Final Examination 2021-2022
F.5 Mathematics Compulsory Part
Paper 1

Time allowed: 2 hours 15 minutes

Name		Class:	(	)

INSTRUCTIONS

1. **Any unauthorized photocopying of the paper or any part of it is strictly prohibited.**

2. This paper consists of THREE sections, A(1), A(2) and B. Each section carries 35 marks.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided.

4. Graph paper and supplementary answer sheets will be supplied on request. Write your name and class on each graph paper and supplementary answer sheet.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

8. No extra time will be given to candidates for writing down their names after the 'Time is up' announcement.
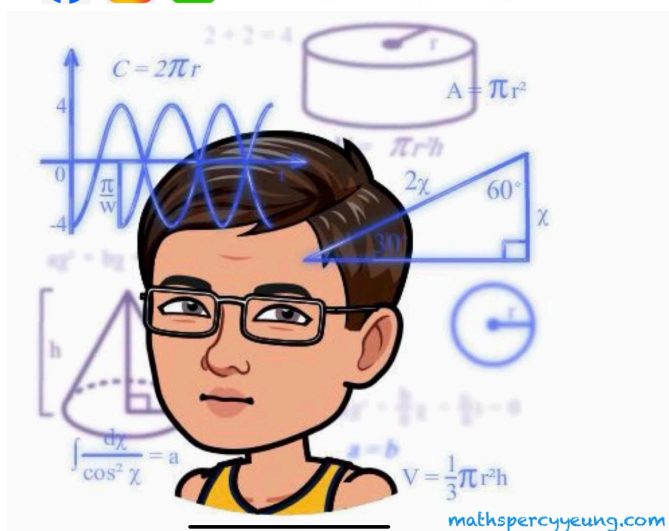
SECTION A(1)   (35 marks)

1.   Simplify $\left(2a^{-4}b^3\right)^6 \left(a^2b^{-9}\right)$ and express the answer with positive indices.   (3 marks)

2.   Let a, b and c be non-zero numbers such that $5a = 6b$ and $\dfrac{a}{c} = \dfrac{9}{7}$. Find $\dfrac{2a - 3c}{b + 5c}$.

   (3 marks)

3. A fair die is thrown twice. A fraction is formed by using the first number thrown as the denominator and the second number thrown as the numerator. Find the probability that the value of the fraction is greater than 1.

(3 marks)

4. Consider the compound inequality

$$2 - \frac{x-1}{3} < x + 5 \quad \text{or} \quad x - 3 \geq 0 \qquad \text{...............(*)}$$

(a) Solve (*).

(b) Write down the least integer satisfying (*).

(4 marks)

5. Factorize
    (a) $9x^2 - 25$,
    (b) $3x^2y - xy - 10y$,
    (c) $9x^2 - 25 - 6x^2y + 2xy + 20y$.

    (4 marks)

6. The coordinates of the points A and B are $(-4, -2)$ and $(3, 6)$ respectively. A' is the reflection image of A with respect to the y-axis. B is rotated anti-clockwise about the origin O through 90° to B'.
    (a) Write down the coordinates of A' and B'.
    (b) Are O, A' and B' collinear? Explain your answer.

    (4 marks)

7.   A hairdryer is sold at a discount of 30% on its marked price. After selling the hairdryer, the profit is $60 and the profit percentage is 40%.
      (a)   Find the cost price of the hairdryer.
      (b)   Find the marked price of the hairdryer.

(5 marks)

8. In Figure 1, a metal frame is formed by cutting the rectangle *EFGH* from a rectangular metal sheet *ABCD*. It is known that all the measurements in the figure are correct to the nearest cm.
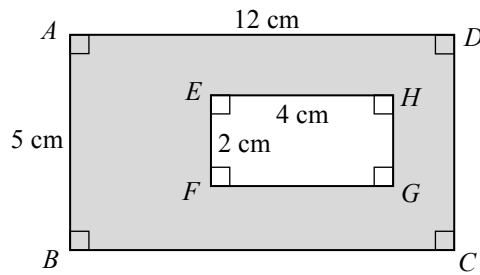

Figure 1

(a) Let x cm^2 be the actual area of *ABCD*. Find the range of values of x.

(b) Someone claims that the area of the frame can be less than 40 cm^2. Do you agree? Explain your answer.

(5 marks)

9.   In Figure 2, *PST* is a straight line. $PR = PQ$ and $\angle QSR = 22°$. Find $\angle RST$.
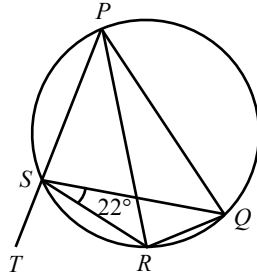
(4 marks)

Figure 2

SECTION A(2) **(35 marks)**

10. The stem-and-leaf diagram below shows the distribution of the ages of the patients waiting in a clinic.

Stem (tens)	Leaf (units)
5	8
6	2 3 3 4 a 6 6 7 7 8
7	0 0 1 2 4 5 6
8	0 2

It is given that the inter-quartile range of the distribution is 8.

(a) Find a. (2 marks)

(b) Now two more patients enter the clinic.

    (i) If the mode of the distribution increases by 1, write down the new median of the distribution.

    (ii) If the mean and the range of the distribution remain unchanged, write down the greatest possible standard deviation of the distribution and the corresponding ages of the two new patients.

(3 marks)

11. The price of a brand A pizza of size x inches is $\$P$. P is partly constant and partly varies as x^2. When $x = 10$, $P = 128$ and when $x = 14$, $P = 224$.

    (a) Find the price of a brand A pizza of size 12 inches.

    (4 marks)

    (b) Someone claims that the total price of two brand A pizzas of size 6 inches is higher than the price of a brand A pizza of size 12 inches. Is the claim correct? Explain your answer.

    (2 marks)

12. Let $p(x)$ be a polynomial and r be a non-zero constant. When $p(x)$ is divided by $x + r$, the quotient is $x^2 - rx + 2r^2$ and the remainder is $-4r^3$.

   (a) Prove that $p(x)$ is divisible by $x - r$. (3 marks)

   (b) Ray claims that all the roots of the equation $p(x) = 0$ are real. Do you agree? Explain your answer. (4 marks)

13. In Figure 3, *TAB* is a semi-circle with diameter *AB*. *AB* is produced to *C* such that *TC* is the tangent to the semi-circle at *T.*
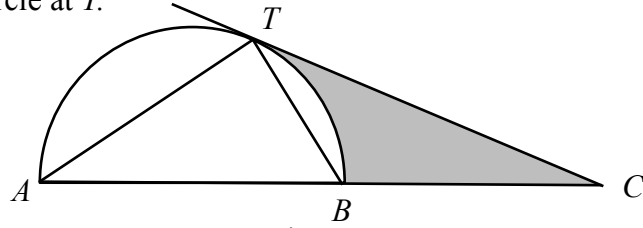

Figure 3

(a) Prove that $\triangle TBC \sim \triangle ATC$.

(2 marks)

(b) Suppose that *TC* = 36 cm and *BC* = 24 cm.
   (i) Find the diameter of the semi-circle.
   (ii) Someone claims that the area of the shaded region is greater than 140 cm². Do you agree? Explain your answer.

(6 marks)

14. The coordinates of points A and B are (2, 16) and (–4, 18) respectively. The circle C passes through A and B and intersects the x-axis at the points P and Q. It is given that $\angle ABP = 90°$.
    (a) Find the coordinates of P. (2 marks)
    (b) Find the equation of C. (3 marks)
    (c) Let R be a moving point in the rectangular coordinate plane such that R is equidistant from P and Q. Denote the locus of R by Γ.
        (i) Describe the geometric relationship between Γ and PQ.
        (ii) Suppose that Γ cuts the x-axis at point D. Someone claims that the area of $\triangle ADQ$ is the same as the area of $\triangle ABQ$. Do you agree? Explain your answer. (4 marks)

SECTION B   (35 marks)

15.   A box contains 3 red balls, 4 yellow balls and 5 blue balls. If 6 balls are randomly selected from the box at the same time,

    (a)   find the probability that exactly 1 red ball is drawn;   (2 marks)

    (b)   find the probability that less than 3 yellow balls are drawn.   (2 marks)

16. The table below shows the scores and standard scores of candidate A and candidate B in a test:

Candidate	Score	Standard score
A	82 marks	8
B	49 marks	−0.25

   (a) Find the mean and standard deviation of the test scores. (2 marks)

   (b) A candidate claims that the range of the test scores does not exceed 32 marks. Is the claim correct? Explain your answer. (2 marks)

17. The graph in Figure 4 shows the linear relation between $\log_4 x$ and $\log_8 y$. The intercept on the horizontal axis of the graph is 6 and the graph passes through the point $(9, -1)$. Express the relation between x and y in the form $y = Ax^k$, where A and k are constants.
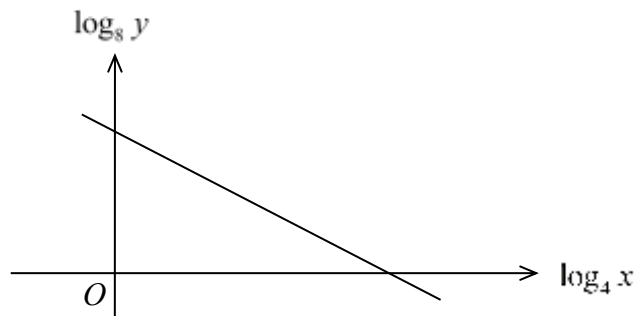
(3 marks)

Figure 4

18. In the rectangular coordinate system, R is a quadrilateral region (including the boundary) bounded by the x-axis, the y-axis, L_1 and L_2, where L_1 and L_2 are two perpendicular straight lines. The slope of L_1 is $\dfrac{5}{2}$. It is given that L_1 and L_2 intersect at the point $(4, -5)$.

(a) It is given that R represents the solution of a system of inequalities. Find the system of inequalities.

(3 marks)

(b) It is given that (a, b) is a point lying in R. Someone claims that the value of $4a + 9b + 36$ may be less than 5. Do you agree? Explain your answer.

(2 marks)

19. Figure 5a shows a triangular cardboard *ABC*. *E* and *F* are points on *AB* such that *AE* = 2.5 cm and *CF* ⊥ *AB*. *D* is a point on *AC* such that *DE* ⊥ *AB*. It is known that *BC* = 6 cm, ∠*ABC* = 60° and ∠*ACB* = 80°.
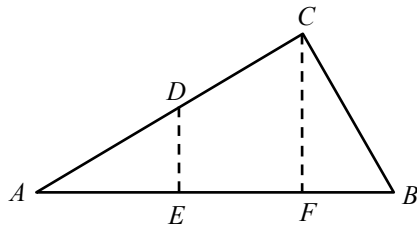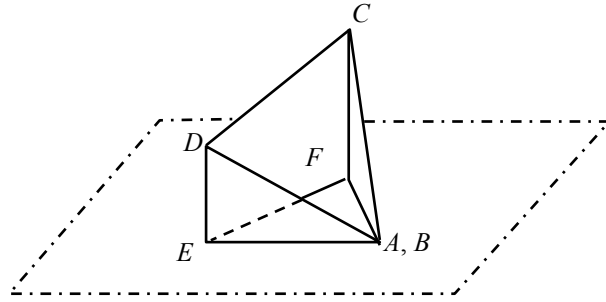

Figure 5a

Figure 5b

(a) Find the lengths of *BF* and *EF*.

(3 marks)

(b) The cardboard is then folded along *CF* and *DE* such that *A* and *B* meet each other. *AE*, *EF* and *BF* lie on the horizontal ground as shown in Figure 5b. The angle between the planes *BCF* and *CDEF* is denoted by α.

(i) Find α.

(ii) The angle between *CE* and plane *BCF* is denoted by β. Someone claims that α > 2β. Do you agree? Explain your answer.

(5 marks)

20. The coordinates of the centre G of the circle C are $(k, 0)$, where $k > -8$. The radius of C is $\sqrt{2}$. Let L be the straight line $x + y + h = 0$, where h is a constant. It is given that L is a tangent to C.

   (a) (i) Find the equation of C in terms of k.

   (ii) It is given that $(k + h)^2 = m$, find the value of m.

   (4 marks)

   (b) L passes through the point $F(-2, -6)$.

   (i) Find k.

   (ii) It is given that L intersects with the x-axis at the point Q. Let R be a point such that G is the in-centre of $\triangle FQR$. Someone claims that $\triangle FQR$ is a right-angled triangle. Do you agree? Explain your answer.

   (7 marks)

END OF PAPER