 marks)

(d) Someone claims that the perimeter of *OMPN* attains its minimum value when the area of *OMPN* attains its maximum value. Do you agree? Explain your answer. (3 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.