

Chapter 9

Indefinite Integration and its Applications

EXERCISE 9A*Level 1*

1. Find $\int \left(-\frac{1}{5}\right)dx$.
2. Find $\int 4x^3 dx$.
3. Find $\int (-2x^{-3})dx$.
4. Find $\int \frac{3}{x^2} dx$.
5. Find $\int \frac{dx}{5x^4}$.
6. Find $\int 8\sqrt{x} dx$.
7. Find $\int 4x^{-\frac{1}{3}} dx$.
8. Find $\int \frac{6dx}{\sqrt[5]{x^3}}$.
9. Find $\int 1.2x^{1.2} dx$.
10. Find $\int (5x^4 + 2x^2 - 3)dx$.
11. Find $\int (3x^4 + 6x^2)dx$.
12. Find $\int \left(\frac{3}{x^3} + 4 - 2x^2\right)dx$.
13. Find $\int 2x^2(x^3 - 4x)dx$.
14. Find $\int (x+1)(3x-2)dx$.
15. Find $\int (2x+3)(4-7x)dx$.

(*) Out syl after HKDSE 2022**Out syl Formula:**

$$\int \operatorname{cosec}^2 x \, dx = -\cot x + C$$

$$\int \sec x \tan x \, dx = \sec x + C$$

$$\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + C$$

Applications of indefinite integrals in some fields such as physics is not required.

16. Find $\int (x - \frac{1}{x})^2 dx$.
17. Find $\int \sqrt{x}(\sqrt{x} - 3)^2 dx$.
18. Find $\int \frac{x^5 - 2x + 3}{6x^3} dx$.
19. Find $\int \frac{(\sqrt{x^3} - \frac{2}{\sqrt{x}})^3}{x^2} dx$.
20. Find $\int \frac{4x^2 - 9}{2x + 3} dx$.
21. Find $\int \frac{x^{\frac{3}{2}} - 8}{\sqrt{x} - 2} dx$.
22. Find $\int \frac{e^x dx}{3}$.
23. Find $\int (x + e)(x - e) dx$.
24. Find $\int \frac{e^{2x} - 4}{e^x - 2} dx$.
25. Find $\int \ln e^{x^2} dx$.
26. Find $\int e^{\ln \sqrt[3]{x}} dx$.
27. Find $\int e^{-\ln 5x} dx$.
28. Find $\int \frac{1}{x \ln 7} dx$.
29. Find $\int (2x + 7)(6 + \frac{5}{2x}) dx$.
30. Find $\int (2x - \frac{1}{x})(\frac{3}{x^2} - 4) dx$.
31. Find $\int \frac{x^3 - 5x + 6}{2x^2} dx$.
32. Find $\int \frac{(3x + 5)^2}{15x^2} dx$.

33. Find $\int \left(\frac{x-2}{2x}\right)^2 dx$.

34. Find $\int \frac{(2\sqrt{x}-7)^2}{x^{\frac{3}{2}}} dx$.

35. Find $\int \sin x(3 - \csc x) dx$.

(*) 36. Find $\int \frac{\cos x}{\cos^2 x - 1} dx$.

Level 2

37. Let $y = \sqrt{x^2 + 4x + 3}$.

(a) Find $\frac{dy}{dx}$.

(b) Hence find $\int \frac{x+2}{\sqrt{x^2 + 4x + 3}} dx$.

38. Let $f(x) = x\sqrt{x-3}$.

(a) Find $f'(x)$.

(b) Hence find $\int \frac{x-2}{\sqrt{x-3}} dx$.

39. Let $y = x^2 e^{-4x}$.

(a) Find $\frac{dy}{dx}$.

(b) Hence find $\int (2x-1)xe^{-4x} dx$.

40. (a) Prove that $\int \frac{xdx}{px^2 + q} = \frac{1}{2p} \ln|px^2 + q| + C$, where p and q are constants, $p \neq 0$ and $x^2 \neq -\frac{q}{p}$.

(b) Hence find the following indefinite integrals.

(i) $\int \frac{xdx}{3-5x^2}$

(ii) $\int \frac{xdx}{(3+2x)(2x-3)}$

41. (a) Prove that $\frac{d}{dx} \left(\frac{1}{\sin x \cos x} \right) = \frac{2}{\cos^2 x} - \frac{1}{\sin^2 x \cos^2 x}$.

(b) Hence find $\int \frac{1}{\sin^2 x \cos^2 x} dx$.

42. (a) Prove that $\frac{1 - \sin x}{1 + \sin x} = 2 \sec^2 x - 2 \tan x \sec x - 1$.

(*) (b) Hence find $\int \frac{1 - \sin x}{1 + \sin x} dx$.

Level 3

43. (a) Prove that $\int \frac{e^{px} dx}{e^{px} + 1} = \frac{1}{p} \ln(e^{px} + 1) + C$, where p is a non-zero constant.

(b) Hence find the following indefinite integrals.

(i) $\int \frac{e^{2x} dx}{e^{2x} + 1}$

(ii) $\int \frac{dx}{e^{2x} + 1}$

44. (a) Find $\frac{d}{dx}(x^x)$, where $x > 0$.

(b) Find $\int x^x(1 + \ln x)dx$, where $x > 0$.

EXERCISE 9B

Level 1

45. Find $\int (x + 2)^2 dx$.

46. Find $\int (8 - x)^6 dx$.

47. Find $\int (2x - 1)^3 dx$.

48. Find $\int \frac{dx}{x + 4}$.

49. Find $\int \frac{dx}{1 - 3x}$.

50. Find $\int \sqrt{2x + 6} dx$.

51. Find $\int \frac{dx}{\sqrt{8-7x}}$.
52. Find $\int (1-2x)^{\frac{1}{3}} dx$.
53. Find $\int \frac{dx}{(5-\frac{x}{3})^4}$.
54. Find $\int \frac{x^2+6x+9}{x^4-18x^2+81} dx$.
55. Find $\int x(x^2+3) dx$.
56. Find $\int x^4 \sqrt{3-2x^5} dx$.
57. Find $\int x^n (px^{n+1}+q)^r dx$, where n, p, q and r are non-zero constants, $n \neq -1$ and $r \neq -1$.
58. Find $\int (x^2+3x-5)^3(2x+3) dx$.
59. Find $\int (3x^2-2)\sqrt{x^3-2x} dx$.
60. Find $\int \frac{x-3}{(x^2-6x+5)^3} dx$.
61. Find $\int \frac{x^2-1}{x^3-3x-5} dx$.
62. Find $\int \frac{x(x+1)(x-1)}{(7+2x^2-x^4)^{\frac{3}{4}}} dx$.
63. Find $\int \frac{1}{x^2} (1+\frac{1}{x})^{\frac{5}{2}} dx$.
64. Find $\int \frac{dx}{x^2 \sqrt{\frac{3}{2}-\frac{2}{x}}}$.
65. Find $\int e^{2x} dx$.
66. Find $\int e^{4x+7} dx$.
67. Find $\int 6e^{3x-2} dx$.
68. Find $\int e^{5-3x} dx$.

69. Find $\int \frac{dx}{7e^{4x-5}}$.
70. Find $\int (e^{5x} + 3e^{-3x})dx$.
71. Find $\int (e^x + e^{-x})^2 dx$.
72. Find $\int (e^x + 1)^3 dx$.
73. Find $\int 2xe^{x^2} dx$.
74. Find $\int x^2 e^{-x^3} dx$.
75. Find $\int (5e^{5x} - 5x^{-5})dx$.
76. Find $\int \frac{\ln 6x}{x} dx$.
77. Find $\int \frac{dx}{2x \ln \frac{x}{2}}$.
78. Find $\int \frac{dx}{\sqrt{x}(4\sqrt{x} - 1)}$.
79. Find $\int 13^x dx$.
80. Find $\int 2^{3-5x} dx$.
81. Find $\int (6^{x+1} - 5x^6)dx$.
82. Find $\int (\frac{e^{3x}}{3} - \frac{4^x}{5})dx$.

Level 2

83. Find $\int \frac{e^x}{e^{2x} + 2e^x + 1} dx$.
84. Find $\int \frac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}} dx$.
85. Find $\int x(2^{x^2+2})dx$.
86. Find $\int \frac{5^{\sqrt{x}-2}}{\sqrt{x}} dx$.

87. Find $\int \frac{\sqrt{\ln \sqrt{x}}}{x} dx$.
88. Find $\int \frac{dx}{6x \ln x^6}$.
89. (a) Express $\log_{10} x$ in terms of $\ln x$.
(b) It is given that $x > 0$, find $\int \log_{10} x^{\frac{1}{x}} dx$.
90. (a) Express $\log_7 x$ in terms of $\ln x$.
(b) It is given that $x > 0$, find $\int \frac{dx}{x \log_7 x^2}$.
91. Find $\int x(x-2)^7 dx$.
92. Find $\int (x+1)(5-x)^{2009} dx$.
93. Find $\int \frac{x+3}{x-1} dx$.
94. Find $\int \frac{2x+3}{x+2} dx$.
95. Find $\int \frac{3x dx}{\sqrt{4x-1}}$.
96. Find $\int \frac{x dx}{(2x+3)^{\frac{5}{3}}}$.
97. Find $\int x^2(3x-1)^{\frac{1}{3}} dx$.
98. Find $\int \frac{x^2 dx}{\sqrt{5-2x}}$.
99. Find $\int \frac{4x^3}{x^2+1} dx$.
100. Find $\int \frac{x^5}{x^3-2} dx$.
101. Find $\int x^5(x^3+5)^{\frac{3}{4}} dx$.
102. Find $\int \frac{2x^3 dx}{(1-2x^2)^{\frac{1}{3}}}$.
103. Find $\int \frac{x(x^2-3)(x^2+1)}{x^2-2} dx$.

104. Find $\int \frac{x^2 - 4x - 12}{x^2 - 4x + 4} dx$.

105. Find $\int \frac{x^2 - 2x + 5}{x^2 + 2x + 1} dx$.

106. It is given that $x > 0$, find $\int \sqrt{x^4 + 2x^2} dx$.

107. Let $x > 0$, using the substitution $u = \frac{1}{x^2}$, find $\int \frac{\sqrt{x^2 + 3}}{x^4} dx$.

108. Let $x > 0$, using the substitution $u = \frac{1}{x}$, find $\int \frac{\sqrt[3]{4x^3 - 2}}{x^5} dx$.

109. (a) Rationalize the denominator of $\frac{4}{\sqrt{x+2} - \sqrt{x-2}}$.

(b) Hence find $\int \frac{4dx}{\sqrt{x+2} - \sqrt{x-2}}$.

110. (a) Prove that $\frac{4}{x^2 - 4} = \frac{1}{x-2} - \frac{1}{x+2}$.

(b) Hence find $\int \frac{dx}{x^2 - 4}$.

111. (a) Let $\frac{2x+1}{[x(x+1)]^2} = \frac{A}{x^2} + \frac{B}{(x+1)^2}$, where A and B are constants. Find the values of A and B .

(b) Hence find $\int \frac{2x+1}{[x(x+1)]^2} dx$.

112. (a) Let $\frac{x-2}{(x+1)^3} = \frac{P}{x+1} + \frac{Q}{(x+1)^2} + \frac{R}{(x+1)^3}$, where P , Q and R are constants. Find the values of P , Q and R .

(b) Hence find $\int \frac{x-2}{(x+1)^3} dx$.

Level 3

113. Let $u = e^{2x} - 1$, find $\int \frac{e^x}{e^x - e^{-x}} dx$.

114. Let $u = 1 - e^{-3x}$, find $\int \frac{dx}{e^{3x} - 1}$.

115. Let $u = \frac{x+1}{x-2}$, find $\int \frac{dx}{x^2 - x - 2}$.

116. Let $u = \frac{2x-5}{2x-3}$, find $\int \frac{dx}{(2x-5)(2x-3)}$.

117. Find $\int (x+1)^2 (x+2)^{\frac{2}{3}} dx$.

118. Find $\int \frac{(x-1)^2}{(2-3x)^{\frac{4}{3}}} dx$.

119. (a) Find $\int \frac{2x+2}{x^2+2x+2} dx$.

(b) Hence find $\int \frac{x^2 dx}{x^2+2x+2}$.

120. (a) Find $\int \frac{x-4}{x^2-8x+32} dx$.

(b) Hence find $\int \frac{x^2 dx}{x^2-8x+32}$.

121. (a) Let $\frac{4x^2+11x-59}{(x+4)(2x-5)} = P + \frac{Q}{x+4} + \frac{R}{2x-5}$, where P , Q and R are constants. Find the values of P , Q and R .

(b) Hence find $\int \frac{4x^2+11x-59}{2x^2+3x-20} dx$.

122. (a) Let $\frac{6x^2+3x-10}{(x^2-2)(x+3)} = \frac{Px+Q}{x^2-2} + \frac{R}{x+3}$, where P , Q and R are constants. Find the values of P , Q and R .

(b) Hence find $\int \frac{6x^2+3x-10}{x^3+3x^2-2x-6} dx$.

EXERCISE 9C

Level 1

123. Find $\int x e^{-x} dx$.
124. Find $\int x^4 \ln x dx$.
125. Find $\int x \ln 7x dx$.
126. Find $\int \ln \sqrt{x} dx$.
127. Find $\int x \sin 7x dx$.
128. Find $\int (x - \pi) \cos 7x dx$.
129. Find $\int x^5 e^{x^3} dx$.

Level 2

130. Find $\int x^2 e^{2x} dx$.
131. Find $\int x^2 \cos 3x dx$.
132. Find $\int \sin \sqrt{x} dx$.
133. Find $\int x(e^{ax} - e^{-ax}) dx$, where $a \neq 0$.
134. Find $\int (px + q)e^{rx} dx$, where p , q and r are constants, and $r \neq 0$.
135. Find $\int x(e^x + \cos 2x) dx$.
136. Find $\int (x + 1)^2 \cos x dx$.
137. Find $\int e^{-x} \cos x dx$.
138. Find $\int e^x \sin 2x dx$.
139. Find $\int e^{hx} \sin kx dx$, where h and k are non-zero constants.
140. Find $\int \cos(\ln x) dx$.

Level 3

141. (a) Find $\int e^{2x} \sin x \, dx$.

(b) Hence find $\int x \sin(\ln x) \, dx$.

142. Let $I_n = \int x^n e^{ax} \, dx$, where n is a positive integer and a is a non-zero constant.

(a) Prove that $I_n = \frac{1}{a} x^n e^{ax} - \frac{n}{a} I_{n-1}$, where $n \geq 1$.

(b) Find $\int x e^x \, dx$.

(c) Hence find $\int x^3 e^x \, dx$.

143. Let $I_n = \int x^n \sin ax \, dx$, where n is a positive integer and a is a non-zero constant.

(a) Prove that $I_n = -\frac{1}{a} x^n \cos ax + \frac{n}{a^2} x^{n-1} \sin ax - \frac{n(n-1)}{a^2} I_{n-2}$, where $n \geq 2$.

(b) Find $\int x \sin x \, dx$.

(c) Hence find $\int x^5 \sin x \, dx$.

**EXERCISE 9D****Level 1**

144. Find $\int \cos(6x + 8) \, dx$.

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145. Find $\int x \sec(x^2) \, dx$.

146. Find $\int \sin 8x \cos 8x \, dx$.

147. Find $\int \sin 3x \cos 4x \, dx$.

148. Find $\int \sin 3x \sin 2x \, dx$.

149. Find $\int \cos 3x \cos 4x \, dx$.

150. Find $\int \sin \frac{3x}{2} \cos \frac{2x}{3} \, dx$.

151. Find $\int \sin \frac{x}{2} \sin \frac{x}{3} \, dx$.

152. Find $\int \cos^8 x \sin^3 x \, dx$.

153. Find $\int \sin^6 x \cos^3 x \, dx$.

154. Find $\int \sin^2 2x \cos 2x \, dx$.

155. Find $\int (2 \sin 2x + 1)^2 \, dx$.

156. Find $\int \sin^2 x \cos^2 x \, dx$.

157. Find $\int \tan^3 x \sec^2 x \, dx$.

158. Find $\int \tan^8 x \sec^2 x \, dx$.

159. Find $\int \sec^4 x \, dx$.

(*) 160. Find $\int \tan^3 2x \, dx$.

(*) 161. Find $\int \cot^2 x \csc^2 x \, dx$.

(*) 162. Find $\int \cot x \csc^3 x \, dx$.

(*) 163. Find $\int \cot 2x \csc^5 2x \, dx$.

(*) 164. Find $\int \cot^3 x \, dx$.

165. (a) Find $\int \frac{a \cos x}{\sqrt{a^2 - a^2 \sin^2 x}} \, dx$, where a is a positive constant and $0 < x < \frac{\pi}{2}$.

(b) Find $\int a \cos x \sqrt{a^2 - a^2 \sin^2 x} \, dx$, where a is a positive constant and $0 < x < \frac{\pi}{2}$.

Level 2

166. Find $\int \cos 3x \sin x \sin 4x \, dx$.

167. Find $\int x \sin x \cos x \, dx$.

(*) 168. Find $\int \tan^2 3x \sec 3x \, dx$.

(*) 169. Find $\int \csc 3x \cot^2 3x \, dx$.

170. Find $\int \sin^2 x \cos^6 x \, dx$.

171. Find $\int \frac{4}{\cot 2x + \tan 2x} \, dx$.

172. Find $\int (1 + \sin \theta)^3 \, d\theta$.

173. (a) Find $\int (\sin 3x - \cos 5x)^2 \, dx$.

(b) Find $\int \sin^2(ax + b) \cos^2(ax + b) \, dx$, where a and b are constants, and $a \neq 0$.

174. Find $\int \cos^4 2x \, dx$.

175. Find $\int \frac{\cos^3 \theta - \cos 3\theta}{\cos \theta} \, d\theta + \int \frac{\sin^3 \theta + \sin 3\theta}{\sin \theta} \, d\theta$.

176. Find $\int \frac{2\sin^3 y}{1 - \cos y} \, dy$.

177. Find $\int \frac{\sin \theta \cos 2\theta}{\sec 3\theta \csc 4\theta} \, d\theta$.

178. (a) Prove that $\frac{1 + \sin 2x - \cos 2x}{1 + \sin 2x + \cos 2x} = \tan x$.

(*) (b) Hence find $\int \frac{1 + \sin 2x - \cos 2x}{\cos^2 x (\sin x + \cos x)} \, dx$.

179. (a) Prove that $\frac{1 + \cos x + \cos 2x}{\sin x + \sin 2x} = \cot x$.

(*) (b) Hence find $\int \frac{1 + \cos x + \cos 2x}{\sin^2 x (1 + 2\cos x)} \, dx$.

180. (a) Consider $\sin \theta = 2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}$ and $\sin^2 \frac{\theta}{2} = \frac{1}{2}(1 - \cos \theta)$.

(i) Prove that $\sin^4 \frac{x}{4} \cos^4 \frac{x}{4} = \frac{1}{16} \sin^4 \frac{x}{2}$.

(ii) Hence prove that $\sin^4 \frac{x}{4} \cos^4 \frac{x}{4} = \frac{1}{64} \left(\frac{3}{2} - 2\cos x + \frac{1}{2} \cos 2x \right)$.

(b) Using the result of (a)(ii), find $\int \sin^4 \frac{x}{4} \cos^4 \frac{x}{4} \, dx$.

181. Find $\int \frac{2\sin^4 2x}{1 - \cos 2x} \, dx$.

182. (a) Find $\int \cos 4x \cos x dx$.

(b) Prove that $\frac{\sin 6x - \sin 2x}{\sin x} = 4 \cos 4x \cos x$.

(c) Hence find $\int \frac{\sin 6x}{\sin x} dx$.

Level 3

183. Let $I_n = \int \tan^n x dx$, where n is a non-negative integer.

(a) Prove that $I_n = \frac{1}{n-1} \tan^{n-1} x - I_{n-2}$, where $n \geq 2$.

(b) Hence find $\int \tan^6 x dx$.

184. (a) Find $\int e^{-x} \sin 2x dx$.

(b) Hence find $\int e^{-x} (\sin x + \cos x)^2 dx$.

EXERCISE 9E

Level 1

185. Find $\int \frac{dx}{\sqrt{25-x^2}}$.

186. Find $\int \frac{dx}{\sqrt{3-x^2}}$.

187. Find $\int \frac{dx}{x^2+25}$.

188. Find $\int \frac{dx}{x^2+3}$.

(*) 189. Find $\int \frac{dx}{x\sqrt{x^2 - 4}}$.

(*) 190. Find $\int \frac{dx}{x\sqrt{25x^2 - 9}}$.

(*) 191. Find $\int \frac{dx}{x^2\sqrt{4 - x^2}}$.

(*) 192. Find $\int \frac{dx}{x^2\sqrt{5 - x^2}}$.

193. Find $\int \frac{dx}{x^2\sqrt{x^2 + 4}}$.

194. Find $\int \frac{dx}{x^2\sqrt{x^2 + 7}}$.

Level 2

195. Find $\int \frac{x^3 dx}{\sqrt{9 - x^2}}$.

(*) 196. Find $\int x^3 \sqrt{x^2 + 16} dx$.

197. Find $\int x^3 \sqrt{9x^2 - 16} dx$.

(*) 198. Find $\int \frac{x^3}{\sqrt{4x^2 - 25}} dx$.

199. Find $\int x^2 (4 - x^2)^{-\frac{3}{2}} dx$.

(*) 200. Find $\int \frac{(9 - x^2)^{\frac{3}{2}}}{x^4} dx$.

Level 3

201. (a) Express $8x - x^2$ in the form of $b - (x - a)^2$, where a and b are constants.

(b) Hence find $\int \frac{dx}{\sqrt{8x - x^2}}$.

(*) 202. (a) Express $x^2 - 4x$ in the form of $(x - a)^2 - b$, where a and b are constants.

(b) Hence find $\int (x - 2)^3 \sqrt{x^2 - 4x} dx$.

(*) 203. Find $\int \frac{x}{x^2 - 2x + 10} dx$.

204. (a) Find $\int \frac{dx}{\sqrt{1 - x^2}}$.

(b) Hence find $\int \sin^{-1} x dx$.

205. Find $\int \frac{x dx}{x^4 + 16}$.

206. Find $\int \frac{dx}{e^x + e^{-x}}$.

207. Find $\int e^x (e^{2x} + 2e^x + 10)^{-\frac{3}{2}} dx$.

EXERCISE 9F

Level 1

- 208.** The slope at any point (x, y) of a curve is $2x + 5$. If the curve passes through $(2, 8)$, find the equation of the curve.
- 209.** The slope at any point (x, y) of a curve is $(3x - 1)(9x - 1)$, and the curve cuts the x -axis at $(1, 0)$. Find the equation of the curve.
- 210.** The slope at any point (x, y) of a curve is $4e^{2x} - e^x$, and the y -intercept of the curve is 5. Find the equation of the curve.
- 211.** The slope at any point (x, y) of a curve is $2x - \frac{30}{x} + \frac{50}{x^2}$. If the curve passes through $(5, 10)$, find the equation of the curve.
- 212.** The slope at any point (x, y) of a curve is $\sin 2x + 2\cos 3x$. If the curve passes through $(\frac{\pi}{2}, 1)$, find the equation of the curve.
- 213.** It is given that $\frac{dy}{dx} = 5e^x + 2(5^x)$. When $x = 0, y = 3$. Express y in terms of x .
- 214.** The slope at any point (x, y) of curve C is given by $\frac{dy}{dx} = -3x^2 + 2x - 1$, and the y -intercept of C is 1.
- (a) Find the equation of C .
- (b) Find the equation of the tangent to C at the point where C cuts the x -axis.
- 215.** It is given that $\frac{dy}{dx} = 6x^2 + kx + 8$, where k is a constant. When $x = 1, \frac{d^2y}{dx^2} = 6$ and $y = 24$.
- (a) Find the value of k .
- (b) Express y in terms of x .
- 216.** It is given that $f'(x) = ax^2 - 3x - 6$, where a is a constant. If $f(-6) = 120$ and $f'(\frac{1}{a}) = -5$, find the value of a and $f(x)$.
- 217.** It is given that $f'(x) = x^2 - ax - 5$, where $a > 0$. If $f(-3) = 4$ and $f'(\frac{1}{a}) = -2$, find the value of a and $f(x)$.
- (*) 218.** s and v are the displacement and velocity of a particle moving on a straight line at time t respectively. Given that $v = 8\sin 4t$, and $s = 3$ when $t = \frac{\pi}{4}$, express s in terms of t .
- (*) 219.** s, v and a are the displacement, velocity and acceleration of a particle moving on a straight line at time t respectively. Give that $a = 12t + 4, s = 2$ and $v = 3$ when $t = 0$, express s in terms of t .

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220. A particle starts moving along the x -axis from $x = 5$. Its velocity at any time t (in seconds) is given by $v = 8t - 3$ where $t \geq 0$.
- Find the velocity and acceleration, in magnitude and direction, when the particle starts to move.
 - Express x in terms of t .
 - Prove that the particle will not pass through the origin at any time.
221. The slope at any point (x, y) of a curve is $\frac{e^x}{e^x + 1}$, and the y -intercept of the curve is $\ln 4$. Find the equation of the curve.
222. The slope at any point (x, y) of a curve is $\frac{x^2 + 1}{\sqrt{2x^3 + 6x + 8}}$, and the x -intercept of the curve is 2. Find the equation of the curve.
223. At any point on a curve, $\frac{d^2y}{dx^2} = 6x - 3$. Find the equation of the curve if it passes through the point $(1, 1)$ and the slope is 1 at that point.
224. At any point on a curve, $\frac{d^2y}{dx^2} = 2x^2 - 3$. If the equation of the tangent to the curve at the point $(3, 1)$ is $x + 2y - 5 = 0$, find the equation of the curve.
225. At any point on a curve, $\frac{d^2y}{dx^2} = e^{2x} + 2e^{-x} + 1$. If the equation of the tangent to the curve at the origin O is $x = 3y$, find the equation of the curve.
226. At any point on a curve, $\frac{d^2y}{dx^2} = (3x - 2)^{-\frac{4}{3}}$. Find the equation of the curve if it passes through the points $(1, -\frac{3}{2})$ and $(22, 5)$.
227. Given that $f''(x) = x^2 + 2$ and $f(0) = f(2) = 1$, find $f(x)$.
228. At any point on a curve, $\frac{d^2y}{dx^2} = 3 + 2x - x^2$. If $(5, 35)$ is the maximum point of the curve, find the equation of the curve.
229. At any point on a curve, $\frac{d^2y}{dx^2} = 6e^{-2x} + 4$. If $(0, 1)$ is the minimum point of the curve, find the equation of the curve.

- 230.** The slope at any point (x, y) of curve C is given by $\frac{dy}{dx} = 4x + 5$, and the straight line $y = 7 - 3x$ is the tangent to the curve at point A .
- (a) Find the coordinates of A .
- (b) Find the equation C .
- 231.** At any point on a curve, $\frac{d^2y}{dx^2} = kx^2 + 3x$, where k is a constant and $\frac{dy}{dx}\bigg|_{x=2} = \frac{dy}{dx}\bigg|_{x=-4}$. It is given that the curve passes through the points $(-2, 5)$ and $(4, 20)$.
- (a) Find the value of k .
- (b) Find the equation of the curve.
- 232.** At any point on a curve, $\frac{d^2y}{dx^2} = kx - 6$, where k is a constant and $\frac{dy}{dx}\bigg|_{x=-2} = \frac{dy}{dx}\bigg|_{x=1}$. If the curve touches the x -axis at $x = -4$, find the equation of the curve.
- (*) **233.** An apple on a tree is 3 m above the ground. Tracy throws a stone up vertically from the ground underneath the apple. After t seconds, the speed of stone is v m/s, where $v = 7 - 8t$. Will the stone hit the apple? Explain briefly.
- (*) **234.** A person drives a car along a straight road from town A to town B without stopping. After travelling for t hours from town A, the velocity of the car is $(120t - 60t^2)$ km/h.
- (a) Find the travelling time.
- (b) Find the distance between town A and town B.
- (c) Find the maximum velocity of the car.

Level 3

- 235. (a)** Show that $\frac{d}{dx}[(x-3)^n(x+1)] = (x-3)^{n-1}[(n+1)x + n - 3]$, where n is a rational number.
- (b) The slope at any point (x, y) of curve C is given by $\frac{dy}{dx} = (x-3)^{2008}(1005x + 1003)$. If C passes through the point $(3, 3)$, find the equation of C .
- 236. (a)** Show that $\frac{d}{dx}[(2x-1)(x-1)^n] = (x-1)^{n-1}[2(n+1)x - n - 2]$, where n is a rational number.
- (b) The slope at any point (x, y) of curve C is given by $\frac{dy}{dx} = (x-1)^{998}(2000x - 1001)$. If the y -intercept of C is 2, find the equation of C .

237. The slope at any point (x, y) of curve C is given by $\frac{dy}{dx} = 3x^2 - 4x - 5$, and the straight line $y = -x - 1$ is the tangent to the curve at point A .

- (a) Find the possible coordinates of A .
- (b) Given that the y -intercept of C is greater than 0, find the equation of C .
- (c) Find the other point of intersection of the tangent and C .

(*) 238. An ant walks along a straight line. Let s cm, v cm/s and a cm/s² be the displacement, velocity and acceleration of the ant at time t s respectively.

- (a) Prove that $a = \frac{1}{2} \frac{d}{ds}(v^2)$.
- (b) Given that when $v = 0$, $s = 3$. If $a = \frac{1}{2s^2}$, express s in terms of v .