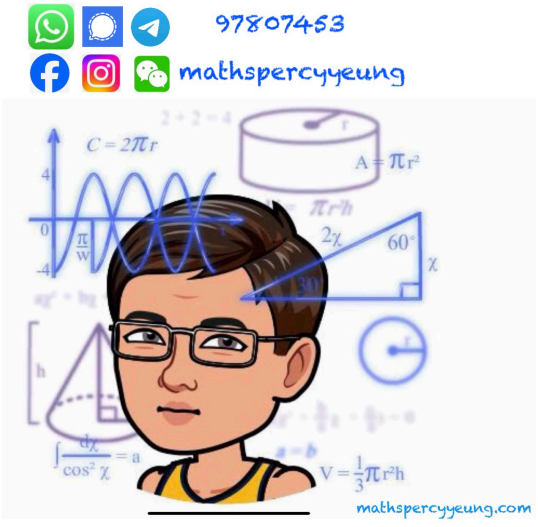
School-Based Exercise (S.B.E)

Chapter 10 Applications of Trigonometry

Name: _____ ( )

Class: _____

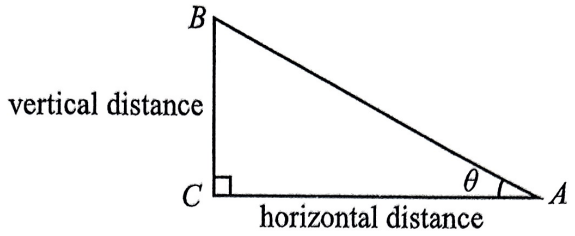


Gradient and Inclination of an Inclined Plane

The figure shows an inclined plane AB. AC lies on the horizontal ground.

(a) Gradient of $AB = \dfrac{\text{vertical distance}}{\text{horizontal distance}}$

$= \dfrac{BC}{AC}$



(b) $\tan\theta$ = gradient of AB

Obviously, the greater the gradient (inclination) of an inclined plane, the steeper is the inclined plane.

Exercise 10A

1. Find the inclination of an inclined plane for each of the following gradients.

(a) 1 : 3 **(b)** $\dfrac{2}{5}$

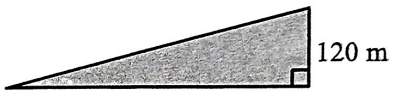
2. For each of the following inclinations of inclined planes, find the gradient in the form of 1 : n, where n is correct to the nearest integer.

(a) 14° **(b)** 9°

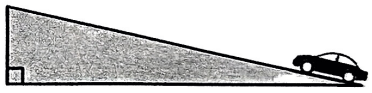
3. The figure shows an inclined path.

(a) Find the gradient of the path in fraction.

(b) Find the inclination of the path.



4. Nick drives along an inclined road of gradient 1 : 5. If the horizontal distance travelled is 75 km, what is the vertical distance travelled?



5. The gradient of a wheelchair ramp is 1 in 12. If the vertical distance of the ramp is 28 cm, find the horizontal distance of the ramp.

6. There are two inclined roads A and B. The average gradient of road A is 1 : 5 while the average inclination of road B is 13°.

(a) Find the average inclination of road A.

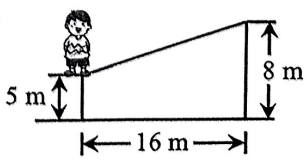
(b) Find the difference in the average inclination between roads A and B.

7. Tom walks up along an inclined plane AB and then walks down along another inclined plane BC. The average inclination of AB is 15° and the average gradient of BC is 1 : 4. Which inclined plane, AB or BC, is steeper? Explain your answer.

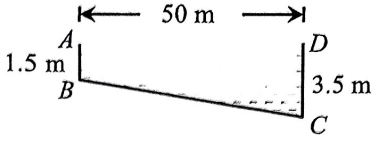
8. On an inclined road, Steven goes up from a vertical level of 5 m to 8 m. The horizontal distance travelled is 16 m.

(a) Find the vertical distance Steven travelled.

(b) Find the average inclination of the road.



9. The figure shows a cross-section of a swimming pool. Find the inclination of the bottom of the swimming pool.

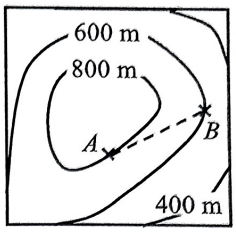


10. The scale of the contour map shown on the right is 1 : 100 000. The length of AB on the map is measured as 1.4 cm.

(a) Find the actual horizontal distance between A and B in m.

(b) Find the actual vertical distance between A and B.

(c) Find the inclination of AB.

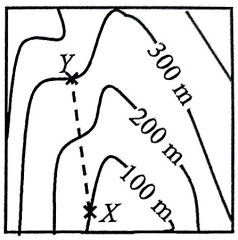


Scale 1 : 100 000

11. The figure shows a contour map with a scale of 1 cm : 200 m. Suppose XY is a straight road. Its length is measured as 1.8 cm on the map.

(a) Find the gradient of XY.

(b) Find the inclination of XY.



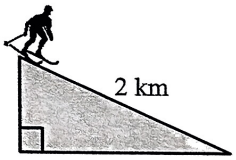
Scale 1 cm : 200 m

12. James walks up a mountain along a straight path of gradient $\frac{2}{9}$.

(a) Find the inclination of the path.

(b) What is the distance walked by James if the vertical distance travelled is 300 m?

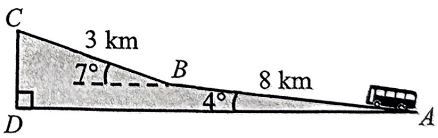
13. Ken is skiing down a straight path of gradient 2 : 3. Find the horizontal distance travelled when he skis 2 km along the path.



14. In the figure, A is at the same horizontal ground level as D. A bus travels 8 km up an inclined road AB with an inclination of 4° and then travels 3 km up another inclined road BC with an inclination of 7°. Find the vertical distance travelled by the bus from its starting point A

(a) when it reaches B,

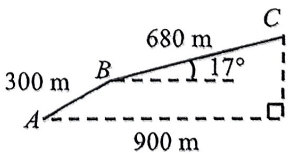
(b) when it reaches C.



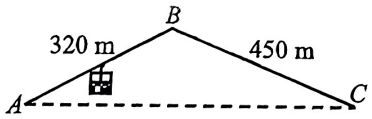
15. In the figure, Jimmy cycles 300 m from A to B. Then he cycles 680 m to C. The inclination of BC is 17°. The horizontal distance between A and C is 900 m.

(a) Find the horizontal distance between A and B.

(b) Find the vertical distance between A and C.



16. In the figure, A is at the same horizontal ground level as C. A cable car starts moving from A, then passes through B and reaches C. $AB = 320$ m, $BC = 450$ m and the gradient of AB is $6 : 11$.

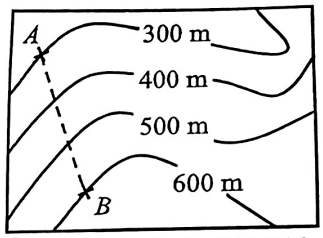


(a) Find the height of B above the ground.

(b) Find the inclination of BC.

(c) Find the horizontal distance between A and C.

17. The figure shows part of a contour map with a scale of $1 : 200\,000$. A straight path connects A and B. The length of AB on the map is measured as 2 cm.

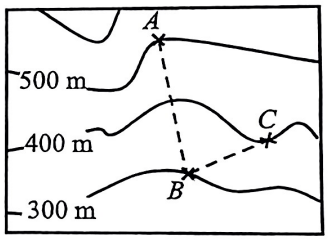


Scale 1 : 200 000

(a) Find the gradient of the straight path AB.

(b) Find the length of the straight path AB.

18. The figure shows a contour map with a scale of $1 : n$. The lengths of two straight paths AB and BC on the map are measured as 1.8 cm and 1.1 cm respectively. It is given that the inclination of the path AB is 15.5°.



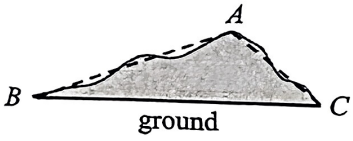
Scale 1 : n

(a) **(i)** Express, in terms of n, the actual horizontal distance of AB in m.

(ii) Find the value of n, correct to the nearest thousand.

(b) Find the inclination of BC.

19. In the figure, the length of BC is 3200 m. The average inclinations of AB and AC are 6° and 12° respectively.



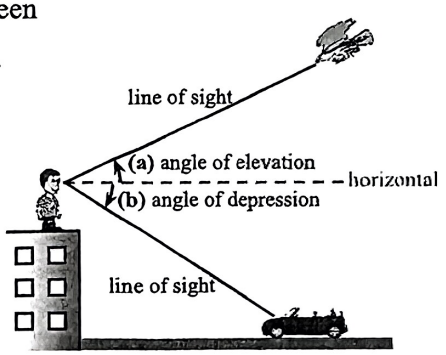
(a) Find the height of A above the ground.

(b) Andy walks up the hill from B to A at a speed of 0.5 m/s and Ben walks up the hill from C to A. Find the speed of Ben such that they reach A at the same time.

Angle of Elevation and Angle of Depression

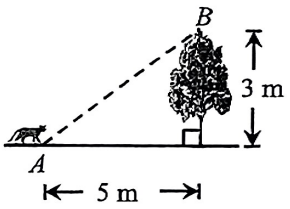
(a) When an observer is looking an object *above* him, the angle between the line of sight and the horizontal is called the *angle of elevation.*

(b) When an observer is looking an object *below* him, the angle between the line of sight and the horizontal is called the *angle of depression.*

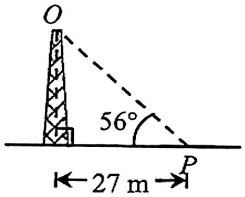


Exercise 10B

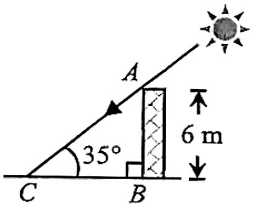
1. In the figure, a cat at A at horizontal ground level looks at the top B of a tree which is 3 m tall. If the horizontal distance between A and B is 5 m, find the angle of depression of A from B.



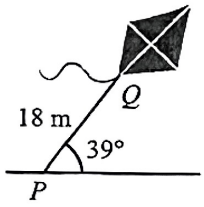
2. In the figure, an observer at P on the horizontal ground sees a man at the top O of a tower. The angle of elevation of O from P is 56°. If P is 27 m from the tower, find the height of the tower.



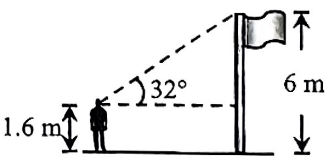
3. The figure shows a wall AB 6 m high. The angle of elevation of the sun is 35°. Find the length of the shadow of the wall CB on the horizontal ground.



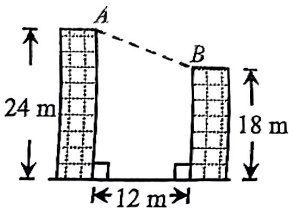
4. In the figure, a kite Q is tied to a point P on the horizontal ground by a string of length 18 m. If the angle of elevation of Q from P is 39°, what is the horizontal distance between P and Q?



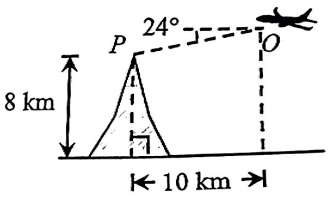
5. In the figure, David is in front of a flagpole, which is 6 m high. David's eye level is 1.6 m above horizontal ground level. The angle of elevation of the top of the flagpole from David is 32°. How far is David from the foot of the flagpole?



6. In the figure, the heights of the two buildings are 24 m and 18 m respectively. The horizontal distance between the two buildings is 12 m. Find the angle of depression of the top B of the shorter building from the top A of the taller building.



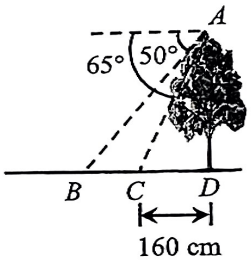
7. In the figure, the height of a mountain is 8 km. The angle of depression of the top P of the mountain from an aeroplane O is 24°. If the horizontal distance between O and P is 10 km, find the height of the aeroplane above the ground.



8. The figure shows a tree AD. B, C and D lie on the same straight road. The angles of depression of B and C from A are 50° and 65° respectively. The distance between C and D is 160 cm.

(a) Find the height of the tree.

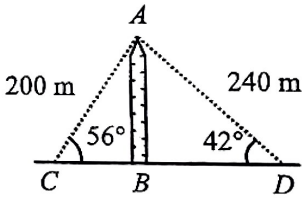
(b) Find the distance between B and D.



9. In the figure, a monument AB is mounted by two wires AC and AD of lengths 200 m and 240 m respectively. A, C and D lie on the same vertical plane. C, B and D are at horizontal ground level. The angles of elevation of A from C and D are 56° and 42° respectively.

(a) Find the distance between B and C.

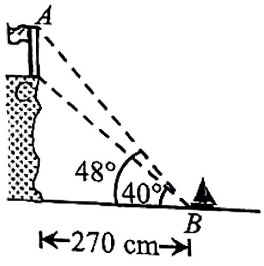
(b) Find the distance between C and D.



10. In the figure, a vertical flagpole AC is on the top of a cliff. The angles of elevation of A and C from a boat at B are 48° and 40° respectively. The horizontal distance between B and C is 270 cm.

(a) Find the height of the cliff above sea level.

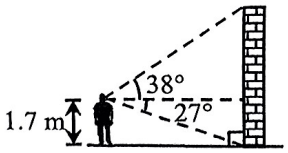
(b) Find the height of the flagpole AC.



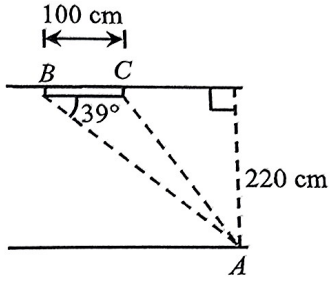
11. In the figure, Johnny's eye level is 1.7 m above horizontal ground level. The angle of elevation of the top of a wall from Johnny is 38°. The angle of depression of the bottom of the wall from Johnny is 27°.

(a) Find the horizontal distance between Johnny and the wall.

(b) Find the height of the wall.



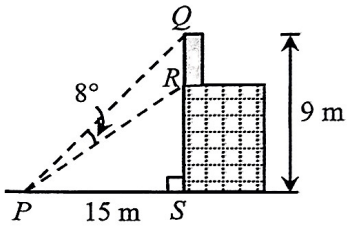
12. In the figure, a light pipe BC of length 100 cm is fixed on the ceiling of a room. The height of the room is 220 cm. The angle of depression of a point A on the ground from one end B of the light pipe is 39°. Find the angle of depression of A from the other end C of the light pipe. (The height of the light pipe is neglected.)



13. The figure shows an advertising board QR at the top of a building RS such that $QS = 9$ m. P is a point on the horizontal ground 15 m away from the bottom of the building. The angles of elevation of the top and the bottom of the advertising board from P differ by 8°.

(a) Find the angle of elevation of R from P.

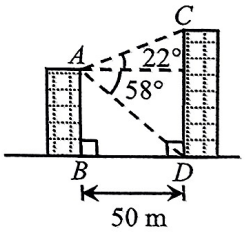
(b) Find the height of the advertising board.



14. In the figure, two buildings AB and CD are 50 m apart. The angle of elevation of C from A is 22°. The angle of depression of D from A is 58°.

(a) Find the height of the building AB.

(b) Find the angle of elevation of C from B.

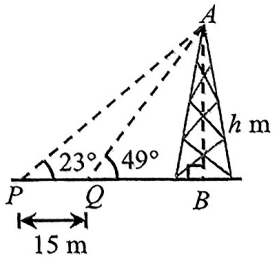


15. The figure shows a transmission tower AB. B, Q and P lie on a horizontal straight road. The angles of elevation of A from P and Q are 23° and 49° respectively. The distance between P and Q is 15 m. Let h m be the height of the tower.

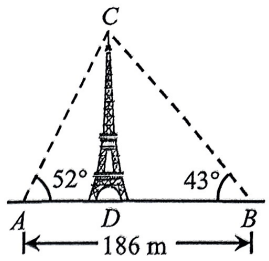
(a) **(i)** By considering $\triangle AQB$, express the distance between Q and B in terms of h.

(ii) By considering $\triangle APB$, express the distance between Q and B in terms of h.

(b) Find the height of the tower.



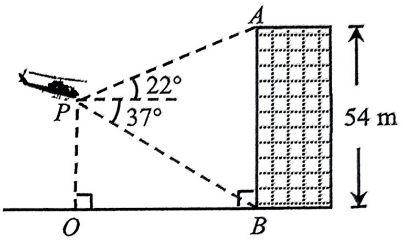
16. The figure shows a tower CD. A and B are at horizontal ground level and the distance between A and B is 186 m. A, B and C lie on the same vertical plane. The angles of elevation of C from points A and B are 52° and 43° respectively. Find the height of the tower.



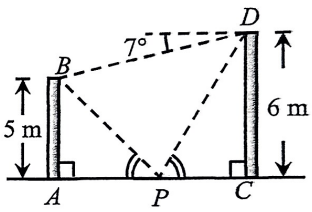
17. In the figure, the height of a building AB is 54 m. A helicopter is rising vertically from O to P at a speed of 32 km/h. The angle of elevation of A from P is 22°. The angle of depression of B from P is 37°. It is given that A, B, O and P lie on the same vertical plane.

(a) Find the vertical distance OP.

(b) Will the helicopter reach P from O in 4 seconds? Explain your answer.

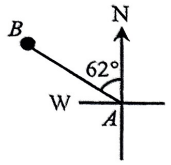


18. The figure shows two vertical poles AB and CD on the same horizontal level. $AB = 5$ m, $CD = 6$ m and the angle of depression of B from D is 7°. P is a point lying on AC such that the angles of elevation of B and D from P are equal. Find $\angle APB$.

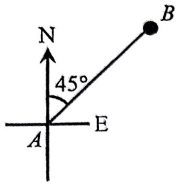


Compass Bearing

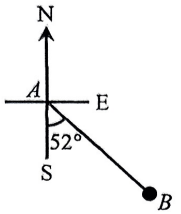
A compass bearing describes a direction using an *acute angle* measured from either *north* or *south*. The following diagrams show how to describe the direction by compass bearing.



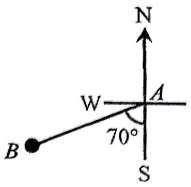
The compass bearing of B from A is N62°W.



The compass bearing of B from A is N45°E.



The compass bearing of B from A is S52°E.

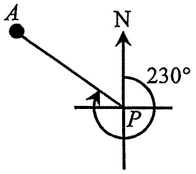


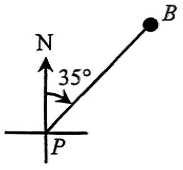
The compass bearing of B from A is S70°W.

True Bearing

A true bearing describe a direction using an angle (from 0° to 360°) measured in a *clockwise direction* from *north*.

e.g. In the figure, the true bearing of A from P is 230°.





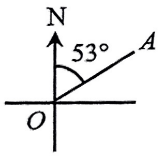
Note: The integral part of the true bearing must be expressed in 3 digits.

For example, the true bearing of B from P is written as 035° instead of 35°.

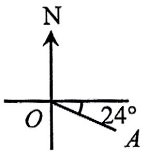
Exercise 10C

1. In each of the following, find the true bearing of A from O.

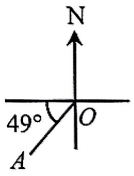
(a)



(b)

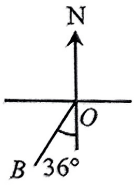


(c)

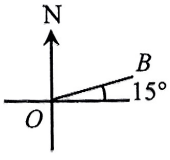


2. In each of the following, find the compass bearing of B from O.

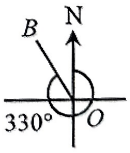
(a)



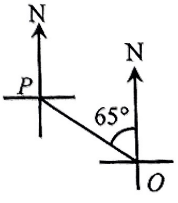
(b)



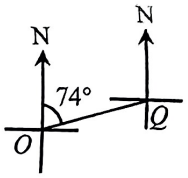
(c)



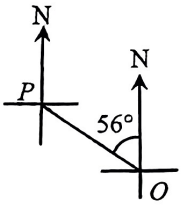
3. In the figure, find the true bearing of O from P.



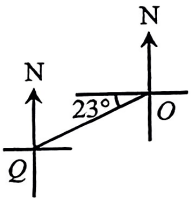
4. In the figure, find the true bearing of O from Q.



5. In the figure, find the compass bearing of O from P.



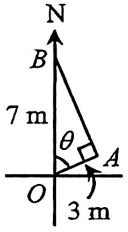
6. In the figure, find the compass bearing of O from Q.



7. Refer to the figure.

(a) Find θ.

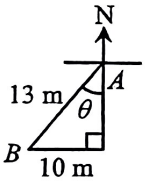
(b) Find the true bearing of A from O.



8. Refer to the figure.

(a) Find θ.

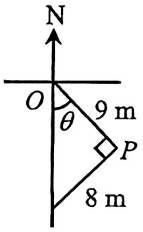
(b) Find the true bearing of B from A.



9. Refer to the figure.

(a) Find θ.

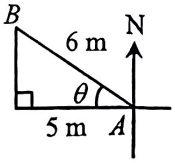
(b) Find the compass bearing of P from O.



10. Refer to the figure.

(a) Find θ.

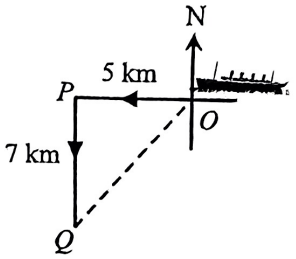
(b) Find the compass bearing of B from A.



11. In the figure, a ship left pier O and sailed 5 km due west to P. It then sailed 7 km due south to reach Q.

(a) Find the compass bearing of Q from O.

(b) Find the distance between O and Q.

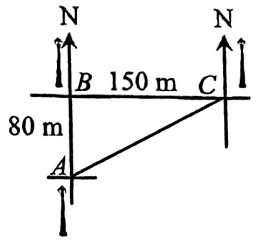


12. In the figure, three lampposts A, B and C are at the same horizontal ground level. A is 80 m due south of B. C is 150 m due east of B. Find the true bearing of

(a) C from A,

(b) A from C.

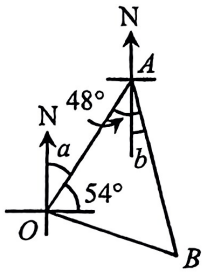
(*Give the answers correct to the nearest degree.*)



13. In the figure, A, B and O are on the same horizontal plane.

(a) Find the values of a and b.

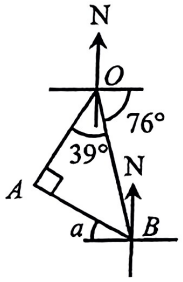
(b) Find the compass bearing of B from A.



14. In the figure, A, B and O are on the same horizontal plane.

(a) Find the value of a.

(b) Find the true bearing of A from B.

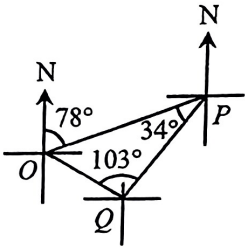


15. In the figure, O, P and Q are on the same horizontal plane.

(a) Find the compass bearing of O from P.

(b) Find the true bearing of P from Q.

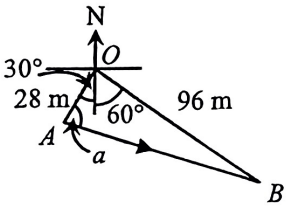
(c) Find the true bearing of O from Q.



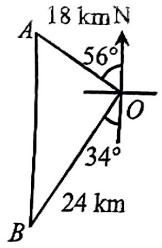
16. In the figure, an athlete starts to run from a point A to point B. A is on a bearing of S30°W and 28 m away from a coach O. B is on a bearing of S60°E and 96 m away from O.

(a) Find the value of a.

(b) Find the true bearing of B from A.



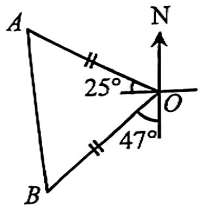
17. Two men A and B left a station O. A travels in the direction N56°W for 18 km. B travels in the direction S34°W for 24 km as shown in the figure.



(a) Find the distance between the two men A and B.

(b) Find the true bearing of B from A.

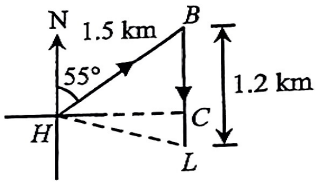
18. In the figure, A, B and O are on the same horizontal plane and $AO = BO$.



(a) Find the compass bearing of O from A.

(b) Find the compass bearing of B from A.

19. Ben takes a walk from his home (H). He walks 1.5 km in the direction N55°E to a bookstore (B). Then he walks 1.2 km due south to a library (L). C is a point on the straight road BL where the distance between H and BL is the shortest.

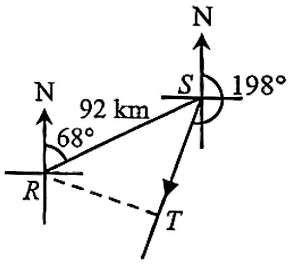


(a) Find HC and CL.

(b) Find the true bearing of the library from his home.

(c) Find the distance between the library and his home.

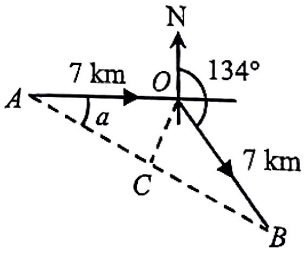
20. In the figure, a submarine at S is 92 km away from a radar R. The true bearing of S from R is 068°. The submarine starts to sail along a course in the direction of 198° at a constant speed. T is a point on the course where the distance between the radar and the submarine is the shortest.



(a) Find the true bearing of T from R.

(b) It is given that the submarine will be detected by the radar within 70 km. Will the submarine be detected? Explain your answer.

21. In the figure, a boat leaves A and sails due east for 7 km to O. It then sails for another 7 km in the direction 134° to B. C is a point on AB where the distance between O and AB is the shortest.



(a) Find the value of a.

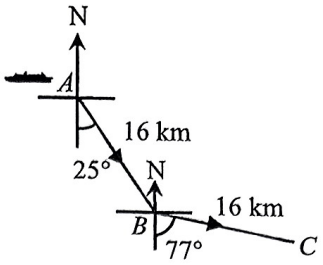
(b) Find the the true bearing of B from A.

(c) Find the distance between A and B.

22. In the figure, a ship sails for 16 km on a bearing of S25°E from A to B, and then sails for 16 km on a bearing of S77°E to C. Then the ship gets straight back to A.

(a) What is the compass bearing of A from C?

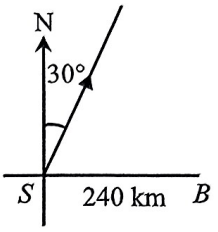
(b) If the average speed of the ship is 15 km/h, find the time, in minutes, it takes to reach A from C.



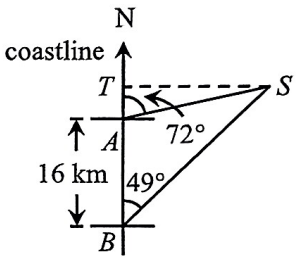
23. In the figure, a ship S is 240 km due west of a buoy B at 9 am. The ship is moving at a speed of 50 km/h in the direction N30°E.

(a) Find the shortest distance between the ship and the buoy.

(b) A sailor claims that the ship will be closest to the buoy at 11:24 am. Do you agree? Explain your answer.



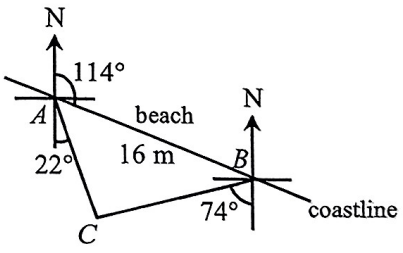
24. In the figure, two lighthouses A and B are located on a straight coastline. A is 16 km due north of B. The true bearings of a ship S from A and B are 072° and 049° respectively. Suppose T is a point on the coastline such that the distance between the ship and the coastline is the shortest. Find the shortest distance between the ship and the coastline.



25. In the figure, A and B are two points on the straight coastline of a beach. B is on a bearing of 114° and 16 m away from A. The compass bearings of a swimmer C from A and B are S22°E and S74°W respectively.

(a) Find $\angle ABC$.

(b) Find the shortest distance between the swimmer and the coastline.



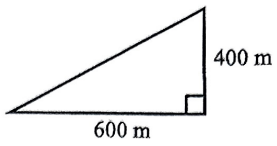
26. An aeroplane flies 18 km in the direction N40°W to a point B from a point A. Then the aeroplane travels 30 km to a point C from B in the direction S10°W.

(a) Find the distance between A and C.

(b) Find the true bearing of C from A, correct to the nearest degree.

MC

1. The figure below shows an inclined plane.



Find the gradient of the inclined plane.

A. 1 : 2 B. 1 : 3

C. 2 : 3 D. 2 : 5

2. Irene walks along an inclined road of gradient $\frac{1}{8}$. If the horizontal distance travelled is 560 m, what is the vertical distance travelled by Irene?

A. 65 m B. 70 m

C. 75 m D. 80 m

3. Find the inclination of an inclined plane of gradient 3 : 10, correct to 3 significant figures.

A. 16.7° B. 18.4°

C. 71.6° D. 73.3°

4. Which of the following inclined roads is the steepest?

A. A road of gradient 1 in 5.

B. A road of gradient 22%.

C. A road of inclination 20°.

D. A road of gradient $\frac{1}{6}$.

5. Joey walks up a hill along a straight road of inclination 15°. If the vertical distance travelled is 24 m, what is the distance walked by Joey?

A. 6.21 m, *cor. to 3 sig. fig.*

B. 24.8 m, *cor. to 3 sig. fig.*

C. 89.6 m, *cor. to 3 sig. fig.*

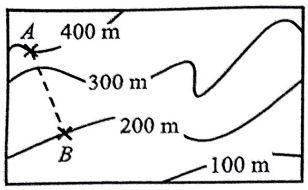
D. 92.7 m, *cor. to 3 sig. fig.*

6. Mr Wong rides a bicycle along an inclined road. When he rides 100 m along the road, the vertical distance travelled is 20 m. Find the inclination of the road, correct to 3 significant figures.

A. 11.5° B. 18.1°

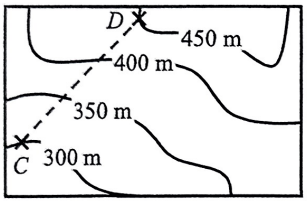
C. 71.9° D. 78.5°

7. The following contour map shows a straight road AB. If the actual horizontal distance of the road is 580 m, find the inclination of the road, correct to the nearest degree.



A. 14°
B. 19°
C. 35°
D. 71°

8. The scale of the following contour map is 1 : 90 000. The length of CD on the map is measured as 1.9 cm. Find the inclination of CD, correct to the nearest degree.



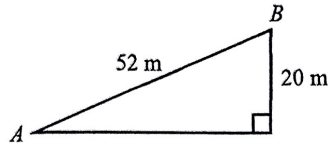
Scale 1 : 90 000

A. 3°
B. 4°
C. 5°
D. 6°

9. Patrick is walking down a straight road of gradient 4 : 7. Find the horizontal distance travelled when he walked 650 m along the road.
   A. 564 m, *cor. to 3 sig. fig.*
   B. 681 m, *cor. to 3 sig. fig.*
   C. 749 m, *cor. to 3 sig. fig.*
   D. 1310 m, *cor. to 3 sig. fig.*

10. A bus travels 2 km down an inclined road with an inclination of 20° and then travels 1 km down another inclined road with an inclination of 5°. Find the height of the bus has fallen from its starting point.
   A. 516 m, *cor. to 3 sig. fig.*
   B. 653 m, *cor. to 3 sig. fig.*
   C. 771 m, *cor. to 3 sig. fig.*
   D. 815 m, *cor. to 3 sig. fig.*

11. The length of the path AB is 52 m and the vertical distance between A and B is 20 m. Find the gradient of the path.



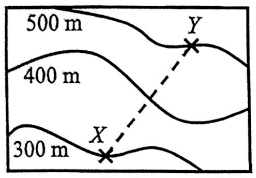
A. $\frac{4}{11}$

B. $\frac{5}{12}$

C. $\frac{5}{13}$

D. $\frac{12}{13}$

12. The figure shows a contour map with a scale of 1 : 60 000. XY is a straight path and the length of XY on the map is measured as 2.8 cm. Find the inclination of the path XY.



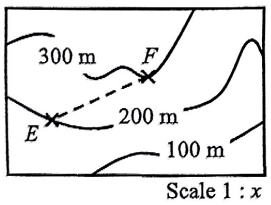
A. 6.04°, *cor. to 3 sig. fig.*

B. 6.79°, *cor. to 3 sig. fig.*

C. 8.31°, *cor. to 3 sig. fig.*

D. 10.7°, *cor. to 3 sig. fig.*

13. The figure below shows a contour map with a scale of 1 : x. The inclination of the straight road EF is 4.4° and the length of EF on the map is measured as 1.2 cm. Find the value of x, correct to the nearest thousand.



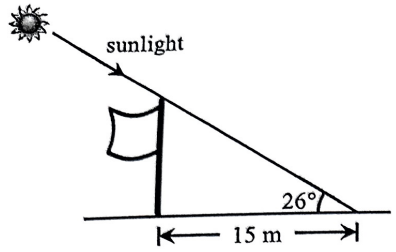
A. 108 000

B. 110 000

C. 113 000

D. 117 000

14. In the figure, the shadow of a flagpole on the horizontal ground is 15 m long. The angle of elevation of the sun is 26°. Find the height of the flagpole, correct to 1 decimal place.

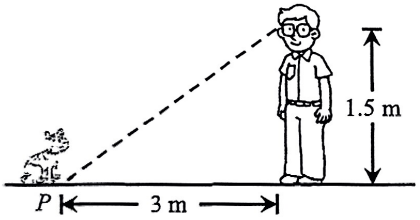


A. 4.9 m
B. 6.6 m
C. 7.3 m
D. 13.5 m

15. If the angle of depression of A from B is 40°, find the angle of elevation of B from A.

A. 20°
B. 40°
C. 60°
D. 80°

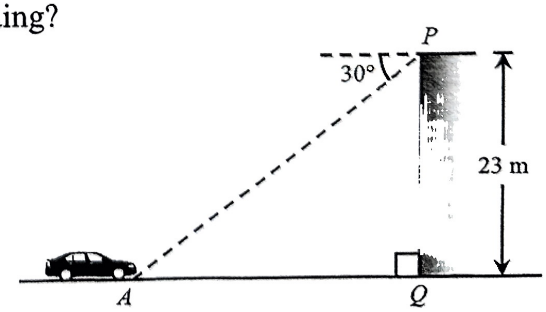
16. In the figure, Charles observes a cat sitting on the horizontal ground. The eye level of Charles is 1.5 m above horizontal ground level and the cat is 3 m away from Charles. Find the angle of depression of the cat from Charles, correct to the nearest degree.



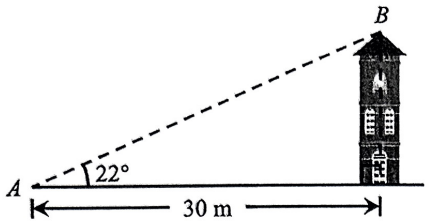
A. 24°
B. 25°
C. 26°
D. 27°

17. In the figure, Michael stands at the top P of a building and he sees a car at A on the horizontal ground. The angle of depression of A from P is 30°. If the height of the building is 23 m, what is the horizontal distance between the car and the building?

A. 13.3 m, *cor. to 3 sig. fig.*
B. 39.8 m, *cor. to 3 sig. fig.*
C. 41.6 m, *cor. to 3 sig. fig.*
D. 54.2 m, *cor. to 3 sig. fig.*

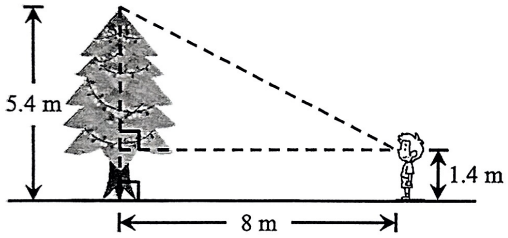


18. In the figure, A is a point on the horizontal ground. The angle of elevation of the top B of a clock tower from A is 22°. If the horizontal distance between A and B is 30 m, find the height of the clock tower, correct to 3 significant figures.



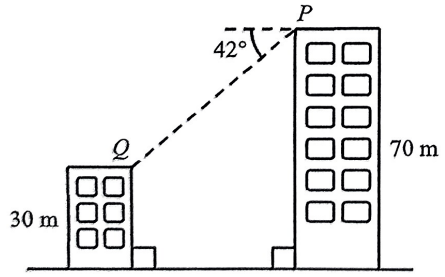
A. 9.38 m
B. 12.1 m
C. 14.5 m
D. 18.0 m

19. In the figure, a boy is 8 m away from the foot of a tree. The height of the tree is 5.4 m and the eye level of the boy is 1.4 m above horizontal ground level. Find the angle of elevation of the top of the tree from the boy, correct to 3 significant figures.



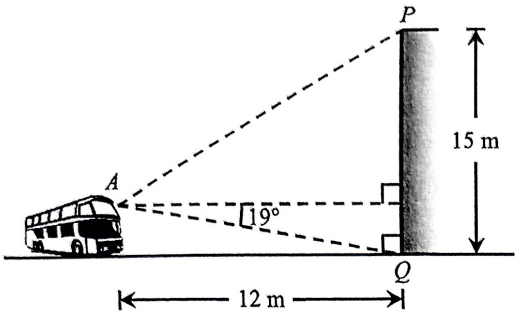
A. 26.6°
B. 28.3°
C. 34.0°
D. 63.4°

20. In the figure, the heights of two buildings are 30 m and 70 m respectively. The angle of depression of the top Q of the lower building from the top P of the higher building is 42°. Find the horizontal distance between two buildings.



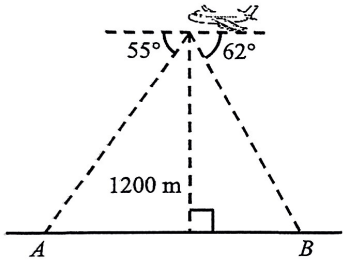
A. 32.8 m, *cor. to 3 sig. fig.*
B. 36.0 m, *cor. to 3 sig. fig.*
C. 44.4 m, *cor. to 3 sig. fig.*
D. 47.2 m, *cor. to 3 sig. fig.*

21. In the figure, the height of a library PQ is 15 m. The angle of depression of Q from A on a bus is 19°. The horizontal distance between A and Q is 12 m. Find the angle of elevation of P from A, correct to the nearest degree.



A. 35°
B. 42°
C. 48°
D. 54°

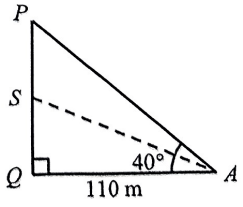
22. In the figure, the angles of depression of points A and B on the horizontal ground from a plane are 55° and 62° respectively. The plane is 1200 m above horizontal ground level. Find the distance between A and B.



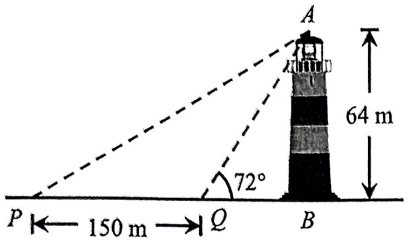
A. 1250 m, *cor. to 3 sig. fig.*
B. 1400 m, *cor. to 3 sig. fig.*
C. 1480 m, *cor. to 3 sig. fig.*
D. 1610 m, *cor. to 3 sig. fig.*

23. In the figure, a person at A on the horizontal ground looks at a building PQ. S lies on PQ such that $PQ = 2PS$. P, S, Q, A lie on the same vertical plane. The angle of elevation of P from A is 40° and the horizontal distance between A and Q is 110 m. Find the angle of elevation of S from A, correct to 1 decimal place.

A. 22.8°
B. 25.3°
C. 26.1°
D. 29.4°

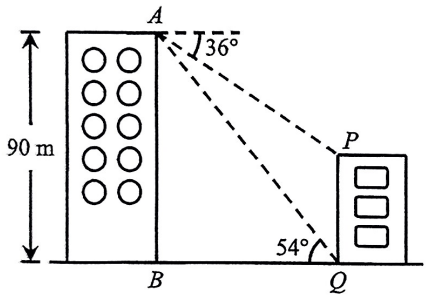


24. The figure shows a lighthouse AB which is 64 m high. Sandy walks 150 m towards the lighthouse from P to Q on the horizontal ground. The angle of elevation of the top A of the lighthouse from Q is 72°. Find the angle of elevation of A from P, correct to the nearest degree.



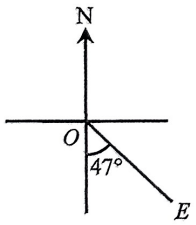
A. 20°
B. 21°
C. 29°
D. 30°

25. In the figure, the height of building AB is 90 m. The angle of depression of P from A is 36° and the angle of elevation of A from Q is 54°. Find the height of the building PQ, correct to 3 significant figures.



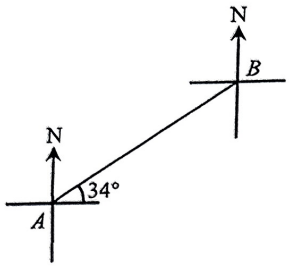
A. 36.1 m
B. 37.6 m
C. 40.8 m
D. 42.5 m

26. Refer to the figure below, find the true bearing of E from O.



A. 043°
B. 047°
C. 133°
D. 227°

27. Refer to the figure below, find the compass bearing of A from B.



A. N34°E
B. N56°E
C. S34°W
D. S56°W

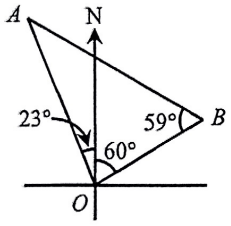
28. Express S14°W as true bearing.

A. 104°
B. 166°
C. 194°
D. 284°

29. If the bearing of Q from P is N53°W, then the bearing of P from Q is

A. S37°W.
B. S53°E.
C. N37°E.
D. S53°W.

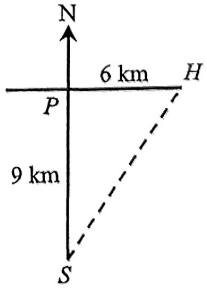
30. In the figure, O, A and B are on the same horizontal plane. Find the true bearing of B from A.



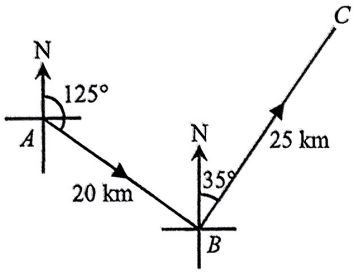
A. 119°
B. 124°
C. 128°
D. 133°

31. In the figure, a school (S) is 9 km due south of a park (P) and a hospital (H) is 6 km due east of P. Find the compass bearing of H from S, correct to the nearest degree.

A. N56°E
B. N34°E
C. S56°W
D. S34°W

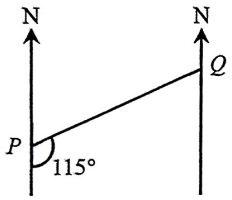


32. In the figure, a bus leaves town A and travels for 20 km in the direction of 125° to reach town B. Then it travels in the direction of 035° for another 25 km to reach town C. Find the distance between town A and town C, correct to 3 significant figures.



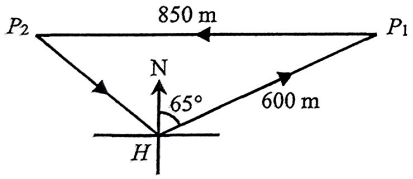
A. 32.0 km
B. 35.6 km
C. 39.1 km
D. 40.3 km

33. Refer to the figure, find the compass bearing of P from Q.



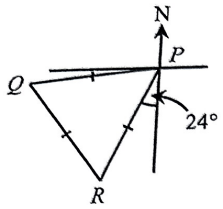
A. N25°E
B. S25°W
C. N65°E
D. S65°W

34. Carol takes a walk from her home (H) after having dinner. She first walks 600 m in the direction N65°E to a pavilion (P_1). Then, she walks 850 m due west to another pavilion (P_2) and walks directly to home. Find the total distance travelled by her, correct to 3 significant figures.



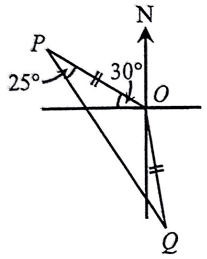
A. 315 m
B. 398 m
C. 1520 m
D. 1850 m

35. In the figure, P, Q and R are on the same horizontal plane such that $\triangle PQR$ is an equilateral triangle. Find the compass bearing of Q from R.



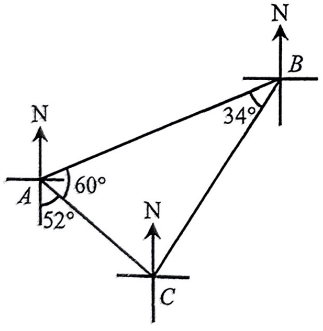
A. N84°W
B. S84°E
C. N36°W
D. S36°E

36. In the figure, $\triangle OPQ$ is an isosceles triangle where $OP = OQ$. Find the compass bearing of P from Q.



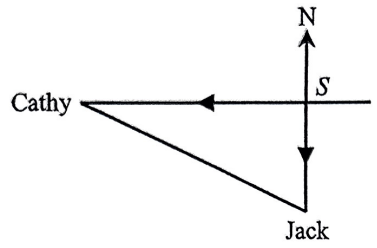
A. N35°E
B. N35°W
C. N40°E
D. N40°W

37. In the figure, A, B and C are on the same horizontal plane. Find the true bearing of C from B.



A. 186°
B. 208°
C. 214°
D. 232°

38. Cathy and Jack leave a station (S) at the same time. Cathy walks due west of the station while Jack walks due south of the station. If the distance travelled by Cathy is triple that by Jack, find the true bearing of Jack from Cathy now. (*Give the answer correct to 3 significant figures.*)



A. 108°
B. 162°
C. 252°
D. 288°