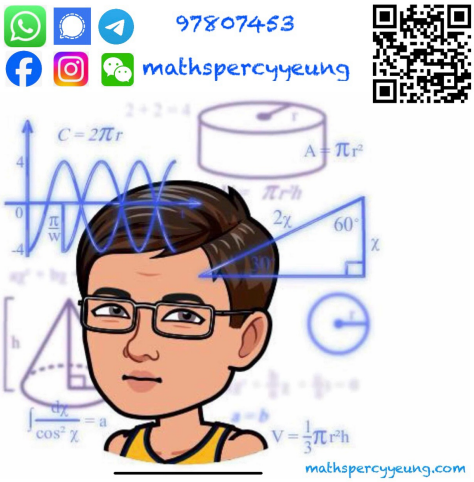
Final Examination 2024 – 2025
S.4 Mathematics
Paper 1

Date : 6 June 2024

Time : 08:30 – 10:00 (1 hour and 30 minutes)



Paper	Section	Marks
1	A(1)	/31
	A(2)	/30
	B	/20
	PP–_____	
	Total	/81
2		/39
Total		/120

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your name, class and class number in the spaces provided on this cover page.

2. This paper consists of **THREE** sections, **A(1), A(2) and B.**

3. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book.

4. Graph paper and supplementary answer sheets will be supplied on request. Write your name, class, class number and question number on each sheet and fasten them **INSIDE** this book.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 **significant figures.**

7. The diagrams in this paper are not necessarily drawn to scale.

8. No extra time will be given to candidates for writing their names, classes or class numbers after the "Time is up" announcement.

SECTION A(1) [31 marks]

1. Simplify $\dfrac{(xy^{-2})^3}{x^{-4}y^5}$ and express your answer with positive indices. **(3 marks)**

2. Factorize

(a) $4a^2-2ab-6b^2$,

(b) $a+b-4a^2+2ab+6b^2$. **(3 marks)**

3. Make k the subject of $\dfrac{3}{k}-\dfrac{2}{h}=1$. **(3 marks)**

4. In **Figure 1**, AB and CD intersect at E. It is given that $AC \parallel DB$.

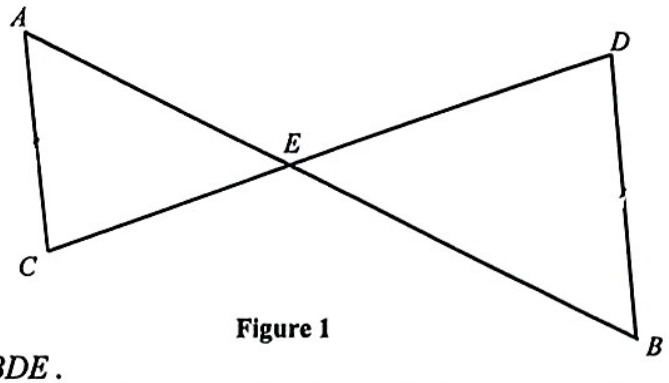


Figure 1

(a) Prove that $\Delta ACE \sim \Delta BDE$. **(2 marks)**

(b) Suppose that $AC = 4$ cm, $DB = 6$ cm and $AB = 20$ cm, find the length of AE. **(2 marks)**

5. Let a, b and c be non-zero numbers such that $\frac{a}{4b}=\frac{1}{5}$ and $\frac{2b+7c}{10a-5c}=1$. Find

(a) $a:b:c$,

(b) $\frac{4a-3b}{9b+2c}$.

(4 marks)

6. Bag A contains two cards numbered 6 and 8 respectively while bag B contains three cards numbered 2, 3 and 4 respectively. Peter is drawing one card from each bag randomly.
   (a) Find the probability that the sum of the two numbers drawn is not less than 10.
   (b) If the sum of the two numbers drawn is not less than 10, Peter can get 6 tokens; otherwise he cannot get any tokens. Write down the expected number of tokens Peter gets.

(3 marks)

7. Let $f(x)=(x-1)(x+k)$, where k is a constant. If $f(-1)=14$, find the value of $f(0)$.

(3 marks)

8. Let $f(x)=12x^2+4kx+12+3k$, where k is a positive constant. It is given that the equation $f(x)=0$ has equal roots.

(a) Find the value of k.

(b) Solve the equation $f(x)=-kx-2k$. **(4 marks)**

9. **(a)** L_1 is a line parallel to the y-axis and passes through $A(-5, 2)$. Write down the equation of L_1.

(b) L_2 is a line perpendicular to the y-axis and passes through $B(-3, 6)$. Write down the equation of L_2.

(c) The equation of L_3 is $x + 2y + 3 = 0$. L_4 is perpendicular to L_3 and passes through the origin. Find the equation of L_4. **(4 marks)**

SECTION A(2) **[30 marks]**

10. **Figure 2** shows the graph of the function $y = ax^2 + bx + c$, where a, b and c are constants. If the minimum value of the function is -50, find the values of a, b and c. **(4 marks)**

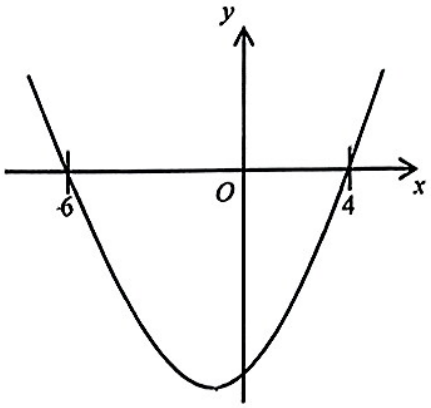


Figure 2

11. Two straight lines $L_1 : 2x-3y+12=0$ and $L_2 : kx-y+15=0$ intersect at point A. L_2 is parallel to the straight line $L_3 : y=-3x+6$.

(a) Write down the value of k. **(1 mark)**

(b) Write down the coordinates of A. **(1 mark)**

(c) L_1 cuts the x-axis at B and L_2 cuts the x-axis at C. D is a point on line segment BC such that $BD:DC=4:7$. Find the equation of AD. **(4 marks)**

12. It is given that f(x) partly varies as $(x-1)$ and partly varies as x^2. Suppose that $f(2)=45$ and $f(-3)=70$.

(a) Express f(x) in form of ax^2+bx+c.

(b) If the graph of y = f(x) cuts the x-axis at A and B and cuts the y-axis at C, find the area of ΔABC. **(6 marks)**

13. The base radius and the height of a solid metal right circular cone X are 45 cm and 200 cm respectively. X is divided into a right circular cone Y and a frustum Z by a plane which is parallel to its base, such that the curved surface area of Z is 24 times the curved surface area of Y.

(a) Express the volume of Z in terms of π. **(3 marks)**

(b) Z is melted and recast into 2 identical solid spheres. A student claims that the total curved surface area of these two spheres is less than 3.4 m^2. Is the claim correct? Explain your answer. **(4 marks)**

You may use this space to sketch the diagrams and put down the relevant information.
Sketching is not carrying any marks. It will not be marked.

14. Let $f(x) = 3x^3 + ax^2 - x + 7$, where a is a constant. It is given that when $f(x)$ is divided by $x^2 - 3x - 2$, the quotient is $bx - 1$, where b is a constant.

(a) Write down the value of b. **(1 mark)**

(b) Find the remainder when $f(x)$ is divided by $x^2 - 3x - 2$. **(3 marks)**

(c) Someone claims that all the roots of the equation $f(x) - 2x - 5 = 0$ are rational numbers. Do you agree? Explain your answer. **(3 marks)**

SECTION B [20 marks]

15. Solve $8(5^{x+2}) = 7^{2-3x}$ and give the answer correct to 3 significant figures if necessary.

(3 marks)

16. Solve the following equations.

(a) $3^{x+1}+3^{2-x}-28=0$, **(3 marks)**

(b) $(\log_4 x)^2-\log_2 x^2-12=0$. **(3 marks)**

17. The graph in **Figure 3** shows the linear relation between $\log_9 y$ and $\log_3 x$. The intercepts on the vertical and horizontal axes of the graph are 8 and 4 respectively.

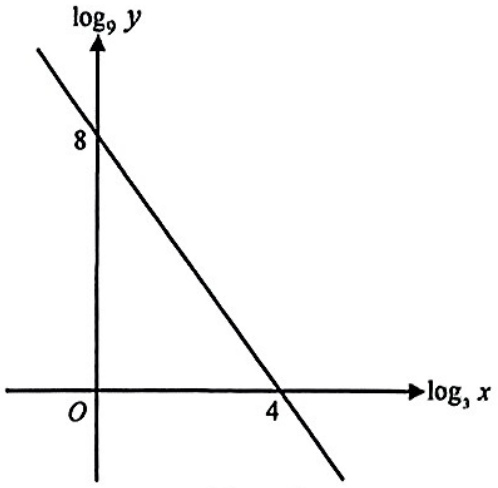


Figure 3

Express the relation between x and y in the form $y = \left(\frac{A}{x}\right)^4$, where A is a constant. **(4 marks)**

18. Let $f(x) = 2x^2 + 4x - 12kx + 18k^2 - 10k + 3$, where $k < -1$. Denote the vertex of the graph of $y = f(x)$ by P.

(a) Using the method of completing the square, express, in terms of k, the coordinates of P. **(2 marks)**

(b) Denote the origin by O. Q is a point on the y-axis such that $J(0, -12.5)$ is the circumcentre of ΔOPQ.

(i) Find the coordinates of P. **(2 marks)**

(ii) Prove that P is the orthocentre of ΔOPQ. **(3 marks)**

You may use this space to sketch the diagrams and put down the relevant information.
Sketching is not carrying any marks. It will not be marked.