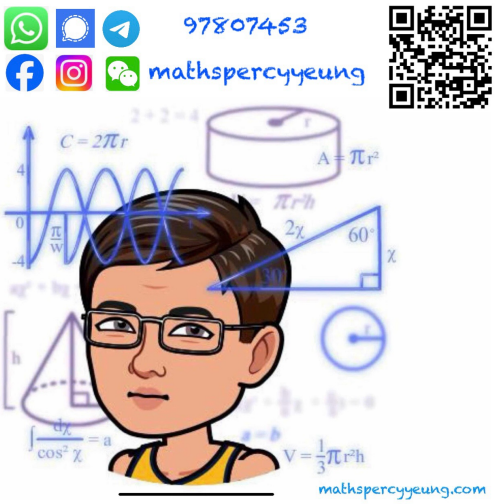

MATHEMATICS Compulsory Part
PAPER 1
Question-Answer Book

8:25 am – 10:25 am (2 hours)

This paper must be answered in English

INSTRUCTIONS

(1) After the announcement of the start of the examination, you should first write your Name, Class and Class Number in the space provided on Page 1.

(2) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

(3) Unless otherwise specified, all working must be clearly shown.

(4) Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.

(5) No extra time will be given to candidates for writing names or filling in the question number boxes after the 'Time is up' announcement.

(6) The full mark of this paper is 90.

1. Simplify $\dfrac{\left(m^{-2}n^{3}\right)^{4}}{n^{7}}$ and express your answer with positive indices. (3 marks)

2. (a) Factorize $4x^{2}-8x+3$.

(b) Factorize $4x^{2}-8x+3-4xy+6y$.

(3 marks)

3. It is given that $z=\dfrac{xy-2}{y+3x}$, express y in terms of x and z.

(3 marks)

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4. Solve $2\log x = \log(x+20)+1$. (4 marks)

5. Solve $3^{2x} = 4^{x+1}$. (3 marks)

6. In Figure 1 , OAB is an isosceles triangle with $OA = OB = x$.
C is a point on AO such that $BC \perp AO$. If $AC = 1$ and $BC = 4$,
find x.

(3 marks)

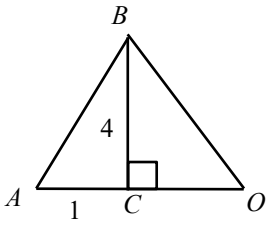

Figure 1

Answers written in the margins will not be marked

7. In Figure 2, $ABCD$ is a circle. AC and BD intersect at S. AC is produced to a point T. It is given that $\angle ABD = 60^\circ$, $\angle CTD = 38^\circ$ and $\angle BDC = \angle CDT$. Find $\angle DST$. (4 marks)

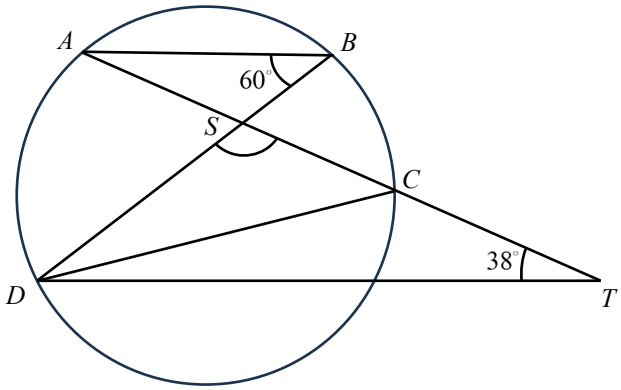

Figure 2

8. In Figure 3, $ABCD$ is a semi-circle with AD as a diameter. $\overset{\frown}{AC} : \overset{\frown}{CD} = 5:1$. Find $\angle ABC$. (4 marks)

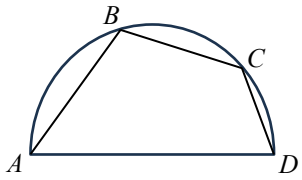

Figure 3

Answers written in the margins will not be marked

9. In Figure 4, $\angle CAB = 45^\circ$, $\angle ABC = 30^\circ$, $AC = 8\sqrt{2}$ cm, $BD = 13$ cm and $CD = 17$ cm.

(a) Find BC.

(b) Find the area of $ABDC$.

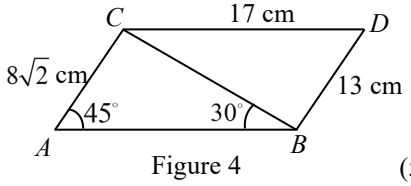

Figure 4

(5 marks)

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10. Let $i = \sqrt{-1}$.

(a) Express $\dfrac{11-2i}{3+4i}$ in the form of $a+bi$, where a and b are real constants.

(b) It is given that $\left(\dfrac{11-2i}{3+4i}\right)(k+i)$ is real, where k is a real number. Find k.

(5 marks)

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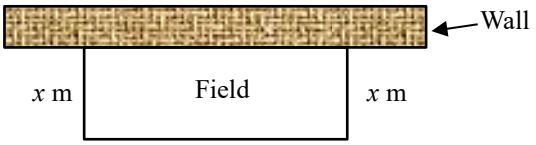
11. (a) Using method of completing square, express $10x - x^2$ in the form $a(x-h)^2 + k$, where a, h, k are constants.

(b)



A fence of length 20 m is used to bound a rectangular field next to a wall as shown (Only three sides of the field are bounded by the fence). It is known that two sides of the fence are of equal length x m. Abby claims that the area bounded can be greater than 50 m^2. Do you agree? Explain your answer.

(4 marks)

Answers written in the margins will not be marked

12. It is given that α and β are roots of $x^2 - 6x + 2 = 0$.

(a) Find the value of $\alpha^2 + \beta^2$.

(b) Form a quadratic equation with roots $\frac{\alpha - 1}{\beta}$ and $\frac{\beta - 1}{\alpha}$.

(5 marks)

Answers written in the margins will not be marked

13. The straight lines $L_1 : hx + 3y + 36 = 0$ and $L_2 : 6x - 8y + k = 0$ are perpendicular to each other.

(a) Find the value of h.

(b) L_1 intersects the x-axis and y-axis at A and B respectively. If L_2 divides AB in the ratio 1 : 5, find k.

(7 marks)

Answers written in the margins will not be marked

14. It is given that $f(x) = \dfrac{1}{x^2 - 3x + 2}$ and $g(x) = f(x) + 2f(2x)$.

(a) Show that $g(x) = \dfrac{3}{(2x-1)(x-2)}$.

(b) Solve $g(x) = f(x-1)$. (5 marks)

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15. Let $p(x)$ be a cubic polynomial. When $p(x)$ is divided by $x-3$, the remainder is 216. When $p(x)$ is divided by x^2+3, the quotient and the remainder are $4x+15$ and $-31x+c$ respectively, where c is a constant.

(a) Find c.

(b) Prove that $x+5$ is a factor of $p(x)$.

(c) Someone claims that all the roots of the equation $p(x)=0$ are real numbers. Do you agree? Explain your answer.

(8 marks)

Answers written in the margins will not be marked

16. Let $y = ax^b$, where a and b are constants.

(a) Express $\log_8 y$ as a linear function of $\log_2 x$.

(b) It is given that the intercept on the vertical axis of the graph of $\log_8 y$ against $\log_2 x$ is 3 and $y = 8$ when $x = 8$. Find the value of y when $x = 4$.

(6 marks)

Answers written in the margins will not be marked

17. Let $f(x) = \dfrac{\sin x + 3\sin(270^\circ - x)}{\cos(90^\circ + x)}$ for $0^\circ < x < 180^\circ$.

(a) Solve $f(x) = 2$.

(b) Someone claims that $f(x) = k$ has solution(s) for any real value of k. Do you agree? Explain your answer.

(7 marks)

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18. In Figure 5, ABC is a straight line and AD is a tangent to the circle BCD. CD is produced to E such that $AD = AE$.

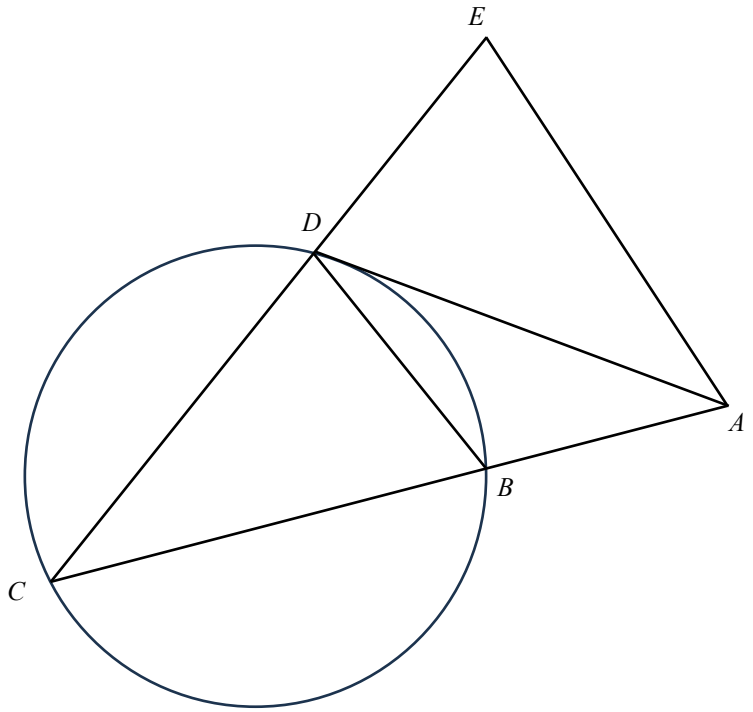

Figure 5

(a) Prove that $ABDE$ is a cyclic quadrilateral.

(b) Let O and M be the centre of circle $ABDE$ and the mid-point of AB respectively.

It is given that $BC : BD : CD = 10 : 7 : 9$ and $OM = 22$.

(i) Find the value of $\cos \angle BCD$.

(ii) By considering $\angle BOM$ or otherwise, find the radius of circle $ABDE$.

(iii) Let F be a point such that AE and AD are tangents to circle DEF. Let K be the centre of circle DEF.

(I) Someone claims that K lies on the circle $ABDE$. Do you agree? Explain your answer.

(II) Find OK.

(11 marks)

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