

School SY

FIRST TERM EXAMINATION (2022 - 2023)
Mathematics Compulsory Part
Paper 1

Form 4

Time: 1 hour 30 minutes

Total marks: 72 marks

Answer ALL questions.

Unless otherwise specified, all working steps must be clearly shown.

Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.

1. Simplify $\frac{(a^3b^{-4})^2}{ab}$ and express your answer with positive indices. (3 marks)

2. Factorize

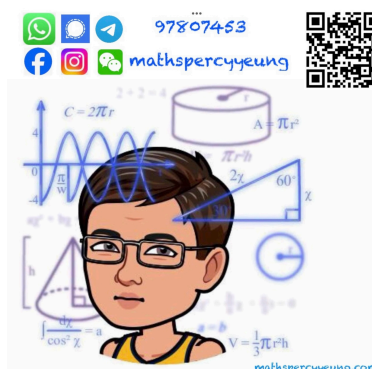
(a) $a^2 - 6ab - 7b^2$,

(b) $a^2 - 6ab - 7b^2 + 4a - 28b$.

3. Find the exact roots of the following quadratic equations.

(a) $2x^2 - 7x - 15 = 0$

(b) $x^2 - 2 + 3x = 0$



4. Simplify $\frac{1}{x+1} + \frac{3}{x^2 - x - 2}$. (3 marks)

5. Let L be the straight line passing through the points $(-3, 0)$ and $(5, -4)$.

(a) Find the equation of L .

(b) If the straight line $x + y + 1 = 0$ and L intersect at P , find the coordinates of P .

(5 marks)

6. The quadratic equation $2x^2 + 5x + (k + 1) = 0$ has two distinct real roots. If k is an integer, find the maximum value of k .

(4 marks)

7. Form a quadratic equation in x with roots $3 - 3i$ and $3 + 3i$, where $i = \sqrt{-1}$.

(4 marks)

8. Let $i = \sqrt{-1}$.

(a) Simplify $\frac{5}{1-2i}$ and express the answer in the form $a + bi$, where a and b are real numbers.

(b) If x and y are real numbers and $\frac{5}{1-2i} + (x + yi) = y + 4i$, find the values of x and y .

(6 marks)

9. Let $f(x)$ be a polynomial. When $f(x)$ is divided by $x^2 - 4x - 2$, the quotient and the remainder are $x - 1$ and 5 respectively. Find the remainder when $f(x)$ is divided by $x - 2$.

(3 marks)

10. The cost (\$C) of producing n watches by a certain company is given by: $C = 2n^2 - 100n + 3200$, where n is the number of watches produced per day. Using the method of completing the square, find the minimum daily cost and the corresponding number of watches produced per day. (4 marks)
11. If α and β are the roots of the quadratic equation $x^2 + kx + 10 = 0$ and $(\alpha - \beta)^2 = 10$, find the value(s) of k . (4 marks)
12. Let $f(x) = 2x^3 + x^2 + hx - 6$, where h is a constant. It is given that $f(x)$ is divisible by $x + 3$.
 (a) Find h .
 (b) Someone claims that all the roots of the equation $f(x) = 0$ are rational. Do you agree? Explain your answer. (5 marks)
13. $A(-2, 64)$ and $B(78, 4)$ are two points on the rectangular coordinate plane. L is the straight line that is perpendicular to AB and passes through the origin.
 (a) Find the equation of L .
 (b) $C(a, b)$ is a point such that the orthocentre of $\triangle ABC$ lies on L and $AB = BC$. It is given that $a > 0$.
 (i) Find the coordinates of C .
 (ii) Find the coordinates of the orthocentre of $\triangle ABC$. (11 marks)
14. It is given that the curve $y = -x^2 + 6x + 16$ cuts the x -axis at $A(a, 0)$ and $B(b, 0)$, where $a < b$. Denote the vertex of the curve by V .
 (a) Find the coordinates of A , B and V .
 (b) $D(c, d)$ lies on the curve and VD intersects the x -axis at M .
 (i) Express d in terms of c .
 (ii) It is given that BD is parallel to VA .
 (I) Find the coordinates of D .
 (II) Find the ratio of the area of $\triangle VAM$ to the area of $\triangle DBM$. (12 marks)

END OF PAPER