S.2 Mathematics

School-Based Exercise (S.B.E.)

Chapter 10
Pythagoras' Theorem and Irrational Numbers

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mathspercyyeung

$C = 2\pi r$

$A = \pi r^2$

$\pi r^2 h$

2χ $60°$ χ

$\int \frac{d\chi}{\cos^2 \chi} = a$

$V = \frac{1}{3}\pi r^2 h$

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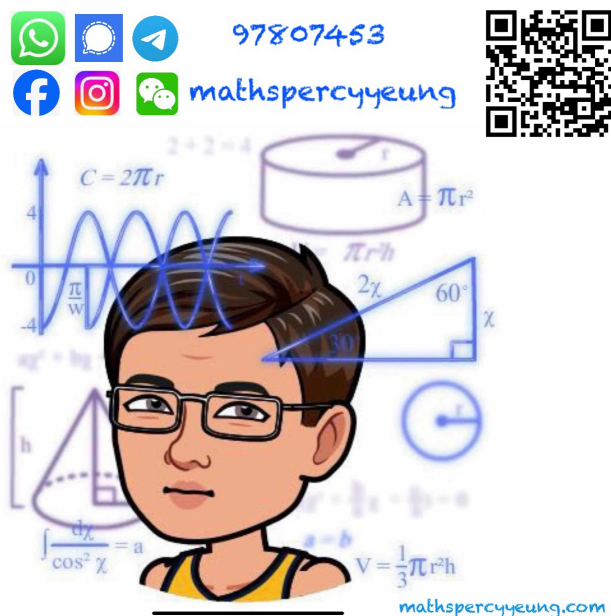
Square Roots and nth Roots

If $x^2 = a$, then x is a square root of a.

e.g. : Since $7^2 = 49$, 7 is a square root of 49.   (i.e. $7 = \sqrt{49}$)

Since $(-7)^2 = 49$, -7 is also a square root of 49.   (i.e. $-7 = -\sqrt{49}$)

$\sqrt{49}$ is called the positive square root of 49 and $-\sqrt{49}$ is called the negative square root of 49.

Exercise 10A

Write down the square roots of each of the following numbers without using a calculator. **(3 – 5)**

1. (a) 9   (b) 36   (c) 49

2. (a) 64   (b) 121   (c) 400

3. (a) 25   (b) 169   (c) 196

Evaluate each of the following expressions without using a calculator. **(4 – 15)**

4. $-\sqrt{24+25}$ 5. $\sqrt{6\times24}$ 6. $\sqrt{43-27}$

7. $\sqrt{64}+\sqrt{9}$ 8. $\sqrt{121}-\sqrt{36}$ 9. $-\sqrt{625}\times\sqrt{4}$

10. $\sqrt{24^2+7^2}$ 11. $\sqrt{17^2-15^2}$ 12. $\sqrt{11^2-5^2+2^2}$

13. $\sqrt{13^2+6^2-3^2}$ 14. $\sqrt{2^2+4^2+5^2+6^2}$ 15. $\sqrt{12^2-9^2+6^2+1}$

Find the value of each of the following expressions. **(16 – 18)**

(Give the answers correct to 3 significant figures.)

16. (a) $\sqrt{19}$ (b) $\sqrt{41}$ (c) $-\sqrt{88}$

17. (a) $\sqrt{4.4}$ (b) $\sqrt{8.1}$ (c) $\sqrt{0.1}$
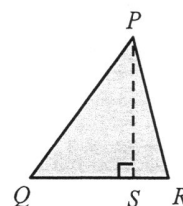
18. (a) $\sqrt{\dfrac{1}{7}}$ (b) $\sqrt{\dfrac{3}{5}}$ (c) $\sqrt{\dfrac{7}{3}}$

19. The area of a square is 16 cm². Find the length of a side of the square.

20. In the figure, QSR is a straight line and $PS = QR$.

    If the area of $\triangle PQR$ is 18 cm², find the length of QR.

Write down the square roots of each of the following numbers without using a calculator. **(21 – 24)**

21. (a) 15^2 (b) 1.73^2 (c) $(-8)^2$

22. (a) $\left(\dfrac{2}{5}\right)^2$ (b) $\dfrac{16}{25}$ (c) $\dfrac{100}{441}$

23. (a) 0.25 (b) 5.76 (c) 3.24

24. (a) 9^3 (b) 2^4 (c) $\dfrac{1}{4^3}$

Evaluate each of the following expressions without using a calculator. **(25 – 30)**

25. $\dfrac{\sqrt{36}}{7}$ **26.** $\dfrac{25-\sqrt{16}}{9}$ **27.** $\dfrac{\sqrt{49}+\sqrt{121}}{2}$

28. $\dfrac{25+\sqrt{225}}{\sqrt{36}}$ **29.** $\dfrac{\sqrt{25+144}}{\sqrt{17^2}}$ **30.** $\dfrac{\sqrt{29^2-21^2}}{16}$

Find the value of each of the following expressions. **(31 – 33)**

(*Give the answers correct to 3 significant figures.*)

31. (a) $\dfrac{\sqrt{10}-3}{5}$ (b) $\dfrac{\sqrt{2}+\sqrt{3}}{4}$ (c) $\sqrt{5^2-3}$

32. (a) $5-\sqrt{\dfrac{3}{7}}$ (b) $\dfrac{1}{9}+\sqrt{0.79}$ (c) $\sqrt{0.7}-\sqrt{0.5}$

33. (a) $\dfrac{1}{\sqrt{5}+3}$ (b) $\dfrac{1}{\sqrt{3}-1}$ (c) $\dfrac{\sqrt{2}}{3-\sqrt{7}}$

Write down the cube roots of each of the following numbersr. **(34 – 35)**

34. (a) $19\,863$ (b) $8\,000$ (c) $2\,744$

35. (a) $\dfrac{216}{1331}$ (b) $2\dfrac{10}{27}$ (c) 6.959

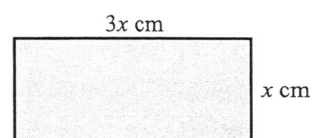Find the value of each of the following expressions.

(*Give the answers correct to 3 significant figures.*)

36. (a) $\dfrac{\sqrt[4]{1+2^3}}{3}$

(b) $\dfrac{4+\sqrt{2}}{\sqrt[3]{-3}}$

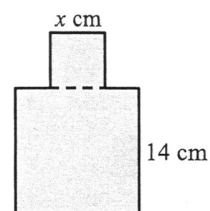(c) $\dfrac{\sqrt{20}+\sqrt[5]{-20}}{\sqrt[4]{2+\sqrt{5}}}$

37. In the figure, the area of the rectangle is 243 cm². Find the value of x.

3x cm
x cm

38. The polygon in the figure is made up of a larger square and a smaller square. The area of the polygon is 232 cm².

(a) Find the value of x.

(b) Find the perimeter of the polygon.

x cm
14 cm

39. The dimensions of a rectangular metal sheet are $(2x + 3)$ cm × $(2x - 3)$ cm. The area of the metal sheet is 16 cm².

(a) Find the value of x.

(b) Find the perimeter of the metal sheet.

40. If $a > 0$, simplify $\sqrt{a^2 + 2a + 1}$.

41. If $\sqrt{x^2 + y} = z$, write down two sets of possible positive integral values of x, y, z.

Pythagoras' Theorem

For a right-angled triangle with sides a, b and c, where c is the length of the side opposite to the right angle,

Then $a^2 + b^2 = c^2$ [Reference: *Pyth. theorem*]
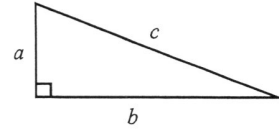
Exercise 10B

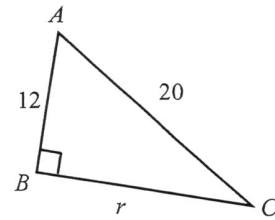Find the unknown in each of the following triangles. **(1 – 12)**

1.
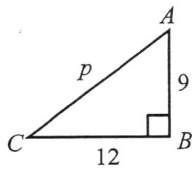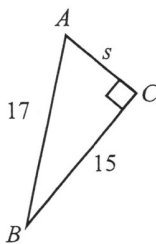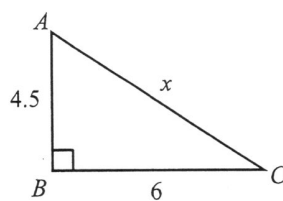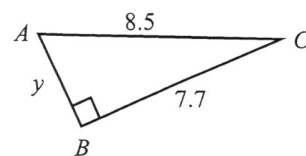
2.
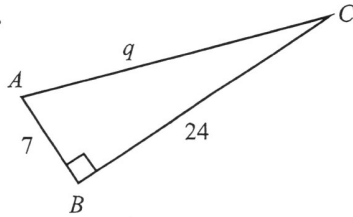
3.

4.
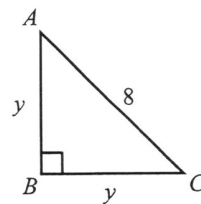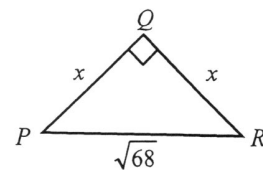
5.

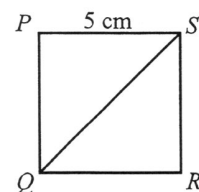**6.**

7.
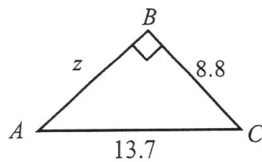
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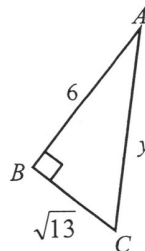
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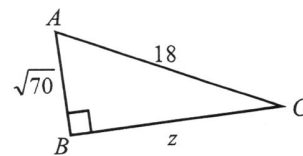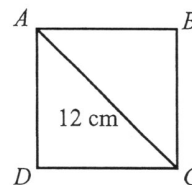
10.
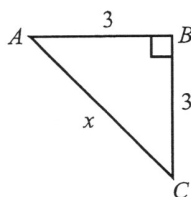
11.

12.

13. In the figure, *PQRS* is a square of side 5 cm. Find the length of diagonal *QS*.
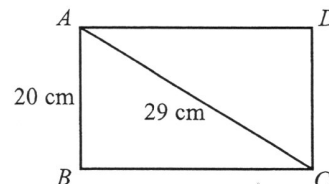
14. In the figure, the length of a diagonal of a square $ABCD$ is 12 cm. Find the length of a side of the square.
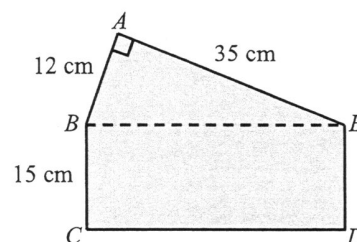
15. In the figure, $ABCD$ is a rectangle, where $AB = 20$ cm and $AC = 29$ cm.

    (a) Find the length of BC.
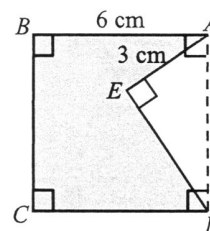
    (b) Find the perimeter of the rectangle.

16. The figure on the right is made up of a right-angled triangle ABE and a rectangle $BCDE$.

    (a) Find the length of BE.

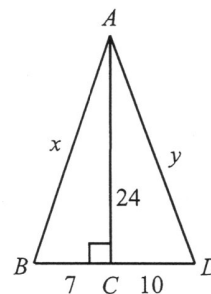   **(b)** Find the area of the figure.

17. In the figure, a pentagon $ABCDE$ is formed by cutting away a right-angled triangle ADE from a square $ABCD$.
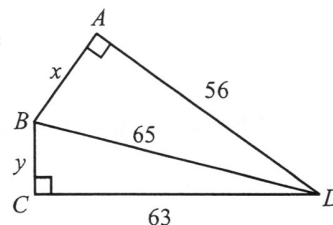
    (a) Find the length of ED.
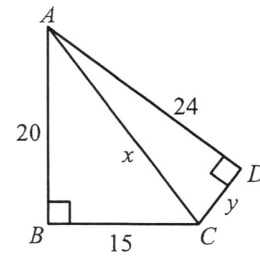
    (b) Find the area of $ABCDE$.

18. In the figure, C is a point on BD such that $AC \perp BD$, $BC = 7$, $CD = 10$ and $AC = 24$. Find the values of x and y.
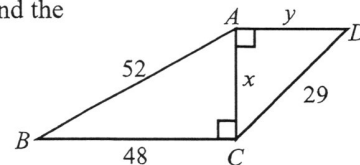
19. In the figure, $\triangle ABD$ and $\triangle BCD$ are two right-angled triangles. Find the values of x and y.
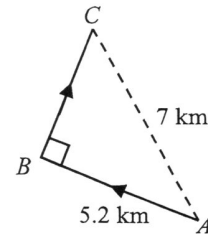
20. In the figure, $\triangle ABC$ and $\triangle ACD$ are two right-angled triangles. Find the values of x and y.
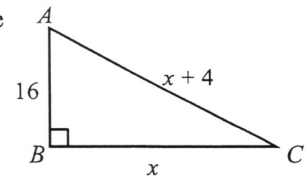
21. In the figure, $\triangle ABC$ and $\triangle ACD$ are two right-angled triangles. Find the values of x and y.

22. The figure shows the walking path of a charity walk which has two sections AB and BC. John is one of the participants and his mother sponsors him $48 for every km he finishes in the walk. It is given that A and C are 7 km apart. The first section AB is 5.2 km long.

    (a) Find the length of the second section BC.

    (b) If John finishes the whole walk, find the total amount sponsored by John's mother.

23. In the figure, $\angle ABC = 90°$, $AB = 16$, $BC = x$ and $AC = x + 4$. Find the value of x.

24. In the figure, BCD is a straight line. $\angle ABD = 90°$, $AB = 12$, $AC = 24.1$ and $CD = 23.3$.
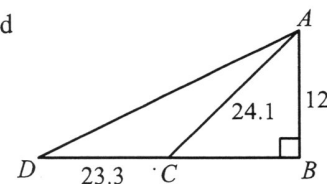
    (a) Find the length of BC.

    (b) Find the length of AD.

25. In the figure, S is a point on PR such that $QS \perp PR$ and $QS = RS$. If $PS = 7$ and $PQ = 9$, find the value of x.

6
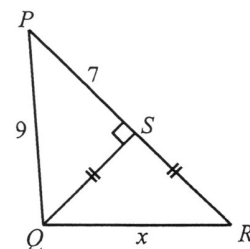
26. In the figure, △ADE and △BCD are two right-angled triangles. ABD and CDE are straight lines. BC = 10, CD = 6, DE = 5 and AE = 13.
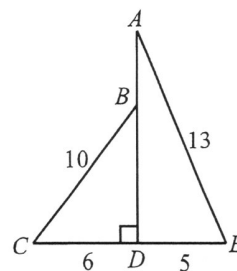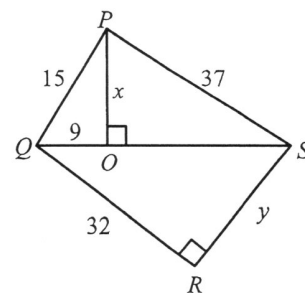
    (a) Find the length of BD.
    (b) Find the length of AB.

27. In the figure, QOS is a straight line. ∠POS = ∠QRS = 90°. Find the values of x and y.

28. The figure shows a trapezium ABCD with AD = 3 cm, BC = 6 cm and DC = 5 cm. Find the length of AB.
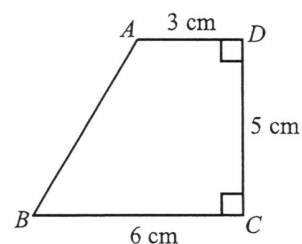
29. In the figure, PQRS is a trapezium with PQ = 27 m, PS = 37 m and SR = 15 m. Find the area of PQRS.
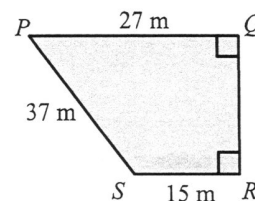
30. In the figure, △ABC is an isosceles triangle with AB = AC = 19 cm and BC = 28 cm. Find the height of △ABC with base BC.
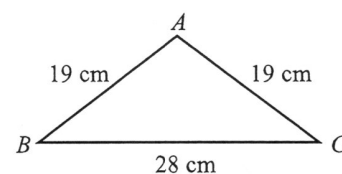
31. In the figure, △PQR is an isosceles triangle with PQ = QR = 25 cm and PR = 14 cm. Find the area of △PQR.

7

32. In the figure, $\angle BAD = \angle BCD = 90°$. $AB = 36$ cm, $BC = 68$ cm and $AD = 77$ cm. Find the area of $ABCD$.
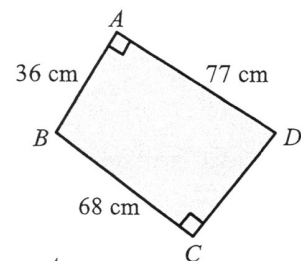
33. In the figure, the area of $\triangle ABC$ is 990 cm².

    (a) Find the length of the hypotenuse of $\triangle ABC$.

    (b) Find the perimeter of $\triangle ABC$.

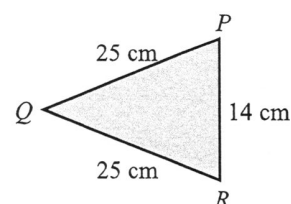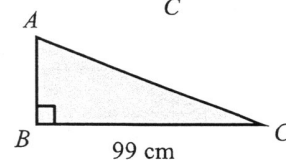**34.** In the figure, $\angle ABC = 90°$, $AB = 3BC$ and $AC = 7$ cm. Find the area of $\triangle ABC$.
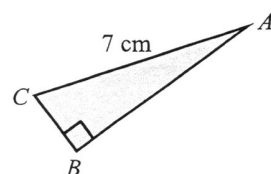
35. In the figure, $\angle PQR = 90°$, $PQ : QR = 7 : 24$ and $PR = 80$ cm.

    (a) Find the perimeter of $\triangle ABC$.

    (b) Find the area of $\triangle ABC$.
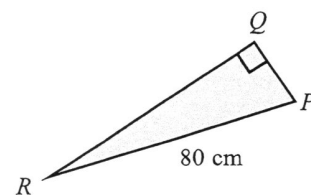
36. The figure shows a framework $ABCDEF$.

    (a) Find the length of a stretched string connecting C and F.

    (b) Find the length of a stretched string connecting A and F.
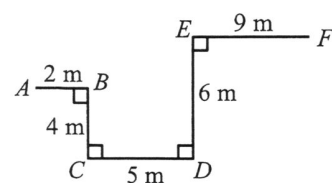
Converse of Pythagoras' Theorem

For $\triangle ABC$ with sides a, b and c,

if $c^2 = a^2 + b^2$,

then $\triangle ABC$ is a right-angled triangle with $\angle C = 90°$. [Reference: *converse of Pyth. theorem*]
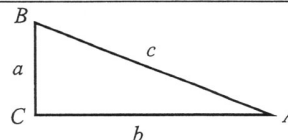
Note: In practical, we will check if the square of the length of the longest side of a triangle equals the sum of the squares of the lengths of the other two sides to determine whether it is a right-angled triangle.

Exercise 10C

Determine whether each of the following triangles is a right-angled triangle. If it is, identify which angle is the right angle. **(1 – 9)**

1.

2.

3.

4.

5.

6.

7.

8.

9.
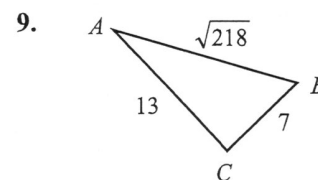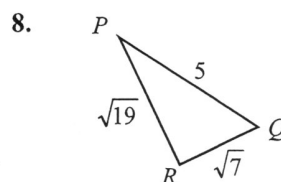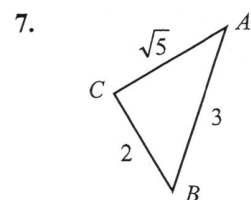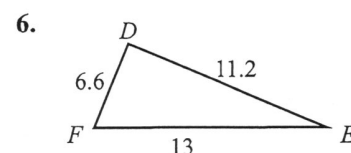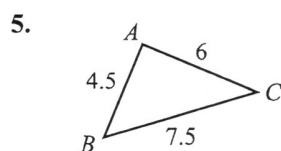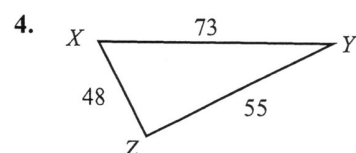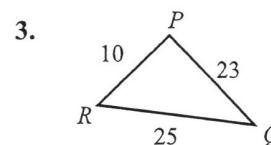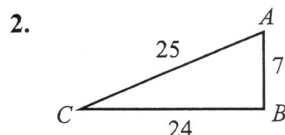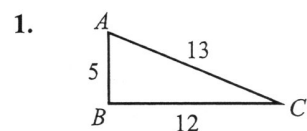
10. In $\triangle ABC$ as shown on the right, $AB = 16$ cm, $BC = 30$ cm and $AC = 34$ cm.
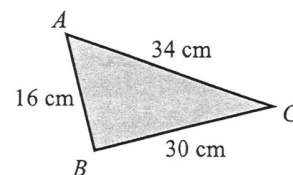
    (a) Show that $\triangle ABC$ is a right-angled triangle.

    (b) Find the area of $\triangle ABC$.

11. In $\triangle ABC$ as shown on the right, $AB = 2k$ cm, $AC = 1.5k$ cm and $BC = 2.5k$ cm.
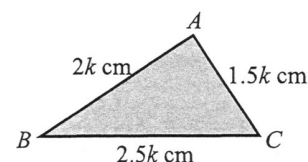
    (a) Show that $\triangle ABC$ is a right-angled triangle.

    (b) Express the area of $\triangle ABC$ in terms of k.

12. In $\triangle PQR$ as shown on the right, $PQ = 17k$ cm, $QR = 15k$ cm and $PR = 8k$ cm, where k is a positive number.
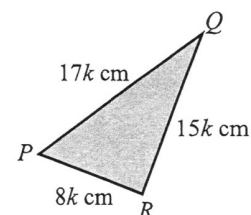
    (a) Show that $\triangle PQR$ is a right-angled triangle.

    (b) If the area of $\triangle PQR$ is 240 cm^2, find the value of k.

13. In $\triangle DEF$ as shown on the right, $DE = 45$ cm, $EF = 53$ cm and $DF = 28$ cm.

    (a) Show that $\triangle DEF$ is a right-angled triangle.

    (b) If the area of a square is equal to that of $\triangle DEF$, find the length of a side of the square.

    (*Give the answer correct to 3 significant figures.*)
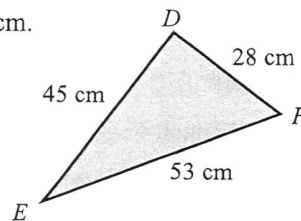
14. In the figure, D is a point on AC such that $BD \perp AC$. $AD = 5$, $BD = 12$ and $CD = 28.8$.

    (a) Find the lengths of AB and BC.

    (b) Is $\triangle ABC$ a right-angled triangle? Explain your answer.
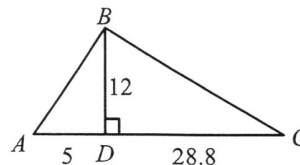
15. In the figure, PSR is a straight line. $QR = 2.4$, $RS = 3.2$, $QS = 4$ and $\angle PQS = \angle QPS$.

    (a) Show that $\triangle PQR$ is a right-angled triangle.

    (b) Find the length of PQ.

    (*Give the answer correct to 3 significant figures.*)
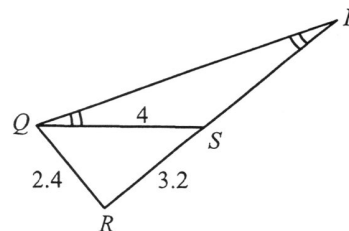
16. In the figure, $PQ = 8.5$ cm, $QR = 4$ cm, $RS = 6$ cm and $PS = 4.5$ cm.

    (a) Find the length of PR.

    (b) Show that $\triangle PRS$ is a right-angled triangle.

    (c) Find the area of quadrilateral $PQRS$.
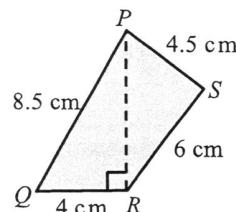
17. In the figure, $PQ = \sqrt{33}$ m, $QR = \sqrt{145}$ m, $RS = 12$ m and $PS = 16$ m.

    (a) Show that $\triangle QRS$ is a right-angled triangle.

    (b) Find the area of quadrilateral $PQRS$.

   (*Give the answer correct to 3 significant figures.*)
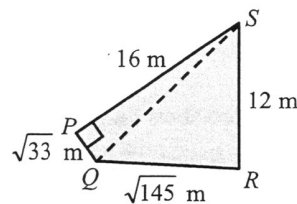
18. In the figure, D and E are two points on BC and AC respectively. $AC = 14$ cm, $BD = 16.5$ cm, $CD = 6$ cm and $DE = 10$ cm. It is given that $AC : AE = 7 : 3$.

    (a) Show that $\triangle ABC$ is a right-angled triangle.

    (b) Find the perimeter and the area of quadrilateral $ABDE$.
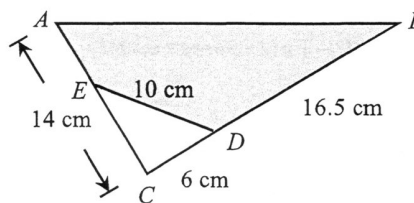
19. The figure shows a triangle ABC, where x and y are positive numbers. Show that $\triangle ABC$ is a right-angled triangle.

10
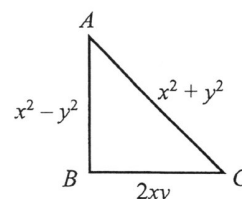
Applications of Pythagoras' Theorem and its Converse

Exercise 10D

1. For a 12-inch 4 : 3 tablet computer, the length of the diagonal of the monitor is 12 inches and the ratio of the length to the width is 4 : 3. By taking 1 inch as 2.54 cm, find the length and the width of the monitor in cm.

2. In the figure, a ladder AB of length 85 cm leans against a vertical wall. The foot B of the ladder is on the horizontal ground and it is 36 cm apart from the foot C of the wall.
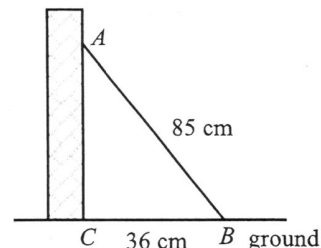
   (a) Find the distance between A and C.

   (b) Martin pushes the foot of the ladder in the horizontal direction 23 cm towards the wall. He claims that the top of the ladder will rise by 7 cm. Do you agree? Explain your answer.

3. A designer designs a square logo of side 9 cm.

   (a) Find the length of the diagonal of the logo.

   (b) The designer makes a photocopy of the logo by enlarging the length of its diagonal to 16 cm. Find the area of the logo of the photocopy.

4. The figure shows a facility $ABCD$ of height 4 m. The tops of the two slides AB and DC are 3 m apart, and their bottoms are 10 m apart. It is given that slide AB is 6 m long. Find the length of the slide DC.
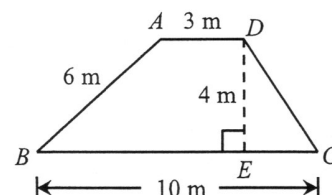
5. The figure shows a paper card ABC in the form of a right-angled triangle with $AC = 8$ cm. The paper card is folded along a line segment BD so that the vertex A falls on the side BC at a point A'. It is given that $A'C = 4$ cm.
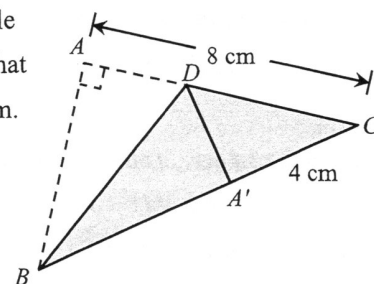
   (a) Find the length of $A'D$.

   (b) Find the area and the perimeter of $\triangle BCD$.

6. The figure shows a triangular playground XYZ. Kelly walks 140 steps from X to Y and 171 steps from Y to Z, then 221 steps back to X.
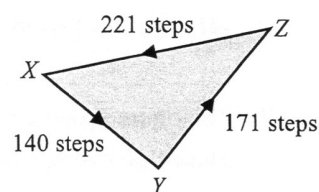
   (a) Is the shape of the playground a right-angled triangle? Explain your answer.

   (b) If Kelly walks a total distance of 212.8 m, find the area of the playground.

7. In the figure, a pole AB 80 cm long is erected on the horizontal ground and its position is fixed by two steel wires AC and AD of lengths 82 cm and 89 cm respectively. If B and C are 18 cm apart, find the distance between C and D.
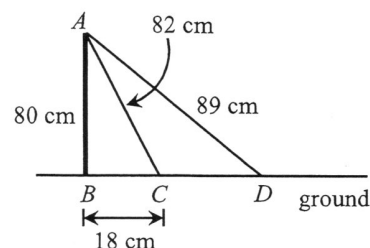
8. Two cars X and Y leave a car park A at the same time. Car X travels along a straight road AB at a constant speed of 96 km/h. Car Y travels along another straight road AC at a constant speed of 110 km/h. After 1.5 hours, the distance between the two cars is 219 km as shown.
    (a) Are the roads AB and AC perpendicular to each other? Explain your answer.
    (b) If the two cars continue to travel for another 2 hours, find the distance between the two cars.
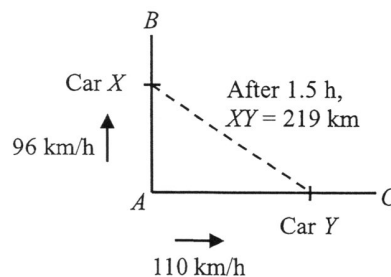
9. The figure shows a triangular swimming pool ABC, where $AB = 45$ m, $BC = 28$ m and $AC = 53$ m.
    (a) Find the area of the swimming pool.
    (b) Now, a lane rope is placed to connect B and a point D on AC in the swimming pool. Find the shortest length of the lane rope.
    (*Give the answer correct to 3 significant figures.*)
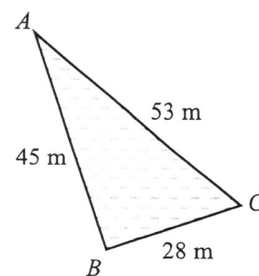
Properties of Surd

For any positive numbers a and b, $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$ and $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$.

(e.g. $\sqrt{4 \times 9} = \sqrt{4} \times \sqrt{9} = 2 \times 3 = 6$ $\qquad \sqrt{\dfrac{16}{49}} = \dfrac{\sqrt{16}}{\sqrt{49}} = \dfrac{4}{7}$)

A surd is said to be in its *simplest* form $p\sqrt{q}$ (q is an integer) if q has no square number factor other than 1.

(e.g. $3\sqrt{5}$ is a surd in its simplest form but $3\sqrt{8}$ is not.)

Exercise 10F(1)

Express each of the following surds in its simplest form. **(1 – 7)**

1. (a) $\sqrt{20}$ (b) $\sqrt{75}$ (c) $\sqrt{99}$

2. (a) $\sqrt{147}$ (b) $\sqrt{180}$ (c) $\sqrt{252}$

3. (a) $\sqrt{200}$ (b) $\sqrt{294}$ (c) $\sqrt{432}$

4. (a) $\sqrt{48} \times \sqrt{3}$ (b) $\sqrt{12} \times \sqrt{75}$ (c) $\sqrt{18} \times \sqrt{32}$

5. (a) $\sqrt{24} \times \sqrt{27}$ (b) $\sqrt{33} \times \sqrt{55}$ (c) $5\sqrt{52} \times 2\sqrt{39}$

6. (a) $\dfrac{\sqrt{150}}{\sqrt{6}}$ (b) $\sqrt{180} \div \sqrt{80}$ (c) $-\dfrac{\sqrt{112}}{2\sqrt{7}}$

7. (a) $\dfrac{\sqrt{128}}{4}$ (b) $\dfrac{\sqrt{125}}{15}$ (c) $\dfrac{\sqrt{90}}{30}$

Evaluate each of the following expressions without using a calculator. **(8 – 9)**

8. (a) $\sqrt{784}$ (b) $\sqrt{1024}$ (c) $\sqrt{0.0004}$

9. (a) $\sqrt{0.0121}$ (b) $\sqrt{8\dfrac{1}{36}}$ (c) $\sqrt{2\dfrac{34}{81}}$

Express each of the following surds in its simplest form. **(10 – 13)**

10. (a) $\sqrt{800}$ (b) $\sqrt{360}$ (c) $\sqrt{864}$

11. (a) $\sqrt{\dfrac{125}{36}}$ (b) $\sqrt{1\dfrac{7}{25}}$ (c) $\sqrt{3\dfrac{53}{64}}$

12. (a) $\sqrt{2} \times \sqrt{0.06} \times \sqrt{27}$ (b) $\sqrt{24} \times \sqrt{15} \times \sqrt{20}$

13. (a) $\dfrac{\sqrt{21}}{\sqrt{6} \times \sqrt{32}}$ (b) $\dfrac{\sqrt{14} \times \sqrt{50}}{\sqrt{63}}$

Rationalization of denominators and addition/subtraction of surds

Given an expression with $\sqrt{a}$ in the denominator, we can rationalize the denominator of the expression as follow:

$$\frac{K}{\sqrt{a}} = \frac{K \times \sqrt{a}}{\sqrt{a} \times \sqrt{a}} = \frac{K\sqrt{a}}{a}$$

(e.g. $\quad \frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$)

If the numbers inside the radical signs of surds in their simplest form are the same, they are called **like surds**

The result of addition and subtraction of like surds can be simplified : $a\sqrt{p} \pm b\sqrt{p} = (a \pm b)\sqrt{p}$

(e.g. $\quad 2\sqrt{5} + 4\sqrt{5} = (2 + 4)\sqrt{5} == 6\sqrt{5}$)

Exercise 10F(2)

1. (a) $3\sqrt{2} + 5\sqrt{2}$ 　　(b) $8\sqrt{3} - \sqrt{3}$ 　　(c) $\sqrt{11} - 6\sqrt{11}$

2. (a) $\sqrt{72} + \sqrt{50}$ 　　(b) $\sqrt{125} - \sqrt{80}$ 　　(c) $\sqrt{112} - \sqrt{448}$

3. (a) $7\sqrt{12} + \sqrt{27}$ 　　(b) $13\sqrt{18} - \sqrt{8}$ 　　(c) $8\sqrt{63} + 3\sqrt{175}$

Rationalize the denominator of each of the following expressions. **(4 – 6)**

4. (a) $\dfrac{1}{\sqrt{15}}$ 　　(b) $\dfrac{10}{\sqrt{10}}$ 　　(c) $\dfrac{7}{\sqrt{21}}$

5. (a) $\dfrac{8}{\sqrt{12}}$ 　　(b) $-\dfrac{25}{\sqrt{45}}$ 　　(c) $\dfrac{98}{\sqrt{28}}$

6. (a) $\sqrt{\dfrac{20}{7}}$ 　　(b) $\sqrt{\dfrac{8}{17}}$ 　　(c) $\sqrt{3\dfrac{1}{5}}$

7. (a) Rationalize the denominator of $\dfrac{1}{\sqrt{2}}$.

   (b) It is given that $\sqrt{2} \approx 1.414$. Estimate the value of $\dfrac{1}{\sqrt{2}}$ without using a calculator.

8. **(a)** Rationalize the denominator of $\sqrt{\dfrac{2}{5}}$.

    (b) It is given that $\sqrt{10} \approx 3.162$. Estimate the value of $10 \times \sqrt{\dfrac{2}{5}}$ without using a calculator.

Simplify each of the following expressions. **(9 – 12)**

9. **(a)** $\sqrt{28} + 3\sqrt{7} - \sqrt{63}$

    (b) $5\sqrt{27} - \sqrt{48} - \sqrt{108}$

10. **(a)** $\sqrt{162} - \sqrt{98} + 4\sqrt{50}$

    (b) $6\sqrt{54} + 2\sqrt{150} - 7\sqrt{24}$

11. **(a)** $2\sqrt{245} - 4\sqrt{125} + 3\sqrt{180}$

    (b) $3\sqrt{224} - 7\sqrt{126} - 5\sqrt{350}$

12. **(a)** $\sqrt{98} \times \sqrt{104} + \sqrt{52}$

    (b) $\dfrac{3\sqrt{132}}{\sqrt{66}} - \sqrt{32}$

In each of the following, simplify the expression and rationalize the denominator of the result if necessary. **(13 – 21)**

13. **(a)** $(\sqrt{12} + \sqrt{27})\sqrt{3}$

    (b) $\sqrt{18}(\sqrt{54} - \sqrt{24})$

14. **(a)** $\sqrt{125}(\sqrt{20} + 2\sqrt{45})$

    (b) $(3\sqrt{48} - \sqrt{75})\sqrt{135}$

15. **(a)** $\dfrac{4\sqrt{52} + \sqrt{117}}{5\sqrt{13}}$

    (b) $\dfrac{\sqrt{224} - 3\sqrt{126}}{\sqrt{28}}$

16. **(a)** $(\sqrt{8} + \sqrt{3})(\sqrt{8} - \sqrt{3})$

    (b) $(\sqrt{6} + \sqrt{10})(\sqrt{15} - \sqrt{9})$

17. **(a)** $(1 + \sqrt{3})^2$

    (b) $(\sqrt{2} + \sqrt{5})^2$

18. **(a)** $(\sqrt{7} - 4)^2$

    (b) $(\sqrt{10} - \sqrt{6})^2$

19. **(a)** $\dfrac{1}{\sqrt{5}} + \sqrt{5}$

    (b) $\dfrac{\sqrt{3}}{6} + \dfrac{6}{\sqrt{3}}$

20. **(a)** $\dfrac{6}{\sqrt{3}} + \sqrt{108}$

    (b) $\sqrt{117} - \dfrac{26}{\sqrt{52}}$

21. **(a)** $\dfrac{8}{\sqrt{15}} + \dfrac{\sqrt{60}}{3}$

    (b) $\dfrac{\sqrt{20}}{4} - \dfrac{9}{\sqrt{180}}$

22. Write down two possible values of q such that the value of $\sqrt{12} \times \sqrt{q} \times \sqrt{18}$ is a rational number.

23. **(a)** Simplify $(3+\sqrt{5})(3-\sqrt{5})$.

   (b) Rationalize the denominator of $\dfrac{3+\sqrt{5}}{3-\sqrt{5}}$.

24. **(a)** If a is a positive number, simplify $(\sqrt{a+1}+\sqrt{a})(\sqrt{a+1}-\sqrt{a})$.

   (b) Simplify each of the following expressions.

   (i) $\dfrac{1}{\sqrt{1}+\sqrt{2}}+\dfrac{1}{\sqrt{2}+\sqrt{3}}+\dfrac{1}{\sqrt{3}+\sqrt{4}}$

   (ii) $\dfrac{1}{6+\sqrt{37}}+\dfrac{1}{\sqrt{37}+\sqrt{38}}+\cdots+\dfrac{1}{\sqrt{62}+\sqrt{63}}+\dfrac{1}{\sqrt{63}+8}$

Multiple Choice Questions

1. Which of the following is a surd?

    A. $\sqrt{16}$

    B. $\sqrt{\dfrac{1}{49}}$

    C. $\sqrt{\dfrac{81}{1\,000}}$

    D. $\sqrt{0.09}$

2. Which of the following is a surd?

    A. $\sqrt{289}$

    B. $\sqrt{0.36}$

    C. $\sqrt{\dfrac{12}{27}}$

    D. $\sqrt{\dfrac{111}{400}}$

3. Which of the following has the smallest value?

    A. $\sqrt{13}$

    B. $\sqrt{\dfrac{100}{7}}$

    C. $\sqrt{3^2 + 4^2}$

    D. $\sqrt{10^2 - 9^2}$

4. Which of the following has the greatest value?

    A. $-\sqrt{85}$

   B. $-\sqrt{\dfrac{1\,010}{11}}$

    C. $-\sqrt{4^2 + 8^2}$

    D. $-\dfrac{26}{\sqrt{3^2 - 1^2}}$

5. Which of the following pairs of numbers have the same integral part?

A. $\sqrt{90}$ and $\sqrt{101}$

B. $\sqrt{123}$ and $\sqrt{143}$

C. $\sqrt{201}$ and $\sqrt{226}$

D. $\sqrt{689}$ and $\sqrt{759}$

6. Between which two consecutive numbers does $\sqrt{147}$ lie?

A. 11 and 12

B. 12 and 13

C. 13 and 14

D. 14 and 15

7. Which of the following numbers lie between 20 and 21?

I. $\sqrt{414}$

II. $\sqrt{440}$

III. $\sqrt{444}$

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

8. Without using a calculator, arrange the following numbers in ascending order.

$$17, \sqrt{290}, \sqrt{\frac{577}{2}}$$

A. $17 < \sqrt{290} < \sqrt{\frac{577}{2}}$

B. $\sqrt{290} < 17 < \sqrt{\frac{577}{2}}$

C. $\sqrt{\frac{577}{2}} < 17 < \sqrt{290}$

D. $\sqrt{\frac{577}{2}} < \sqrt{290} < 17$

9. Without using a calculator, arrange the following numbers in descending order.
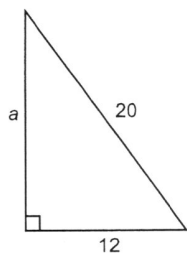
$$16, \sqrt{248}, 17, \sqrt{254}$$

    A. $17 > \sqrt{254} > \sqrt{248} > 16$

    B. $17 > 16 > \sqrt{254} > \sqrt{248}$

    C. $\sqrt{254} > 17 > 16 > \sqrt{248}$

    D. $\sqrt{254} > 17 > \sqrt{248} > 16$

10. If the area of a square is $256\,cm^2$, find the length of the side of the square.

    A. 14 cm

    B. 16 cm

    C. 32 cm

    D. 64 cm

11. If the area of a square poster is $841\,cm^2$, find the perimeter of the poster.

    A. 29 cm

    B. 58 cm

    C. 116 cm

    D. 210.25 cm

12. Find the value of a in the figure.
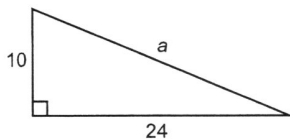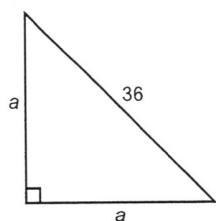

    A. 17

    B. 24

    C. 26

    D. 34

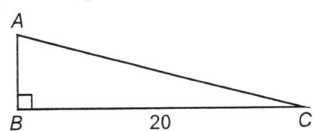13. Find the value of a in the figure.

    A. 13

    B. 16

    C. 20

    D. 23

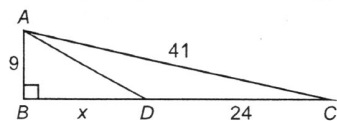14. Find the value of a in the figure.

    A. 6

    B. 18

    C. 25.4 (corr. to 3 sig. fig.)

    D. 25.5 (corr. to 3 sig. fig.)

15. In the figure, $AC = 4AB$, $AC =$

    A. 20.7 (corr. to 3 sig. fig.).

    B. 24.

    C. 25.

    D. 46.2 (corr. to 3 sig. fig.).

16. In the figure, BDC is a straight line. Find the value of x.

    A. 8

    B. 16

    C. 39.7 (corr. to 3 sig. fig.)

    D. 40

17. Find the length of AB in the figure correct to 3 significant figures.
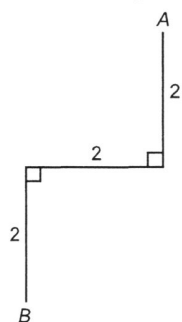
    A. 3.46

    B. 4.47

    C. 5.66

    D. 8.49

18. In the figure, $CD =$
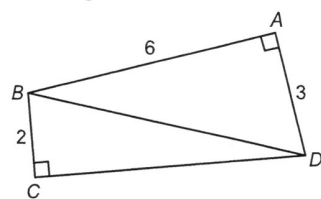
    A. $\sqrt{23}$.

    B. $\sqrt{31}$.

    C. $\sqrt{41}$.

    D. 7.

19. Find the area of $\triangle ABC$ in the figure.
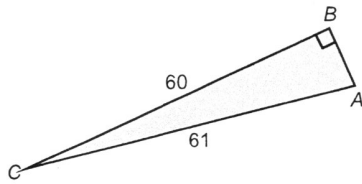
   A. 330
   B. 335.5
   C. 660
   D. 1 830

20. In the figure, find the perimeter of $ABCD$ correct to 3 significant figures.
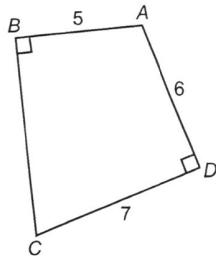
   A. 23.4
   B. 25.7
   C. 26.2
   D. 28.5

21. Find the area of trapezium $ABCD$ in the figure.

   A. $4.25\,\mathrm{m}^2$
   B. $4.5\,\mathrm{m}^2$
   C. $6\,\mathrm{m}^2$
   D. $7.5\,\mathrm{m}^2$

22. Find the length of AE in the figure correct to 3 significant figures.
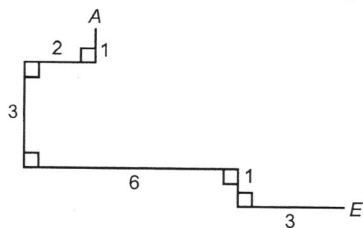
A. 8.60
B. 10.4
C. 14.2
D. 16.0

23. In the figure, D is a point on BC such that $AD \perp BC$. Find the length of AD.
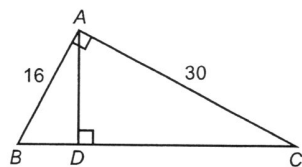
A. 12
B. 14
C. 14.1 (corr. to 3 sig. fig.)
D. 18.9 (corr. to 3 sig. fig.)

24. In the figure, $\triangle ABC \sim \triangle PQR$.
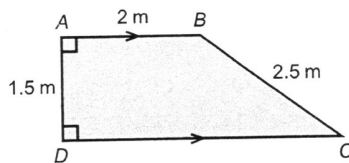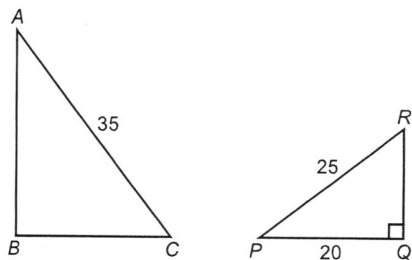

Which of the following must be true?
I. $\angle ABC = 90°$
II. $AB = 28$
III. Perimeter of $\triangle ABC = 84$

A. I and II only
B. I and III only
C. II and III only
D. I, II and III

25. In the figure, which of the following is a right angle?
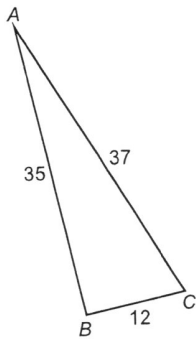
A. ∠BAC

B. ∠ABC

C. ∠ACB

D. None of the above

26. In the figure, which of the following is a right angle?
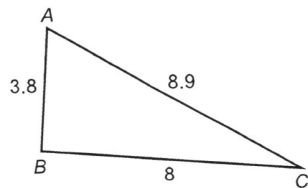
A. ∠BAC

B. ∠ABC

C. ∠ACB

D. None of the above

27. Which of the following is/are right-angled triangle(s)?

I.
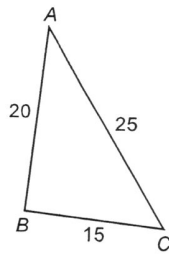
II.
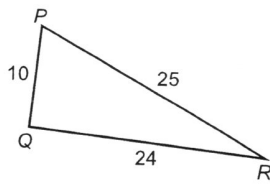
III.
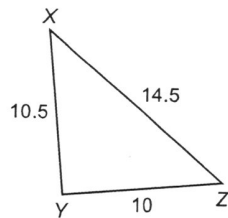
A. I only

B. I and II only

C. I and III only

D. II and III only

28. Which of the following is/are right-angled triangle(s)?

I.
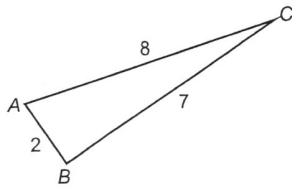
II.
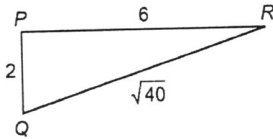
III.
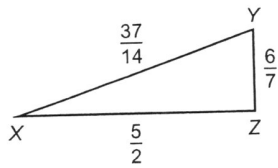
A. II only

B. III only

C. I and II only

D. II and III only

29. Find the area of $\triangle ABC$ in the figure.
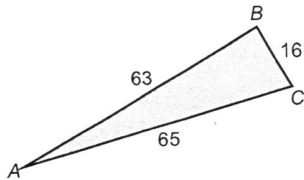
A. 504

B. 520

C. 1 008

D. 2 047.5

30. In the figure, D is a point on BC such that $AD \perp BC$.
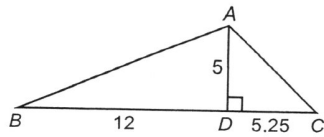
Which of the following must be true?

I. $AB = 13$

II. $AC = 7.25$

III. $\triangle ABC$ is a right-angled triangle.

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

31. In the figure, $\triangle ABC \sim \triangle PQR$.
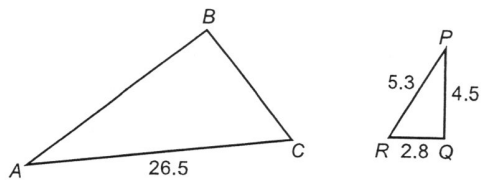
Which of the following must be true?

I. $\angle PQR = 90°$

II. $BC = 14$

III. Area of $\triangle ABC = 315$

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

32. In the figure, ADC is a straight line. Which of the following is/are right-angled triangle(s)?
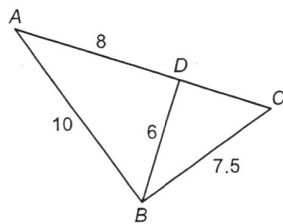
    I. $\triangle ABD$

    II. $\triangle BCD$

    III. $\triangle ABC$

    A. I only

    B. II only

    C. I and II only

    D. I, II and III

33. Which of the following can be put inside a rectangular box of length 20 cm and width 10 cm horizontally?

    I.   A rod of length 22 cm

    II. A rod of length 22.5 cm

    III. A rod of length 23 cm

    A. I only

    B. I and II only

    C. I, II and III

    D. None of the above

34. In the figure, a flagpole AB of 3 m high is perpendicular to the ground. Find the length of the shadow CB of the flagpole.
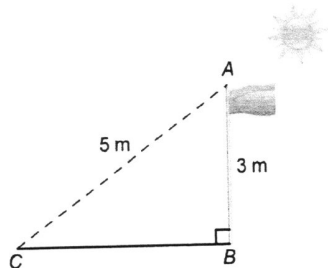
    A. 3 m

    B. 4 m

    C. 5 m

    D. 5.83 m (corr. to 3 sig. fig.)

35. Find the length of *AB* in the figure.
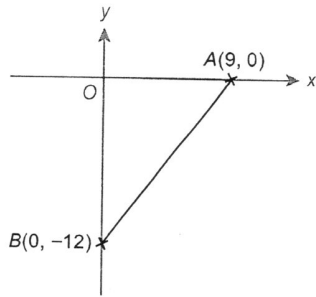
    A. 7.94 units (corr. to 3 sig. fig.)

    B. 9 units

    C. 12 units

    D. 15 units

36. In the figure, $A(-3, 4)$, $B(-3, -2)$ and $C(5, -2)$ are vertices of a right-angled triangle *ABC*. Find the perimeter of $\triangle ABC$.
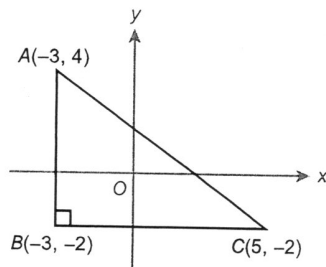
    A. 10 units

    B. 24 units

    C. 30 units

    D. 40 units

37. The diagonal of a square lawn is $\sqrt{18}$ km. Find the area of the lawn.

    A. 6 km^2

    B. 9 km^2

    C. 17.0 km^2 (corr. to 3 sig. fig.)

    D. 18 km^2

38. A ladder of 3 m long leans against a vertical wall and its upper end is 2 m above the ground. If the upper end of the ladder slides down by 1 m, find the distance moved by the lower end of the ladder to the left correct to 2 decimal places.
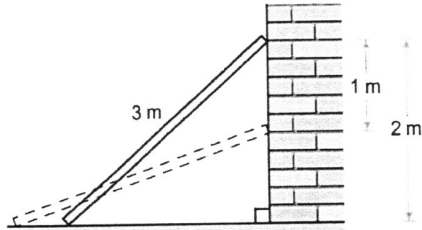
   A. 0.59 m

   B. 1.00 m

   C. 2.24 m

   D. 2.83 m

39. At 10 a.m., a train is 20 km due north of a building. The train travels due west at a constant speed and it will be 100 km away from the building at noon. Find the speed of the train correct to the nearest integer.

   A. 40 km/h

   B. 49 km/h

   C. 51 km/h

   D. 98 km/h

40. The figure shows a right prism $ABCDEF$. The base ABC is a triangle with $AB = 6$ cm, $BC = 17.5$ cm and $AC = 18.5$ cm. If $CD = 13$ cm, find the volume of the prism.
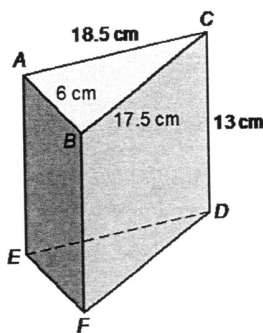
   A. 546 cm^3

   B. 682.5 cm^3

   C. 721.5 cm^3

   D. 2 104.375 cm^3

41. The perimeter of a rectangular carpet is 56 cm and the ratio of the length to the width is 4 : 3. Find the length of the diagonal of the carpet.

   A. 5 cm

   B. 20 cm

   C. 28 cm

   D. 40 cm

42. In the figure, ADC is a straight line on the horizontal ground. Two ropes are fixed at points A and D, and connected to the top of a rod BC.
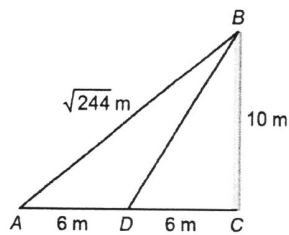
   Which of the following must be true?

   I. $BC \perp AC$

   II. $AB = 2BD$

   III. The shortest distance from C to BD is 5 m.

   A. I only

   B. I and III only

   C. II and III only

   D. I, II and III

43. Simplify $\sqrt{13^3}$.

   A. $\sqrt{13}$

   B. $13\sqrt{13}$

   C. 169

   D. $169\sqrt{13}$

44. Simplify $\sqrt{6^8}$.

   A. 216

   B. $216\sqrt{6}$

   C. 1 296

   D. $1\,296\sqrt{6}$

45. Simplify $\sqrt{3^5 \cdot 7^4}$.

    A. $21\sqrt{3}$

    B. $441\sqrt{3}$

    C. $441\sqrt{7}$

    D. $194\,481\sqrt{21}$

46. Simplify $\sqrt{1\,296}$.

    A. $12\sqrt{3}$

    B. $18\sqrt{2}$

    C. $18\sqrt{3}$

    D. 36

47. Simplify $\sqrt{147}$.

    A. $3\sqrt{7}$

    B. $7\sqrt{3}$

    C. $9\sqrt{7}$

    D. $49\sqrt{3}$

48. Simplify $\sqrt{\dfrac{2}{98}}$.

    A. $\dfrac{1}{7}$

    B. $\dfrac{1}{\sqrt{7}}$

    C. $\dfrac{\sqrt{2}}{7}$

    D. $\dfrac{2}{\sqrt{7}}$

49. Simplify $\sqrt{\dfrac{99}{64}}$.

    A. $\dfrac{9}{8}$

    B. $\dfrac{33}{8}$

    C. $\dfrac{3\sqrt{11}}{8}$

    D. $\dfrac{9\sqrt{11}}{8}$

50. Simplify $\sqrt{121a^6}$ where $a > 0$.

    A. $11a^2$

    B. $11a^3$

    C. $121a^2$

    D. $121a^3$

51. Simplify $\sqrt{\dfrac{a^3}{900}}$ where $a > 0$.

    A. $\dfrac{a}{30}$

    B. $\dfrac{a\sqrt{a}}{30}$

    C. $\dfrac{a}{90}$

    D. $\dfrac{a\sqrt{a}}{90}$

52. Simplify $\sqrt{6} \times \sqrt{24}$.

    A. 12

    B. 15

    C. 24

    D. $12\sqrt{6}$

53. Simplify $\dfrac{\sqrt{108}}{\sqrt{6}}$.

   A. $2\sqrt{3}$

   B. $3\sqrt{2}$

   C. 9

   D. 18

54. Simplify $\dfrac{\sqrt{28}\times\sqrt{14}}{\sqrt{7}}$.

   A. $2\sqrt{14}$

   B. 8

   C. $4\sqrt{14}$

   D. 28

55. Which of the following must be correct?

   I. $\sqrt{111}=3\sqrt{37}$

   II. $\dfrac{\sqrt{500}}{\sqrt{25}}=20$

   III. $\sqrt{2^{5}}=\sqrt{10}$

   A. I only

   B. I and II only

   C. II and III only

   D. None of the above

56. If $a=\sqrt{3}$ and $b=\sqrt{30}$, which of the following must be correct?

   I. $b\sqrt{10}=a$

   II. $a\sqrt{10}=b$

   III. $ab=9\sqrt{10}$

   A. I only

   B. II only

   C. II and III only

   D. I, II and III

57. Rationalize the denominator of $\dfrac{2}{\sqrt{3}}$.

    A. $\dfrac{2\sqrt{3}}{3}$

    B. $\sqrt{6}$

    C. $2\sqrt{6}$

    D. $6\sqrt{3}$

58. Rationalize the denominator of $\dfrac{10}{\sqrt{2}}$.

    A. $\sqrt{5}$

    B. $2\sqrt{5}$

    C. $5\sqrt{2}$

    D. $10\sqrt{2}$

59. Rationalize the denominator of $\dfrac{\sqrt{2}}{\sqrt{26}}$.

    A. $\dfrac{1}{13}$

    B. $\dfrac{\sqrt{13}}{13}$

    C. $\sqrt{13}$

    D. 13

60. Rationalize the denominator of $\dfrac{5\sqrt{3}}{\sqrt{20}}$.

    A. $\dfrac{3}{4}$

    B. $\dfrac{\sqrt{3}}{2}$

    C. $\dfrac{\sqrt{15}}{2}$

    D. $10\sqrt{15}$

61. Rationalize the denominator of $\sqrt{\dfrac{8}{11}}$.

    A. $\dfrac{2\sqrt{22}}{11}$

    B. $\dfrac{2\sqrt{11}}{11}$

    C. $\dfrac{\sqrt{22}}{11}$

    D. $\dfrac{\sqrt{11}}{11}$

62. Rationalize the denominator of $\dfrac{\sqrt{5}}{\sqrt{2}\times\sqrt{6}}$.

    A. $\dfrac{5}{12}$

    B. $\dfrac{\sqrt{15}}{6}$

    C. $\dfrac{5\sqrt{3}}{6}$

    D. $2\sqrt{15}$

63. Simplify $\sqrt{80}+\sqrt{20}$.

    A. 10

    B. $4\sqrt{5}$

    C. $6\sqrt{3}$

    D. $6\sqrt{5}$

64. Simplify $\sqrt{384}-\sqrt{216}$.

    A. $2\sqrt{6}$

    B. $8\sqrt{2}$

    C. $2\sqrt{42}$

    D. $28\sqrt{6}$

65. Simplify $2\sqrt{338} - 15\sqrt{2}$.

   A. $9\sqrt{2}$

   B. $11\sqrt{2}$

   C. $52 - 15\sqrt{2}$

   D. $323\sqrt{2}$

66. Simplify $\dfrac{2\sqrt{5}}{5} - 10\sqrt{\dfrac{1}{125}}$.

   A. 0

   B. $\dfrac{\sqrt{5}}{5}$

   C. $\dfrac{2\sqrt{5}}{5}$

   D. $\dfrac{3\sqrt{5}}{5}$

67. Simplify $\sqrt{300} - \sqrt{75} + \sqrt{27}$.

   A. $2\sqrt{6}$

   B. $8\sqrt{3}$

   C. $6\sqrt{7}$

   D. $84\sqrt{3}$

68. Simplify $\sqrt{18a} + \dfrac{2\sqrt{a}}{\sqrt{2}}$ where $a > 0$.

   A. $8\sqrt{a}$

   B. $4\sqrt{2a}$

   C. $10\sqrt{2a}$

   D. $2\sqrt{5a}$

69. Simplify $(\sqrt{343} - \sqrt{32})(\sqrt{175} + \sqrt{2})$.

   A. 55

   B. $237 - 13\sqrt{14}$

   C. 237

   D. $1\,699 - 13\sqrt{14}$