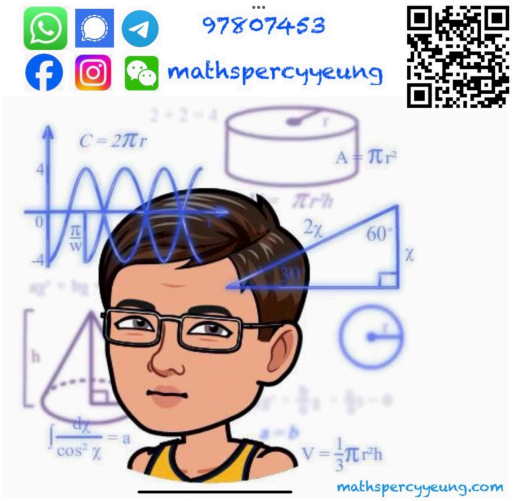
2023-2024-F4

MATHEMATICS



School SY

F.4 FIRST EXAMINATION 2023 – 2024

MATHEMATICS

Question-Answer Book

90 minutes

This paper must be answered in English

FULL MARKS: 90

INSTRUCTIONS

(1) After the announcement of the start of the exam, you should first write your name, class and class number in the spaces provided on Page 1.

(2) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

(3) There are **TWO** sections, A and B, in this paper. You are advised to finish Section A in about 30 minutes.

(4) In Section A, choose the best answer and write the appropriate letter in the box provided.

(5) In Section B, unless otherwise specified, all working must be clearly shown.

(6) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(7) The diagrams in this paper are not necessarily drawn to scale.

Section A: Multiple Choice Questions (30 marks)

1. Round off $0.047\dot{7}\dot{8}$ to 4 significant figures.

A. 0.0478 B. 0.0479 C. 0.04787 D. 0.04788 ☐

2. $\dfrac{27^x}{(3^x)^2} =$

A. 3^{-x}. B. 3. C. 3^x. D. 3^{3x-x^2}. ☐

3. $3+2a-a^2 =$

A. $-(a+1)(a+3)$. B. $-(a+1)(a-3)$. C. $-(a+3)(a-1)$. D. $-(a-1)(a-3)$. ☐

4. Which of the following is an irrational number?

A. $\sin 30^\circ$ B. π^0 C. $\sqrt{12}-\sqrt{3}$ D. $\sqrt{12}\times\sqrt{3}$

☐

5. $(i-1)^2 =$

A. -2. B. 0. C. $-2i$. D. $2-2i$. ☐

6. Find the range of k such that the quadratic equation $x^2+8x+k=0$ has no real roots.

A. $k<-16$ B. $k<16$ C. $k>-16$ D. $k>16$ ☐

7. Let k be a constant. Find the maximum value of $-x^2+6x+k$.

A. $k-9$ B. $k+9$ C. k D. 3

8. If $(x-3k)^2=9k^2$, then

A. $x=0$. B. $x=6k$.

C. $x=6k$ or $x=12k$. D. $x=0$ or $x=6k$.

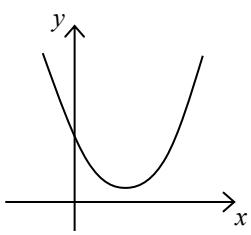
9. If the straight lines $L_1: 2x-hy+h=0$ and $L_2: x-y+2k=0$ have infinitely many points of intersection, then

A. $h=-2$ and $k=-1$. B. $h=-2$ and $k=\frac{-1}{2}$.

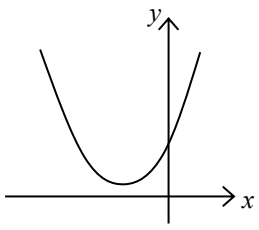
C. $h=2$ and $k=1$. D. $h=2$ and $k=\frac{1}{2}$.

10. If $a<0$, which of the following may represent the graph of $y=\left(\frac{x}{a}+1\right)^2+1$?

A.



B.



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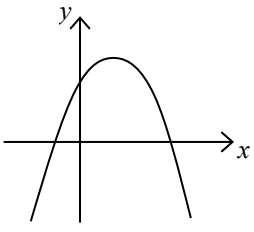
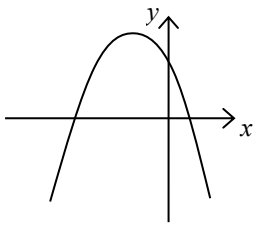


D.



11. Let $m^2+m=n^2+n=-2$ where $m \neq n$. Then $m^2-n=$

A. -3. B. -1. C. 1. D. 3.

12. If the equation $3x^2-kx+3=x$ has two equal negative roots, then $k=$

A. -7. B. -6. C. 5. D. 6.

13. If the roots of the equation $x^2+3kx+2k^2-k-1=0$ are α and β, where k is a constant, then $(\alpha-\beta)^2=$

A. k^2+4k+4. B. k^2-4k-4.

C. $5k^2-2k-2$. D. $5k^2+2k+2$.

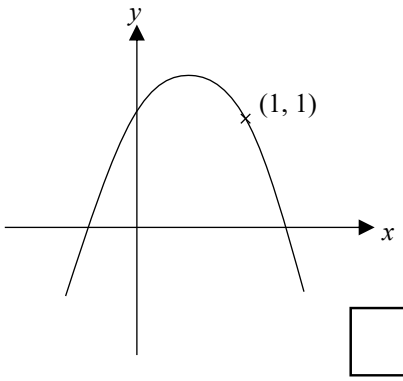
14. The figure shows the graph of $ay=x^2+bx+c$ which passes through $(1, 1)$. Which of the following are true?

I. $b<0$

II. $a \geq b+c$

III. $b^2-4c>0$

A. I and II only
B. I and III only
C. II and III only
D. I, II and III



15. The straight line $2x-y+4=0$ cuts the y-axis at the point A and passes through the point B. Let C be a point such that AC is a horizontal line. If the coordinates of the circumcentre of ΔABC are $(4, 8)$, find the slope of BC.

A. -3 B. -2 C. $\frac{-1}{2}$ D. $\frac{-1}{3}$

END OF SECTION A

Section B: Structural Questions (60 marks)

1. Simplify $\frac{(a^3b^{-4})^2}{ab}$ and express your answer with positive indices. (3 marks)

2. Make y the subject of the formula $Ay + C = \frac{y + A}{B}$. (3 marks)

3. Factorize

   (a) $3a^2 + 13a + 4,$

   (b) $6a^2 + 26a + 8 + ab + 4b.$

   (3 marks)

Answers written in the margins will not be marked

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4. Find the exact roots of the following quadratic equations.

(a) $(x-1)(x-3)=3(x-3)$

(b) $(x-1)(x-3)=1$

(5 marks)

5. Let $i=\sqrt{-1}$.

(a) Simplify $\dfrac{5}{2-i}$ and express the answer in the form $a+bi$, where a and b are real numbers.

(b) If the real part and the imaginary part of $\dfrac{5(i-c)}{2-i}$ are equal, where c is a real number, find c.

(5 marks)

Answers written in the margins will not be marked

6. The graph of $y = kx^2 + 4x + 2$ touches the x-axis at the point A.

(a) Find k.

(b) Find the coordinates of A.

(5 marks)

7. Find the coordinates of the point of intersection of $L_1 : x + 3y - 6 = 0$ and $L_2 : 2x - y - 5 = 0$. (3 marks)

Answers written in the margins will not be marked

8. Let α and β be the roots of the equation $x^2 + 2x + 10 = 0$.

(a) Find $\frac{1}{\alpha} + \frac{1}{\beta}$.

(b) Find a quadratic equation with roots $\frac{2}{\alpha}$ and $\frac{2}{\beta}$.

(5 marks)

Answers written in the margins will not be marked

9. It is given that $L_1 : 2x+3y-24=0$ and $L_2 : x-12y+k=0$ intersect at C on the x-axis.

L_1 and L_2 cut the y-axis at A and B respectively.

(a) Find k.

(b) Find the area of ΔABC.

(c) Let P be a point on BC such that the area of ΔAPC is 45. Find the coordinates of P.

(8 marks)

Answers written in the margins will not be marked

10. A wire of length 28 cm is cut into two parts. One part is bent into a square with side x cm. The other part is bent into a rectangle where the ratio of its length to its width is $3:1$. Denote the total area of the square and the rectangle by A cm^2.

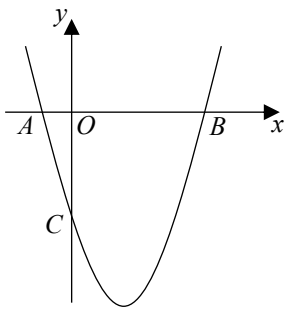
(a) Show that $A = \frac{7}{4}x^2 - \frac{21}{2}x + \frac{147}{4}$.

(b) By the method of completing the square, find the minimum value of A.

(7 marks)

Answers written in the margins will not be marked

11. In the figure, the graph of $y = x^2 - 2hx + k$ cuts the x-axis at points A and B and cuts the y-axis at point C where $h \neq -1$ and $k \neq 0$. It is given that $OB = OC$.



(a) Write down the coordinates of B in terms of k.

(b) Show that $2h + k + 1 = 0$.

(c) The axis of symmetry of the graph of $y = x^2 - 2hx + k$ intersects BC at D.

(i) Express the coordinates of D in terms of h.

(ii) A student claims that AD is an altitude of ΔABC. Do you agree? Explain your answer.

(iii) Find the coordinates of the orthocentre of ΔABC.

(13 marks)

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