SY F4 2022-2023 Final Paper one

Form 4 Time: 2 hours

Total marks: 100 marks

Answer ALL questions.
Unless otherwise specified, all working steps must be clearly shown.
Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.

SECTION A(1) (33 marks)

1. Make y the subject of the formula $\dfrac{3}{x+y}=\dfrac{x}{y-2}$. (3 marks)

2. Simplify $\dfrac{a^5 b}{\left(a^{-2}b^3\right)^4}$ and express your answer with positive indices. (3 marks)

3. (a) Factorize $3x^2y-21x$.

   (b) Factorize $2xy^2-14y-3x^2y+21x$.

   (3 marks)

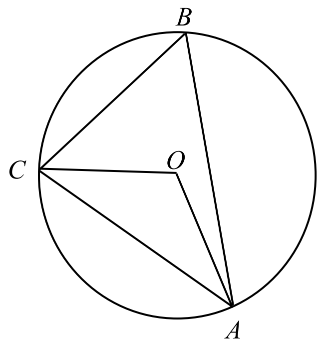
4. Consider the graph of $y=2x^2+6x-k$. Find the range of values of k if the graph has two different x-intercepts.

   (3 marks)

5. Solve $9^{2x+3}=27^{2x}$. (3 marks)

6. In the figure, O is the centre of the circle. If $\overset{\frown}{AB}:\overset{\frown}{AC}=3:2$ and $\angle OAC=40^\circ$, find $\angle BAC$.

   (4 marks)



7. Let $i=\sqrt{-1}$. It is given that $\dfrac{4}{1+ki}=2+ai$, where a and k are real constants. Find the value(s) of a. (4 marks)

8. A straight line $3x+4y-12=0$ cuts the y-axis and the straight line $y=-3$ at A and B respectively. Find the equation of the perpendicular bisector of AB. (5 marks)

9. The difference between two negative numbers is 4 and the sum of their squares is 100. Find the exact value of the smaller number.

   (5 marks)

SECTION A(2) (33 marks)

10. It is given that α and β are the roots of the equation $3x^2+2x+9=0$.

(a) Find the value of $\frac{1}{\alpha}+\frac{1}{\beta}$.

(b) Form a quadratic equation in x with roots $\frac{1}{2\alpha}$ and $\frac{1}{2\beta}$.

(5 marks)

11. Solve $4\sin^2\theta-2\cos^2\theta=\cos(270°-\theta)$ for $0°\le\theta\le 360°$. (5 marks)

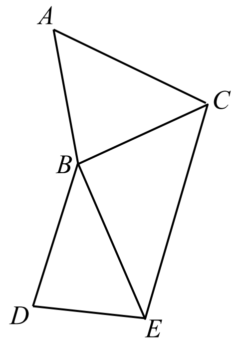
12. Solve $3\log_8 x+\log_2(x+12)=6$. (4 marks)

13. In the figure, it is given that $AB=6$ cm, $AC=8$ cm, $BD=7$ cm, $\angle BAC=55°$, $\angle DBE=50°$ and $\angle BED=60°$.

(a) Find BC and BE.

(b) If $70°<\angle CBE<100°$, find the range of the area of ΔCBE.

(7 marks)



14. Let a and b be constants. It is given that $x-1$ is a factor of $f(x)$. When $f(x)$ is divided by ax^2+x+b, the quotient is $x+1$ and the remainder is 4. When $f(x)$ is divided by $x-2$, the remainder is 28.

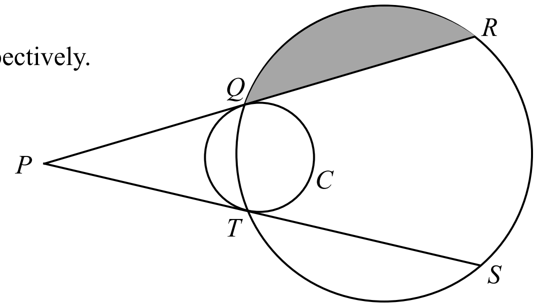
(a) Find the values of a and b.

(b) Solve $f(x-1)=4$. (7 marks)

15. In the figure, PQR and PTS are tangents to the circle C at Q and T respectively. It is given that $PQ=8$, $QR=16$ and radius of C is 3.

(a) Find the radius of the circle $QRST$.

(b) Find the area of the shaded region.



(5 marks)

SECTION B (34 marks)

16. $A(3,4)$, $B(8,4)$, $C(11,10)$ and D are points on a rectangular coordinate system. D lies on the line segment AC such that $\angle ABD = \angle BCD$.

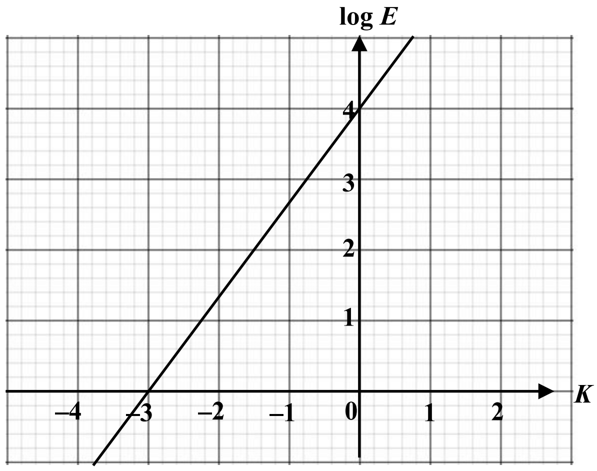
(a) (i) Show that $\Delta ADB \sim \Delta ABC$.

(ii) Hence, find the coordinates of D.

(b) A student claims that the orthocentres of ΔADB, ΔBCD and ΔABC are collinear. Do you agree? Explain your answer.

(7 marks)

17. The relationship between the energy E (measured in Joules) released during an earthquake and the magnitude K (on a K-Scale) is given by the graph of $\log E = mK + c$ in the figure.



(a) Write down the values of c and m.

(b) There are two earthquakes. If the energy released in the second earthquake is 15 times that in the first earthquake, find the increase of magnitude in K-scale.

(5 marks)

18. Let $f(x) = 4x^2 - 72x + 243$. The graph of $y = f(x)$ cuts the x-axis at A and B, it cuts the y-axis at C. Denote the vertex of the graph as V.

(a) Write down the coordinates of C.

(b) Find AB.

(c) Using the method of completing the square, find the coordinates of V.

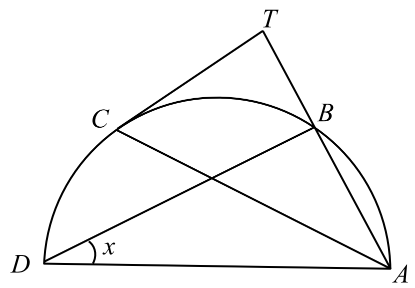
(d) It is given that S lies on the line segment VC and S lies in the first quadrant. Let the x-coordinate of S be a.

(i) Express the y-coordinate of S in terms of a.

(ii) Find the value of a such that the area of ΔCSO is equal to the area of ΔSAB.

(11 marks)

19. In the figure, $ABCD$ is a semicircle. CA is the angle bisector of $\angle BAD$. AB is produced to T such that TC is a tangent to the circle at C. Let $\angle BDA$ be x.



(a) Express $\angle CAD$ in terms of x.

(b) Prove that $TC // BD$.

(c) A student claims that TC can be longer than the radius of the semicircle $ABCD$. Do you agree? Explain your answer.

(d) Let P be the point on CA such that T, C, P and B are concyclic. Let $x = 20°$ and $AD = 10$ cm.

(i) Find $\angle TPB$.

(ii) Find the area of circle $TCPB$.

(11 marks)

END OF PAPER