

F4 2nd Term Examination (2022 – 2023)

Subject: Mathematics (Compulsory Part)

Paper 1

Question - Answer Book

Time: 2 hours

Total Marks: 130

Total No. of Pages: 19

INSTRUCTIONS

- 1. This paper must be answered in English.
- 2. Write your Name, Class and Class Number in the space provided on Page 1.
- 3. This paper consists of THREE sections, A(1), A(2) and B.
- 4. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.
- 5. Supplementary answer sheets will be supplied on request. Write your Name, Class and Class Number, mark the question number box on each sheet, and fasten them INSIDE this book.
- 6. Unless otherwise specified, all working must be clearly shown.
- Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
- 8. The diagrams in this paper are not necessarily drawn to scale.

Simplify $\frac{(mn^4)^3}{m^{-6}n^{17}}$ and express your answer with positive indices.

(3 marks)

2. Make *a* the subject of the formula $\frac{a+3b}{ab-3} = 2$.

(3 marks)

- 3. Factorize
 - (a) $4xy + 6x^2$,
 - **(b)** $4xy + 6x^2 12x 8y$.

(3 marks)

- **4.** The cost of a wallet is \$120. The percentage profit is 25% if the wallet is sold at its marked price.
 - (a) Find the marked price of the wallet.
 - **(b)** If the wallet is sold at a discount of 10% on its marked price, what is the new profit percentage?

5. Cathy and John bought some stamps. If Cathy gives 3 stamps to John, they will have the same number of stamps. If John gives 3 stamps to Cathy, the number of stamps that Cathy has will be 4 times that of John. Find the total number of stamps Cathy and John have.

(4 marks)

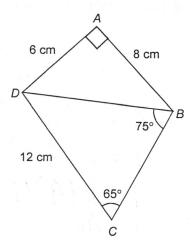
6. Consider the compound inequality

$$-6x \ge 12$$
 and $\frac{2x+5}{3} > 2(x+1)$ (*)

- (a) Solve (*).
- **(b)** Write down the greatest negative integer satisfying (*).

- 7. In the figure, find
 - (a) the length of BD and BC,
 - **(b)** the area of the quadrilateral *ABCD*. (Give your answer correct to 3 significant figures.)

(5 marks)



- **8.** It is given that z varies jointly as (x-2) and y^3 . When x=6 and y=1, z=1.
 - (a) Express z in terms of x and y.
 - **(b)** Find the value of y when x = 10 and z = -16.

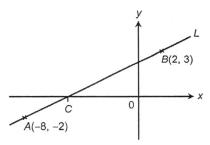
(5 marks)

- **9.** When a polynomial f(x) is divided by $-3x + 3x^2$, the quotient and the remainder are x + 5 and -2x + 3 respectively.
 - (a) Find the polynomial f(x).
 - **(b)** Find the quotient and the remainder of $f(x) \div (3x^2 + 5)$.

(6 marks)

F4 Mathematics (Compulsory Part) Paper 1 2nd Term Examination (2022 – 2023)

10. In the figure, the straight line L passes through two points A(-8, -2) and B(2, 3), and intersects the x-axis at C.

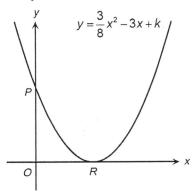


- (a) Find the equation of L.
- (b) (i) Find the coordinates of C.
 - (ii) If L_1 is a straight line passing through C and (-4, -5), find the equation of L_1 .

(6 marks)

Section A(2) (43 marks)

The figure shows the graph of $y = \frac{3}{8}x^2 - 3x + k$ which touches the x-axis at 11. one point R, and cuts the y-axis at P.



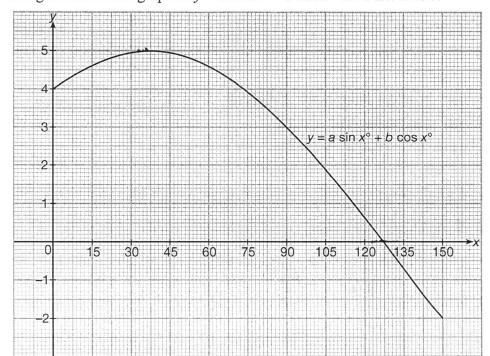
- Find the value of k. (a) (i)
 - (ii) Find the coordinates of P and R.

(5 marks)

(b) It is given that another point Q lying on the graph of $y = \frac{3}{8}x^2 - 3x + k$ such that PQ is parallel to the x-axis. Find the area of $\triangle PQR$.

(2 marks)

12. The figure shows the graph of $y = a \sin x^{\circ} + b \cos x^{\circ}$ for $0 \le x \le 150$.



(a) Find the values of a and b.

(4 marks)

- (b) Find the maximum value of y and the corresponding value of x. (2 marks)
- (c) Using the graph, solve the following equations.
 - (i) $6 \sin x^{\circ} + 8 \cos x^{\circ} = 0$
 - (ii) $3 \sin x^{\circ} + 4 \cos x^{\circ} 2 = 0$

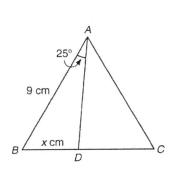
(Give the answers correct to the nearest grid scale.)

- **13.** f(x) is partly constant and partly varies as $(x-1)^2$. It is given that f(0) = 6 and f(4) = -10.
 - (a) Find f(x). (4 marks)
 - **(b)** If V is the vertex of the graph of y = f(x), and the graph cuts the x-axis at A and B.
 - (i) Find the coordinates of A and B.
 - (ii) Find the area of $\triangle VAB$.

(5 marks)

- **14.** Let f(x) = (x-m)(x-n)(3x-1)-24, where m and n are positive integers and m > n. It is given that x-3 is a factor of f(x).
 - (a) (i) Show that (m-3)(n-3) = 3.
 - (ii) Write down the values of m and n. (4 marks)
 - (b) It is given that $f(x) \equiv (x-3)(ax^2+bx+c)$, where a, b and c are constants. Find the values of a, b and c. (4 marks)

15. In Figure (1), $\triangle ABC$ is an equilateral triangle of side 9 cm. D is a point on BC such that $\angle BAD = 25^{\circ}$ and BD = x cm.



9 cm 9 cm 50° D

Figure (1)

Figure (2)

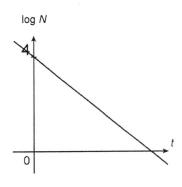
- (a) (i) By considering $\triangle ABD$ and $\triangle ACD$, show that $\frac{x}{\sin 25^{\circ}} = \frac{9-x}{\sin 35^{\circ}}$
 - (ii) Hence, find the value of x correct to 3 decimal places.

(5 marks)

(b) AD is produced to E such that $\angle EBD = 50^{\circ}$ as shown in **Figure (2)**. Find the lengths of BE and CE correct to 3 significant figures.

Section B (43 marks)

16. The number N of a certain kind of insects in a forest t months after the implementation of a insect control program is given by $N = A(1.2)^{-2t}$, where A is a constant. The graph shows the relation between $\log N$ and t. The vertical intercept of the graph is 4 as shown.



(a) Find the value of A.

(3 marks)

(b) How long will the number of the insects be halved? (Give the answer correct to 3 significant figures.)

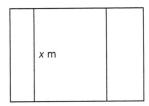
- 17. (a) Express $\frac{15}{3-4i}$ and $\frac{15}{3+4i}$ in the form a+bi, where a and b are real numbers. (4 marks)
 - **(b)** The roots of the quadratic equation $5px^2 9x + q = 0$ are $\frac{15}{3-4i}$ and $\frac{15}{3+4i}$, where $p \neq 0$ and q is a constant.

By considering the sum of roots and the product of roots, find the values of p and q. (4 marks)

18. (a) Let $f(x) = 40x - x^2$. Using the method of completing the square, find the coordinates of the vertex of the graph of y = f(x).

(3 marks)

(b) The length of a piece of wire is 160 m. Jason cuts the wire into three pieces. One piece is used to enclose a rectangular playground of area $A \, \text{m}^2$. The other two pieces of length $x \, \text{m}$ each are used to divide this playground into three rectangular parts as shown in the figure.



- (i) Express A in terms of x.
- (ii) Jason claims that the area of the playground enclosed can be larger than 800 m². Do you agree? Explain your answer.

- **19.** It is given that $\frac{\cos^2(180^\circ x) 3}{5\cos(180^\circ + x)} = \sin(360^\circ x).$
 - (a) Show that $3 \tan^2 x + 5 \tan x + 2 = 0$. (5 marks)
 - **(b)** Hence, solve $\frac{\cos^2(180^\circ x) 3}{5\cos(180^\circ + x)} = \sin(360^\circ x)$ for $0^\circ \le x \le 360^\circ$.

(Give your answers correct to 1 decimal place if necessary.) (4 marks)

- **20.** It is given that the straight line L_1 passes through A(8, 4) and is parallel to the straight line $L_2: 4x + 12y = 21$.
 - (a) Find the equation of L_1 . (3 marks)
 - **(b)** It is known that L_1 is perpendicular to the straight line $L_3: y = kx$.
 - (i) Find the value of k.
 - (ii) Find the coordinates of the intersection of L_1 and L_3 .
 - (iii) Hence, find the shortest distance from A to L_3 . (Leave your answer in surd form.) (9 marks)

