QC F4 2023-2024 Yearly Exam I

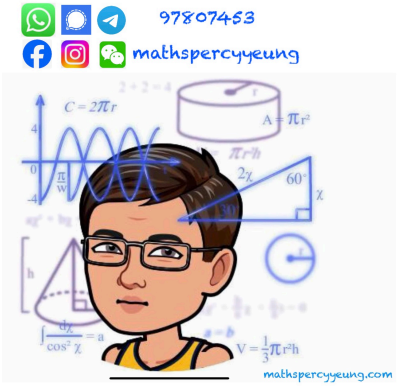
S.4 Mathematics

Compulsory Part (Paper 1)

Question-Answer Book

Date: 7th June 2024

Time: 8:30 am – 10:00 am



INSTRUCTIONS

1. Write your class and class number in the spaces provided on this cover.

2. This paper consists of THREE sections, A(1),A(2) and B. Each section carries equal marks.

3. Attempts ALL questions in this paper. Write your answer in the spaces provided in this Question-Answer Book. Supplementary answer sheets will be supplied on request. Write your class and class number on each sheet and put them inside this book.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

6. The diagrams in this paper are not necessarily drawn to scale.

Class	
Class Number	

	Teacher's Use Only	
	Max. Mark	**Marks**
Section A(1) Question No.	**40**	
1	**4**	
2	**4**	
3	**4**	
4	**6**	
5	**7**	
6	**7**	
7	**8**	
Section A(2) Question No.	**40**	
8	**7**	
9	**7**	
10	**8**	
11	**8**	
12	**10**	
Section B Question No.	**40**	
13a	**4**	
13b	**8**	
14a	**4**	
14b	**9**	
15a	**6**	
15b	**9**	
Total	**120**	

Section A(1) (40 marks)

1. Simplify $\dfrac{6x^3y}{(3xy)^2}$ and express your answer with positive indices. (4 marks)

2. Make p the subject of the formula $\dfrac{p+2}{5q} = \dfrac{p}{3q} - 6$. (4 marks)

Answers written in the margins will not be marked.

3. Factorize

(a) $h^2 + 6h + 9$,

(b) $h^2 + 6h + 9 - h - 3$.

(4 marks)

4. If $\frac{54}{2x^2+5x-7} \equiv \frac{A}{x-1} + \frac{B}{2x+7}$, find the values of constants A and B. (6 marks)

Answers written in the margins will not be marked.

5. Suppose α and β are the roots of the quadratic equation $4x^2 + 6x - 5 = 0$.

(a) Find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ without solving the equation.

(b) Form a quadratic equation in x with integral coefficients whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

(7 marks)

Answers written in the margins will not be marked.

6. (a) Solve the inequality $\frac{x+3}{2} > -3(x-4)$.

(b) Find the smallest integer satisfying both inequalities $\frac{x+3}{2} > -3(x-4)$ and $1-7x \le 13$.

(7 marks)

Answers written in the margins will not be marked.

7. Let $f(x) = 2x^2 + kx + 9$, where k is a constant. When $f(x)$ is divided by $x + k$, the remainder is $6k$.

   (a) Find the value of k.

   (b) Someone claims that the equation $f(x - 1) = 4x + 5$ has an integral root. Do you agree? Explain your answer.

(8 marks)

Answers written in the margins will not be marked.

Section A(2) (40 marks)

8. It is given that $f(x)$ is partly constant and partly varies as x^2. Suppose that $f(2) = 7$ and $f(-1) = 1$.

   (a) Find $f(x)$.

   (4 marks)

   (b) Someone claims that roots of the equation $f(x) = x$ are all rational roots. Do you agree? Explain your answer.

   (3 marks)

Answers written in the margins will not be marked.

9. Suppose $1 + 3i$ is a root of the quadratic equation $x^2 + rx + s = 0$, where r and s are real numbers.

(a) Find the values of r and s. (5 marks)

(b) Is $2 + 6i$ a root of the equation $2x^2 + 2rx + 2s = 0$? Explain your answer. (2 marks)

Answers written in the margins will not be marked.

10. (a) Prove that $\cos(270° + x)\tan x + \cos x = \dfrac{1}{\cos x}$ is an identity.

(5 marks)

(b) Hence, solve $3\cos(270° + x)\tan x + 3\cos x = \dfrac{\sqrt{3}}{\sin x}$ for $0° \le x \le 360°$.

(3 marks)

Answers written in the margins will not be marked.

11. (a) Solve the simultaneous equations $\begin{cases} 9^{a+\frac{1}{2}} - 2^{b+1} = 11 \\ 9^a - 2^b = 1 \end{cases}$. (5 marks)

(b) Solve $\log_{2x-3}(4x-3) = 2$. (3 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

12. Let $f(x) = x^2 + 4kx + 8k^2 - 2k - 3$.

(a) Using the method of completing the square, express, in terms of k, the coordinates of the vertex of the graph of $y = f(x)$. (3 marks)

(b) Suppose the graph of $y = f(x)$ passes through the origin O with vertex V, and cuts the x-axis at A. Someone claims that the area of $\triangle OVA$ can be smaller than 1 square unit. Do you agree? Explain your answer. (7 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Section B (40 marks)

13. It is given that $f(x)$ is a cubic polynomial. When $f(x)$ is divided by $x^2 - x - 2$, the remainder is $4x$. When $f(x)$ is divided by $x - 3$, the remainder is 36 and $f(1) = 0$.

(a) Find $f(x)$.

(4 marks)

(b) Let $g(x) = f(x) + kx + 8$, where k is a constant. It is given that $g(x) \equiv (x - 2)(ax^2 + bx + c)$, where a, b and c are constants.

(i) Find a, b and c.

(ii) Solve the equation $g(x) = 0$.

(8 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

14. It is given that the equation of the straight line L_1 is $3x - 2y + k = 0$. L_1 passes through the point A (4, 3) and cuts the y-axis at B. L_2 is a straight line passing through the point (–12, 5) and intersects L_1 at B.

(a) Find the equation of L_2. (4 marks)

(b) C is a point lying on L_2 such that the circumference of the circumcircle of $\triangle ABC$ is 13π units. It is given that C lies in quadrant IV.

(i) Find the coordinates of C.

(ii) D is a point on L_2 such that the area of $\triangle ABC$: the area of $\triangle ABD = 2 : 1$.
If the x-coordinate of D is negative, find the coordinates of D.

(9 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

15. (a) In Figure 1, S is a point on PR such that $QS \perp PR$. Let $\angle RQS = \theta$, $\angle QPS = 2\theta$.

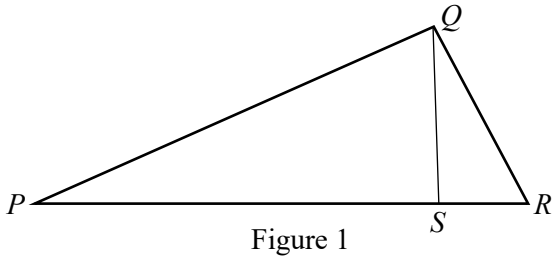


Figure 1

(i) Show that $PQ = PR$.

(ii) Hence, show that $\tan\theta = \dfrac{1-\cos 2\theta}{\sin 2\theta}$.

(6 marks)

(b) In Figure 2, a rectangle $EFGH$ is inscribed in an isosceles triangle ABC with $AC = BC$, $AB = 2a$ cm and $\angle CAB = 15°$, where a is a positive integer. Suppose that $HG = x$ cm.

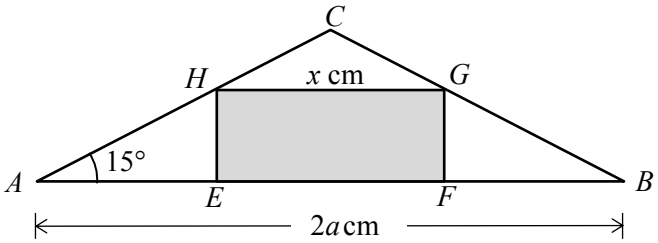


Figure 2

(i) Show that the area of $EFGH$ is $\dfrac{\left(2-\sqrt{3}\right)x}{2}(2a-x)\ \text{cm}^2$.

(ii) Find the least value of a such that the area of $EFGH$ exceeds 2 cm^2.

(9 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

End of Paper