2022-2023 S4
2nd TERM EXAM
MATH EP
M2

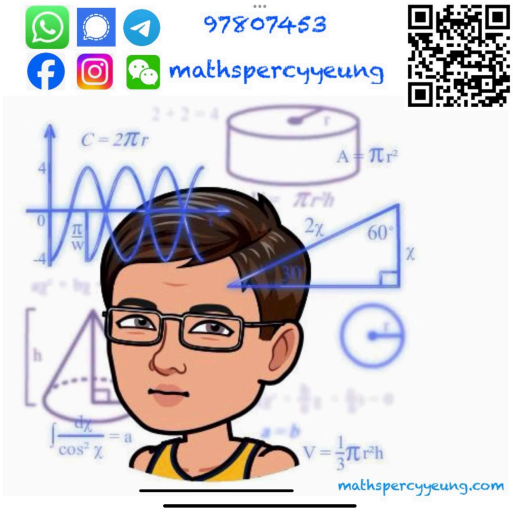
2022 – 2023
S4 Second Term Examination

MATHEMATICS Extended Part

Module 2 (Algebra and Calculus)

Question–Answer Book

16th June, 2023
8:15 am – 9:45 am (1 hour 30 minutes)
This paper must be answered in English



INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.

2. This paper consists of TWO sections, A and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers must be exact.

6. The diagrams in this paper are not necessarily drawn to scale.

Section	Marks
A Total	**/ 46**
B Total	**/ 14**
TOTAL	**/ 60**

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

****

Section A (46 marks)

1. Let $\frac{d}{dx}\left(\sin\frac{x}{3}\right)$ from the first principles. (4 marks)

Answers written in the margins will not be marked

Answers written in the margins will not be marked

2. The coefficient of x in the expansion of $(1+x)^2(1-3x)^n$ is -19, where n is a positive integer. Find the value of n and the coefficients of x^3.

(5 marks)

Answers written in the margins will not be marked

Answers written in the margins will not be marked

3. **(a)** Prove that $\csc 2\theta = \cot\theta - \cot 2\theta$.

(b) Let $f(x) = \dfrac{3 + 2\cot 4x}{\csc 8x + \cot 8x}$.

(i) Express $f(x)$ in the form of $A\tan Bx + C$ where A, B and C are constants.

(ii) Solve the equation $f(x) = 5$, where $0 < x < \dfrac{\pi}{2}$.

(5 marks)

Answers written in the margins will not be marked

Answers written in the margins will not be marked

4. Let $y = y = e^{2x} \sin x$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(b) Let k be a constant. If $\frac{d^2y}{dx^2} - k^2 \frac{dy}{dx} + (k+3)y = 0$ for all real values of x, find the value(s) of k.

(6 marks)

Answers written in the margins will not be marked

Answers written in the margins will not be marked

5. **(a)** Prove that $\cos 3x = 4\cos^3 x - 3\cos x$.

(b) Let $\frac{\pi}{2} < x < \pi$.

(i) Prove that $\frac{\cos 3\left(\frac{5\pi}{6} - x\right)}{\cos\left(\frac{5\pi}{6} - x\right)} = \frac{2\sin 3x}{\sin x - \sqrt{3}\cos x}$.

(ii) Solve the equation $\frac{2\sin 3x}{\sin x - \sqrt{3}\cos x} = 1$.

(8 marks)

Answers written in the margins will not be marked

Answers written in the margins will not be marked

Answers written in the margins will not be marked

Answers written in the margins will not be marked

Answers written in the margins will not be marked

6. **(a)** Using mathematical induction, prove that

$$\frac{1^2+1+1}{1\times 2}+\frac{2^2+2+1}{2\times 3}+\ldots+\frac{n^2+n+1}{n(n+1)}=n+1-\frac{1}{n+1}$$ for all positive integers n.

(b) Using **(a)**, evaluate $\sum_{k=10}^{100}\frac{k^2+k+1}{k(k+1)}$.

(8 marks)

Answers written in the margins will not be marked

Answers written in the margins will not be marked

Answers written in the margins will not be marked

7. Evaluate the following limits:

(a) $\lim_{x\to\infty}\left(\frac{x}{x+3}\right)^{x}$ **(b)** $\lim_{x\to 0}\frac{e^{2x}-e^{-x}}{2x}$

(5 marks)

Answers written in the margins will not be marked

8. Find the equations of the tangents to the curve $C: 3x^2 - xy - y^2 = 9$ which are parallel to the line $5x - 3y + 1 = 0$. (5 marks)

Answers written in the margins will not be marked

Answers written in the margins will not be marked

Answers written in the margins will not be marked

Answers written in the margins will not be marked

Answers written in the margins will not be marked

Section B (14 marks)

9. Let $f(x)=\frac{(x-1)^3}{x^2}$, where $x \neq 0$.

(a) Find the *x*- and *y*-intercept(s) of the graph of $y=f(x)$. (1 mark)

(b) Find $f'(x)$ and prove that $f''(x)=\frac{6(x-1)}{x^4}$ (3 marks)

(c) Find the maximum / minimum point(s) and point(s) of inflexion of the curve $y=f(x)$. (4 marks)

(d) Find all the asymptote(s) of the graph of $y=f(x)$. (3 marks)

(e) Sketch the graph of $y=f(x)$ in Figure 1 on Page 14. (3 marks)

Answers written in the margins will not be marked

Answers written in the margins will not be marked

Answers written in the margins will not be marked

Answers written in the margins will not be marked

Answers written in the margins will not be marked

Answers written in the margins will not be marked

(e)

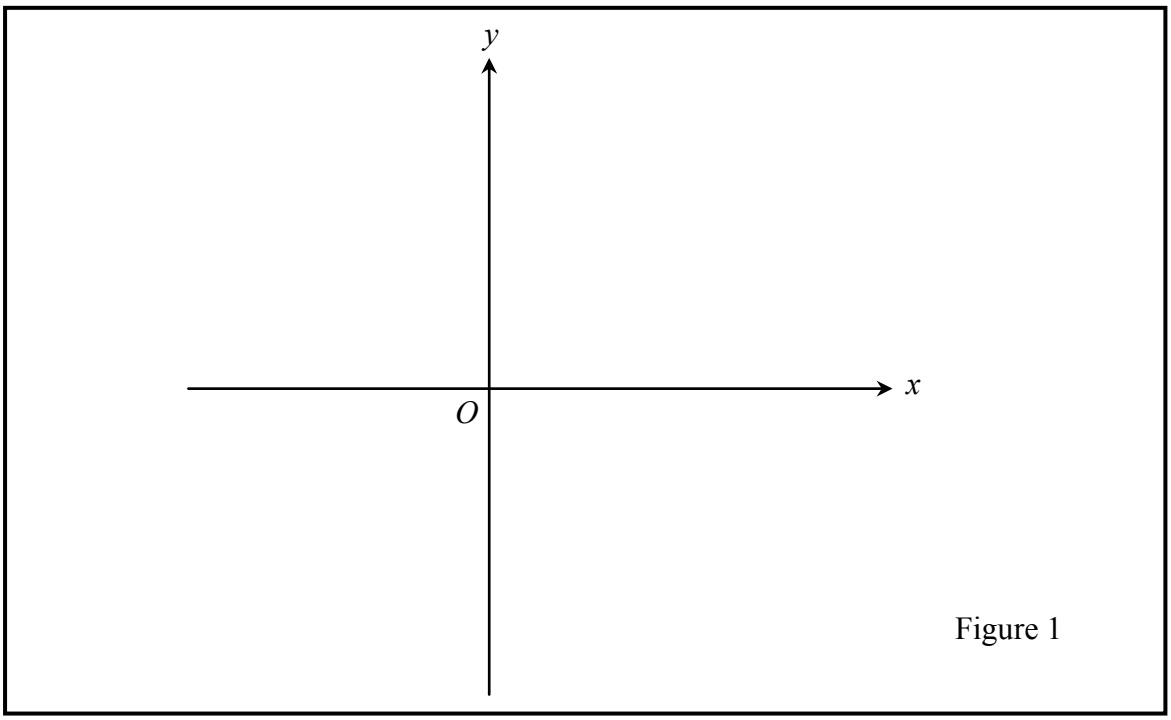


Figure 1

END OF PAPER

Answers written in the margins will not be marked