SY F4 2021-2022 Final Paper one

Form 4

Time: 2 hours
Total marks: 100 marks

Answer ALL questions.
Unless otherwise specified, all working steps must be clearly shown.
Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.

SECTION A(1) (33 marks)

1. Simplify $\dfrac{p^{-9}}{\left(p^{-3}q^{4}\right)^{2}}$ and express your answer with positive indices. (3 marks)

2. (a) Factorize $4x^2-9$.
   (b) Factorize $4x^2-9-6xy+9y$.
   (3 marks)

3. It is given that $\dfrac{2y+3}{xy-2}=\dfrac{1}{x}$. Express x in terms of y. (3 marks)

4. Let $i=\sqrt{-1}$. Express $\dfrac{1+2i}{2-3i}$ in the form of $a+bi$, where a and b are real numbers. (3 marks)

5. The side lengths of two squares are differed by 4 cm and the sum of their areas is $80\ \text{cm}^2$. Find the side length of the smaller square. (4 marks)

6. Solve $4^{x+1}=8^{2x}$. (3 marks)

7. The equation $x^2+kx+k+3=0$ has equal roots. Find the value(s) of k. (4 marks)

8. Let L_1: $x+2y+3=0$ be a straight line. L_2 is another straight line parallel to L_1 and passes through $(-2,\ 3)$.
   (a) Find the equation of L_2.
   (b) Write down the x-intercept of L_2.
   (4 marks)

9. In Figure 1, $ABCD$ is a circle and $\overset{\frown}{AB}:\overset{\frown}{BC}:\overset{\frown}{ADC}=2:3:4$. AC intersects BD at E.

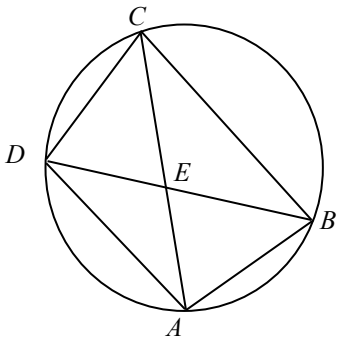
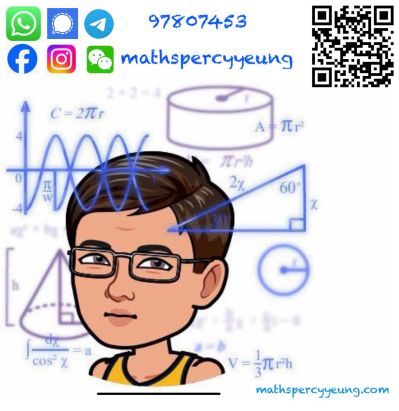


Figure 1

   (a) Find $\angle BAC$.
   (b) If BD is a diameter of the circle, find $\angle CEB$. (6 marks)



SECTION A(2) (33 marks)

10. It is given that α and β are the roots of the equation $2x^2 - 6x - 7 = 0$.

    (a) Find $\alpha^2 + \beta^2$.

    (b) Form a quadratic equation in x with roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

(6 marks)

11. Solve $\log(2x+3) + \log(x+10) = 2$. (4 marks)

12. Solve $\sin(180^\circ + \theta) + 3\cos^2\theta = 1$ for $0^\circ \le \theta < 360^\circ$. (5 marks)

13. Let a, b and c be constants. When $2x^3 + ax^2 - 2x + 14$ is divided by $x^2 - 3x + 4$, the quotient is $2x + b$ and the remainder is $-x + c$. Find the values of a, b and c.

(5 marks)

14. In Figure 2, ABC is a semi-circle. AC cuts the x-axis at D and $BD \perp AC$. It is given that $A = (44,\ 18)$ and $B = (-4,\ 32)$.

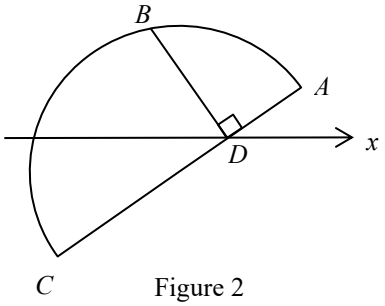


Figure 2

    (a) Find the coordinates of D.

    (b) Find the coordinates of C.

(8 marks)

15. In Figure 3, D is a point on AC. It is given that $\angle CBD = 20^\circ$, $\angle CAB = 30^\circ$, $AB = 5$ and $AD = 2$.

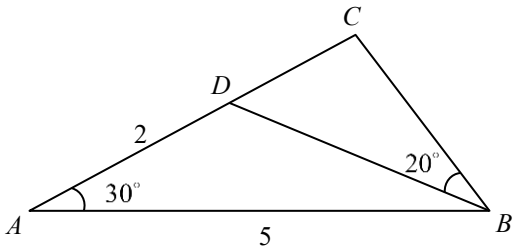


Figure 3

    (a) Find BD.

    (b) Find $\angle ACB$.

(5 marks)

SECTION B (34 marks)

16. Let $y = 16\left(10^{-3t}\right)$.

    (a) Express $\log y$ in the form $a + bt$ where a and b are constants.

    (b) Write down the slope of the graph of $\log y$ against t.

(3 marks)

17. (a) Show that $2x+3$ is a factor of $4x^3-25x-24$.

(b) The population P (in million) of bacteria in a sample t hours after 13:00 is given by $P=\dfrac{4(2)^{3t}+1}{2^t+1}$ million where $0\le t\le 10$. When will the number of bacteria reach 25 million? Express your answer to the nearest minute.

(7 marks)

18. In Figure 4, the graph of $y=x^2-2x+5$ passes through A. B is the vertex of the graph. Let $C(p,\ q)$ be a point on the graph.

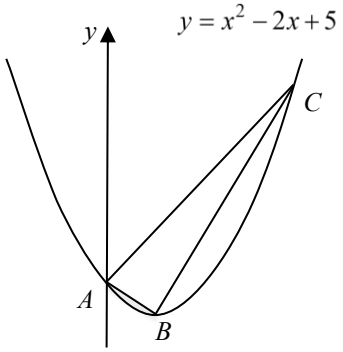


Figure 4

(a) Write down the coordinates of A.

(b) Find the coordinates of B.

(c) Express q in terms of p.

(d) It is given that $AC\perp AB$.

(i) Find the coordinates of C.

(ii) Find the distance from A to BC.

(11 marks)

19. In Figure 5, AB is the common chord of circle $ABCD$ and circle $ABEF$. AC intersects BD at G and AE intersects BF at H. It is given that BF is tangent to circle $ABCD$. Let $\angle ABF=a$ and $\angle EBF=b$.

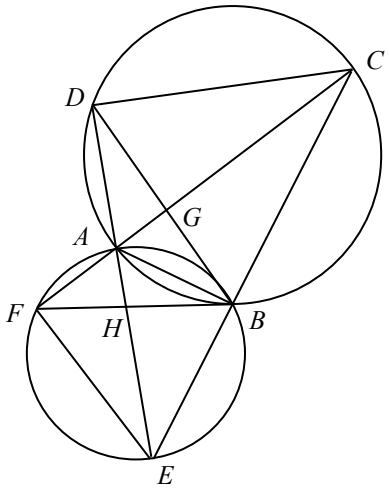


Figure 5

(a) Show that $\angle CBD=b$.

(b) Show that $\angle BDC=b$.

(c) It is given that $AHBG$ is cyclic.

(i) Find the value of b.

(ii) If $a = 45^\circ$ and $AB = 4$, find the area of quadrilateral $ABCD$.

(13 marks)

END OF PAPER