2022-2023 2nd Term Examination
F.4 Mathematics Extend Part (Module 2)
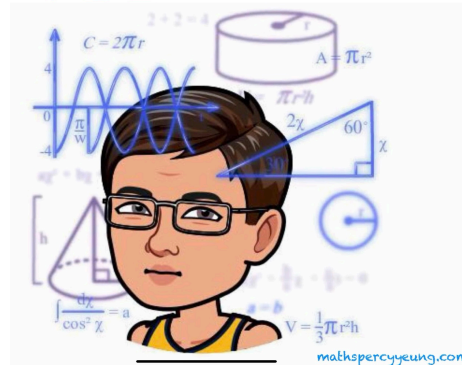
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Date: 13/06/2023

Time: 08:15-10:00

No. of pages: 19

Stationery required:

Score: /70

2 rough papers

1. This Paper consists of TWO sections, A and B.
2. Attempt **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
3. Unless otherwise specified, all working must be clearly shown.
4. The diagrams in this paper are not necessarily drawn to scale.
5. Unless otherwise specified, numerical answers must be exact.

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

Section A (40 marks)

1. Let $q(x) = \dfrac{3x}{2x^2 - 5}$. Prove that $q(1 + h) - q(1) = \dfrac{2h^2 + 7h}{2h^2 + 4h - 3}$.

    Hence, find $q'(1)$ from first principles.

    (4 marks)

2. (a) Using mathematical induction, prove that

$$\sum_{r=1}^{n+1} \frac{1}{r(r+1)} = \frac{n+1}{n+2}$$

for all positive integers n.

(b) Hence, evaluate $\displaystyle\sum_{r=100}^{2\,019} \left(\frac{4}{r+1}\right)\left(\frac{25}{r}\right)$.

(7 marks)

3. Let n be a positive integer. In the expansion of $(1+3x)^n \left(x - \dfrac{4}{x} \right)^2$, the constant term is

5 176.

   (a)   Find the value of n.

   (b)   Find the coefficient of x^2 in the expansion.

(5 marks)

4. (a) Prove that $\sin 3x = 3\sin x - 4\sin^3 x$.

    (b) Using (a), solve the equation $\csc^3 x + 6\csc^2 x - 8 = 0$, where $\dfrac{\pi}{3} < x < \dfrac{2\pi}{3}$.

(5 marks)

5. Let $y = -xe^{-\frac{3}{x^2}}$, where $x \neq 0$.

(a) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$.

(b) Someone claims that the graph of $y = -xe^{-\frac{3}{x^2}}$ has three points of inflexion. Do you agree? Explain your answer.

(6 marks)

6. $P\left(\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$ is a point on the curve $4x^2 - 2x \cos y - y^2 = 0$.

    (a) Find the value of $\dfrac{dy}{dx}$ at P.

    (b) Find the equation of the tangent to the curve at P.

(6 marks)

7. Let $y = \sqrt{\dfrac{3x-4}{4x+5}}$, $x > \dfrac{4}{3}$. Find $\dfrac{dy}{dx}$ by logarithmic differentiation.

(4 marks)

8. Find $\dfrac{d^2y}{dx^2}$ of the function $y = e^x \sin 2x$.

(3 marks)

Section B (30 marks)

9.   Let $f(x) = 2x(3-x)^2$.

    (a)   For the graph of $y = f(x)$, find

          (i)   the x- and y-intercepts,   (2 marks)

          (ii) $f'(x)$ and $f''(x)$,   (2 marks)

          (iii)   the maximum and minimum points, and   (4 marks)

          (iv)   the point of inflexion.   (2 marks)

    (b)   Sketch the graph of $y = f(x)$ for $-1 \le x \le 4$.

                                                         (2 marks)

10. Water is pumped at a constant rate of 1 600 cm^3/s into an empty container, whose shape is an inverted right pyramid with a square base. The length of each side of the square base is $64\sqrt{2}$ cm and the height of the container is 96 cm.

(a) Find the rate at which the depth of water is rising when the depth of water in the container is 20 cm. (4 marks)

(b) Find the rate at which the depth of water is rising after 90 seconds.

(*Give the answer correct to 3 significant figures.*)

(3 marks)

11.   In the figure, *AB* represents a vertical building of height 10 m and *BC* represents a

vertical flagpole of height 5 m. A point P on the horizontal ground is x m from the building. The angles of elevation of B and C from P are θ and ϕ respectively.

Let $\angle BPC = \alpha$.   (Remark : θ, ϕ and α are in radian measure)

(a)   Show that $\tan \alpha = \dfrac{5x}{x^2 + 150}$.   (2 marks)

(b)   Using the result of (a), express $\dfrac{d\alpha}{dx}$ in terms of x.   (3 marks)

(c)   Hence, find the maximum value of $\angle BPC$ correct to 3 significant figures.

(6 marks)

END OF PAPER