Chapter 14

Introduction to Vectors

EXERCISE 14A

(*) **Out syl after HKDSE 2022**
Scalar Triple Products

$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$

$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = (\mathbf{b} \times \mathbf{c}) \cdot \mathbf{a} = (\mathbf{c} \times \mathbf{a}) \cdot \mathbf{b}$

volume of a parallelepiped

Level 1

1. Express each of the following as a single vector.

(a) $\overrightarrow{BC} + \overrightarrow{AB}$

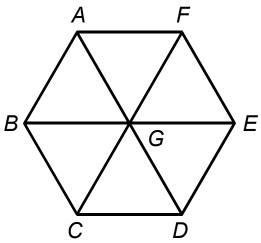
(b) $\overrightarrow{AC} - \overrightarrow{AB}$

2. Express each of the following as a single vector.

(a) $\overrightarrow{PQ} + \overrightarrow{QR} + \overrightarrow{RS}$

(b) $\overrightarrow{PR} + \overrightarrow{SR} - \overrightarrow{QR} - \overrightarrow{SQ}$

3. In the figure, *ABCDEF* is a regular hexagon. Its diagonals *AD*, *BE* and *CF* intersect at *G*.

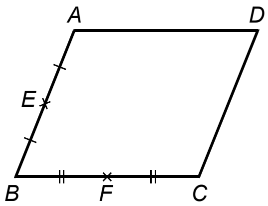


Express each of the following as a single vector.

(a) $\overrightarrow{CG} + \overrightarrow{EF}$

(b) $\overrightarrow{BC} + \overrightarrow{DE} - \overrightarrow{EF}$

4. In the figure, $ABCD$ is a parallelogram. E and F are the mid-points of AB and BC respectively.

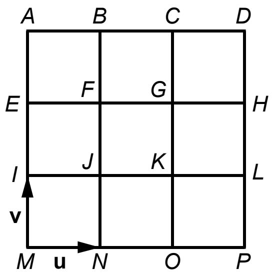


Express each of the following as a single vector.

(a) $\frac{1}{2}\overrightarrow{BC} + \overrightarrow{CD}$

(b) $\overrightarrow{BD} - 2\overrightarrow{EA}$

5. In the figure, square $ADPM$ is composed of nine identical small squares.



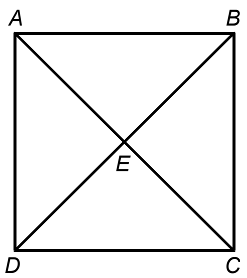
Express each of the following vectors in terms of **u** and **v**.

(a) $\overrightarrow{EH}$

(b) $\overrightarrow{BJ}$

(c) $\overrightarrow{BL}$

6. In the figure, $ABCD$ is a square. AC and BD intersect at E.

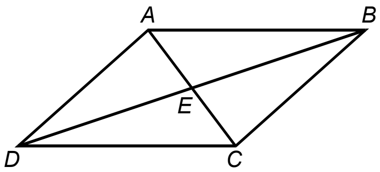


Let $\overrightarrow{DC} = \mathbf{a}$ and $\overrightarrow{DA} = \mathbf{b}$. Express each of the following vectors in terms of **a** and **b**.

(a) $\overrightarrow{AC}$

(b) $\overrightarrow{DE}$

7. In the figure, $ABCD$ is a parallelogram. AC and BD intersect at E.

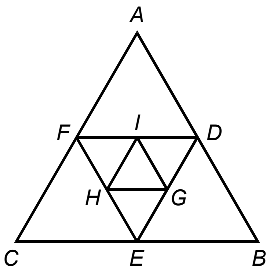


Let $\overrightarrow{DC} = \mathbf{a}$ and $\overrightarrow{DA} = \mathbf{b}$. Express each of the following vectors in terms of **a** and **b**.

(a) $\overrightarrow{AE}$

(b) $\overrightarrow{BD}$

8. In the figure, ABC, DEF and GHI are triangles. D, E, F, G, H and I are the mid-points of AB, BC, CA, DE, EF and FD respectively.

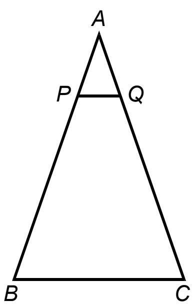


Let $\overrightarrow{HG} = \mathbf{a}$ and $\overrightarrow{HI} = \mathbf{b}$. Express each of the following vectors in terms of **a** and **b**.

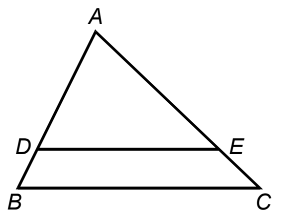
(a) $\overrightarrow{AC}$

(b) $\overrightarrow{AB}$

9. In ΔABC, P and Q are points on AB and AC respectively. It is given that $PB = 3AP$ and $QC = 3AQ$, prove that $PQ // BC$ and $PQ = \frac{1}{4}BC$ by using vectors.



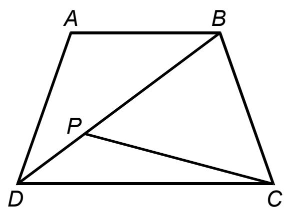
10. In ΔABC, D and E are points on AB and AC respectively such that $AD = \frac{3}{4}AB$ and $AE = \frac{3}{4}AC$. Let $\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{AC} = \mathbf{q}$.



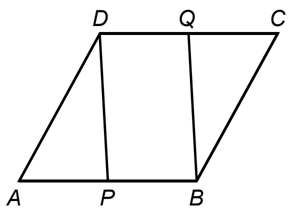
(a) Express $\overrightarrow{BC}$ and $\overrightarrow{DE}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

(b) Prove that $\overrightarrow{DE} = \frac{3}{4}\overrightarrow{BC}$.

11. In the figure, $ABCD$ is a quadrilateral. P is a point on BD such that $BP = 2DP$. Let $\overrightarrow{CB} = \mathbf{u}$ and $\overrightarrow{CD} = \mathbf{v}$. Express $\overrightarrow{CP}$ in terms of $\mathbf{u}$ and $\mathbf{v}$.



12. In the figure, $ABCD$ is a parallelogram. P and Q are the mid-points of AB and DC respectively. Prove that $PBQD$ is a parallelogram by using vectors.



Level 2

13. Let $\mathbf{a}$ and $\mathbf{b}$ be two non-zero vectors and they are not parallel to each other.

(a) If $3\mathbf{x} = \mathbf{y}$, where $\mathbf{x} = 2\mathbf{a} + n\mathbf{b}$ and $\mathbf{y} = 2k\mathbf{a} + 3\mathbf{b}$, find the values of k and n.

(b) If $3\mathbf{x} = 5\mathbf{y}$, where $\mathbf{x} = k\mathbf{a} + n\mathbf{b}$ and $\mathbf{y} = 3\mathbf{a} + \frac{6}{k}\mathbf{b}$, find the values of k and n.

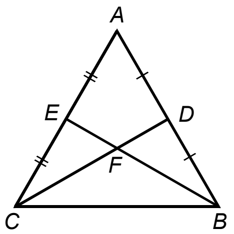
14. Let **a** and **b** be two non-zero vectors and they are not parallel to each other.

(a) If $\mathbf{x} = 2\mathbf{y}$, where $\mathbf{x} = (k+3)\mathbf{a} + 4\mathbf{b}$ and $\mathbf{y} = 3\mathbf{a} + n\mathbf{b}$, find the values of k and n.

(b) If $\mathbf{x} = 2\mathbf{y} - \mathbf{z}$, where $\mathbf{x} = 2k\mathbf{a} + 5\mathbf{b}$, $\mathbf{y} = \mathbf{a} + k\mathbf{b}$ and $\mathbf{z} = -n\mathbf{a} + \mathbf{b}$, find the values of k and n.

15. Let **a** and **b** be two non-zero vectors and they are not parallel to each other. If $\mathbf{x} = 4\mathbf{y}$, where $\mathbf{x} = k^2\mathbf{a} + n^2\mathbf{b}$ and $\mathbf{y} = (k-1)\mathbf{a} - (2n+4)\mathbf{b}$, find the values of k and n.

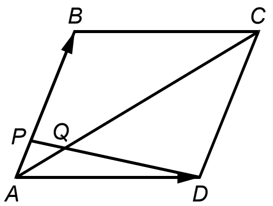
16. In the figure, ABC is a triangle. D and E are the mid-points of AB and AC respectively. BE and CD intersect at F. Let $\overrightarrow{AB} = \mathbf{p}$ and $\overrightarrow{AC} = \mathbf{q}$.



(a) Express $\overrightarrow{BE}$ and $\overrightarrow{CD}$ in terms of **p** and **q**.

(b) Express **p** and **q** in terms of $\overrightarrow{BE}$ and $\overrightarrow{CD}$.

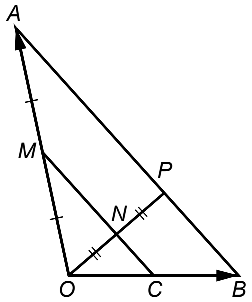
17. In the figure, $ABCD$ is a parallelogram. P is a point on AB. PQD and AQC are straight lines. It is given that $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{AD} = \mathbf{b}$, $PB = 3AP$ and $QC = 4AQ$.



(a) Express $\overrightarrow{AQ}$, $\overrightarrow{DQ}$ and $\overrightarrow{DP}$ in terms of **a** and **b**.

(b) If $\overrightarrow{DQ} = r\overrightarrow{DP}$, find the value of r.

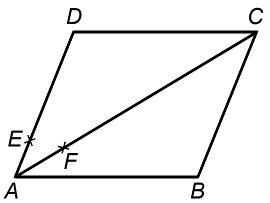
18. In the figure, P is a point on AB such that $AP = 3PB$. M and N are the mid-points of OA and OP respectively. Let $\overrightarrow{OA} = 5\mathbf{a}$ and $\overrightarrow{OB} = 5\mathbf{b}$.



(a) Express $\overrightarrow{AB}$, $\overrightarrow{OP}$ and $\overrightarrow{MN}$ in terms of **a** and **b**.

(b) MN produced meets OB at C. If $\overrightarrow{OC} = k\mathbf{b}$, find the value of k.

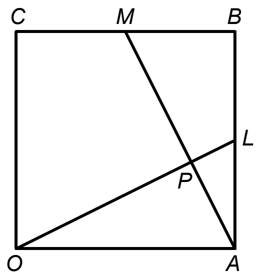
19. In the figure, $ABCD$ is a parallelogram. E and F are points on AD and AC respectively such that $AD = 4AE$ and $AC = 5AF$. Let $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{AD} = \mathbf{b}$.



(a) Express $\overrightarrow{EF}$ and $\overrightarrow{FB}$ in terms of **a** and **b**.

(b) Hence prove that E, F and B are collinear, and find $EF : FB$.

20. In the figure, $OABC$ is a square. M and L are the mid-points of BC and AB respectively. OL and AM intersect at P, $\overrightarrow{OP} = h\overrightarrow{OL}$ and $\overrightarrow{MP} = k\overrightarrow{MA}$. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{b}$.



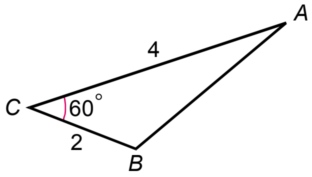
(a) Express $\overrightarrow{OP}$ in terms of h, **a** and **b**.

(b) **(i)** Express $\overrightarrow{MP}$ in terms of k, **a** and **b**.

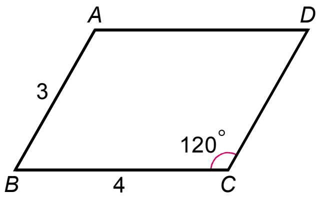
(ii) Express $\overrightarrow{OP}$ in terms of k, **a** and **b**.

(c) Hence find $h : k$.

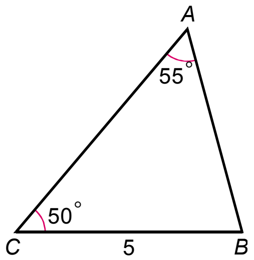
21. In the figure, ABC is a triangle. $AC = 4$, $BC = 2$ and $\angle ACB = 60°$. If $\overrightarrow{CA} = \mathbf{a}$ and $\overrightarrow{CB} = \mathbf{b}$, find $|\mathbf{b} - \mathbf{a}|$.



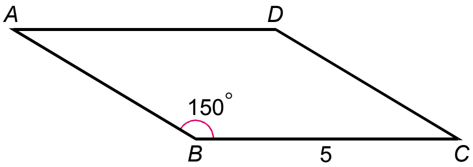
22. In the figure, $ABCD$ is a parallelogram. $AB = 3$, $BC = 4$ and $\angle BCD = 120°$. If $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{BC} = \mathbf{b}$, find $|\mathbf{b} - \mathbf{a}|$.



23. In the figure, ABC is a triangle. $BC = 5$, $\angle ACB = 50°$ and $\angle BAC = 55°$. If $\overrightarrow{CA} = \mathbf{a}$ and $\overrightarrow{CB} = \mathbf{b}$, find $|\mathbf{b} - \mathbf{a}|$. (Give your answer correct to 3 significant figures.)

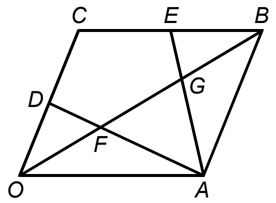


24. In the figure, $ABCD$ is a parallelogram. $BC = 5$, $\angle ABC = 150°$ and the area of $ABCD$ is 10. If $\overrightarrow{BC} = \mathbf{a}$ and $\overrightarrow{BD} = \mathbf{b}$, find $|\mathbf{a} - \mathbf{b}|$.



Level 3

25. In the figure, $OABC$ is a parallelogram. D and E are the mid-points of OC and BC respectively. OB intersects AD and AE at F and G respectively, where $AF = rAD$ and $GE = sAE$. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{b}$.



(a) Express $\overrightarrow{OF}$ in terms of r, $\mathbf{a}$ and $\mathbf{b}$.

(b) Express $\overrightarrow{GB}$ in terms of s, $\mathbf{a}$ and $\mathbf{b}$.

(c) It is given that F and G are the trisection points of OB. Find the values of r and s.

EXERCISE 14B

Level 1

26. Find the modulus and direction of each of the following vectors. (Give your answers correct to 1 decimal place if necessary.)

(a) $3\mathbf{i} + 3\mathbf{j}$

(b) $5\mathbf{i} - 4\mathbf{j}$

27. Find the modulus and direction of each of the following vectors. (Give your answers correct to 1 decimal place if necessary.)

(a) $-2\mathbf{i} + 3\mathbf{j}$

(b) $-5\mathbf{j}$

28. For each of the following, find $\overrightarrow{AB}$, $\left|\overrightarrow{AB}\right|$ and the direction of $\overrightarrow{AB}$. (Give your answers correct to 1 decimal place if necessary.)

(a) $A(5, -2)$, $B(3, -3)$

(b) $A(-2, 1)$, $B(1, 3)$

29. For each of the following, find $\overrightarrow{AB}$, $\left|\overrightarrow{AB}\right|$ and the direction of $\overrightarrow{AB}$. (Give your answers correct to 1 decimal place if necessary.)

(a) $A(2, 6)$, $B(1, 8)$

(b) $A(0, 6)$, $B(6, 0)$

30. In each of the following, $\overrightarrow{AB}$ and the coordinates of one of its end-points are given. Find the coordinates of the other end-point.

(a) $\overrightarrow{AB} = 6\mathbf{i} + 5\mathbf{j}$; $A(2, -1)$

(b) $\overrightarrow{AB} = -3\mathbf{i} + 7\mathbf{j}$; $B(2, 4)$

31. In each of the following, $\overrightarrow{AB}$ and the coordinates of one of its end-points are given. Find the coordinates of the other end-point.

(a) $\overrightarrow{AB} = \mathbf{i} - 4\mathbf{j}$; $A(-3, 7)$

(b) $\overrightarrow{AB} = -2\mathbf{i} - 3\mathbf{j}$; $B(2, 3)$

32. For each of the following, express **a** in terms of **i** and **j**.

(a) $|\mathbf{a}| = 4$, and the direction of **a** is the same as turning **j** 30° in a clockwise direction.

(b) $|\mathbf{a}| = 2$, and the direction of **a** is the same as turning **−i** 45° in an anti-clockwise direction.

33. Find a unit vector which has the same direction as each of the following vectors.

(a) $4\mathbf{i} - 3\mathbf{j}$

(b) $3\mathbf{i}$

34. Find a unit vector which has the same direction as each of the following vectors.

(a) $-7\mathbf{i} - \sqrt{15}\mathbf{j}$

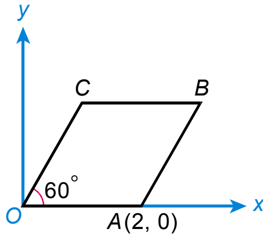
(b) $-2\mathbf{i} + 4\mathbf{j}$

35. If $\mathbf{u} = 2\mathbf{i} - \mathbf{j}$ and $\mathbf{v} = \mathbf{i} + 3\mathbf{j}$, express $-9\mathbf{i} + \mathbf{j}$ in terms of **u** and **v**.

36. If $\mathbf{u} = -3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{v} = \mathbf{i} - 2\mathbf{j}$, express $3\mathbf{i} - 5\mathbf{j}$ in terms of **u** and **v**.

Level 2

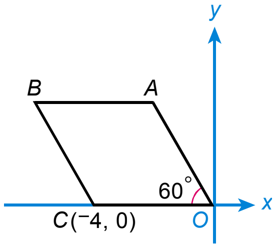
37. In the figure, $OABC$ is a rhombus. The coordinates of A are (2, 0) and $\angle AOC = 60°$.



(a) Express $\overrightarrow{OC}$ in terms of **i** and **j**.

(b) Express $\overrightarrow{OB}$ and $\overrightarrow{AC}$ in terms of **i** and **j**.

38. In the figure, $OABC$ is a rhombus. The coordinates of C are (−4, 0) and $\angle AOC = 60°$.



(a) Express $\overrightarrow{OA}$ in terms of **i** and **j**.

(b) Express $\overrightarrow{OB}$ and $\overrightarrow{AC}$ in terms of **i** and **j**.

39. **(a)** The positive angle between $\overrightarrow{OP}$ and the x-axis is θ, and the modulus of $\overrightarrow{OP}$ is r. Prove that $\overrightarrow{OP} = r(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$.

(b) Express the following vectors in the form of $r(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$, where $r \geq 0$ and $0° \leq \theta < 360°$.

(i) $-5\mathbf{i}$

(ii) $2\mathbf{i} - 2\mathbf{j}$

40. It is given that $\mathbf{u} = 4\mathbf{i} - 3\mathbf{j}$ and $\mathbf{v} = -3\mathbf{i} + 3\mathbf{j}$.

(a) Express **i** and **j** in terms of **u** and **v**.

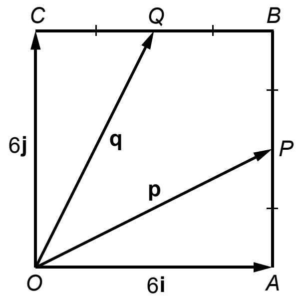
(b) If $\mathbf{i} + 3\mathbf{j} = a\mathbf{u} + b\mathbf{v}$, find the values of a and b.

41. It is given that $\mathbf{u} = 5\mathbf{i} + 2\mathbf{j}$ and $\mathbf{v} = -\mathbf{i} - 4\mathbf{j}$.

(a) Express **i** and **j** in terms of **u** and **v**.

(b) If $4\mathbf{i} - 5\mathbf{j} = a\mathbf{u} + b\mathbf{v}$, find the values of a and b.

42. In the figure, $OABC$ is a square. P and Q are the mid-points of AB and BC respectively. Let $\overrightarrow{OA} = 6\mathbf{i}$, $\overrightarrow{OC} = 6\mathbf{j}$, $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$.



(a) Express $\mathbf{p}$ and $\mathbf{q}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.

(b) Express $\overrightarrow{AB}$ and $\overrightarrow{BC}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

43. $P(2, 5)$ and $Q(6, 2)$ are two given points.

(a) **(i)** Express $\overrightarrow{PQ}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.

(ii) Find $\left|\overrightarrow{PQ}\right|$.

(b) If A is a point on PQ such that $\left|\overrightarrow{PA}\right| = 2$, express $\overrightarrow{PA}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.

44. $P(-1, 2)$ and $Q(2, -3)$ are two given points.

(a) **(i)** Express $\overrightarrow{PQ}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.

(ii) Find $\left|\overrightarrow{PQ}\right|$.

(b) If A is a point on PQ such that $\left|\overrightarrow{PA}\right| = \frac{\sqrt{34}}{2}$, express $\overrightarrow{PA}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.

45. $A(2, 1)$, $B(1, 2)$ and $C(-\frac{k}{2}, 2k)$ are three given points, where k is a constant.

(a) **(i)** Express $\overrightarrow{AB}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.

(ii) Express $\overrightarrow{BC}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and k.

(b) If A, B and C are collinear, find the value of k.

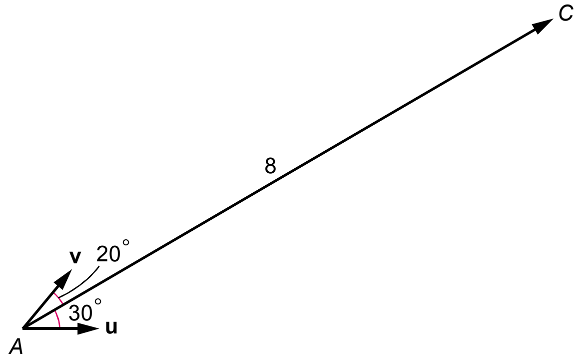
46. $A(-3, -2)$, $B(3, 0)$ and $C(3k, \frac{k}{2})$ are three given points, where k is a constant.

(a) **(i)** Express $\overrightarrow{AB}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.

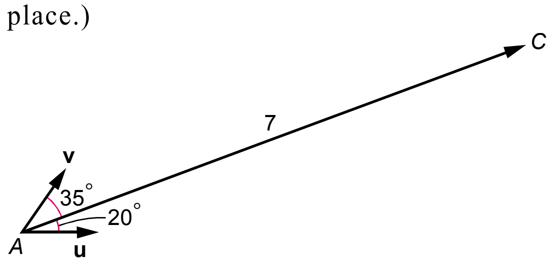
(ii) Express $\overrightarrow{BC}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and k.

(b) If A, B and C are collinear, find the coordinates of C.

47. In the figure, $\left|\overrightarrow{AC}\right| = 8$. **u** and **v** are two unit vectors. The angles **u** and **v** make with $\overrightarrow{AC}$ are 30° and 20° respectively. Express $\overrightarrow{AC}$ in terms of **u** and **v**. (Give your answer correct to 1 decimal place.)

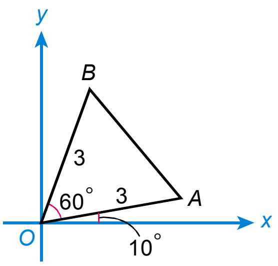


48. In the figure, $\left|\overrightarrow{AC}\right| = 7$. **u** and **v** are two unit vectors. The angles **u** and **v** make with $\overrightarrow{AC}$ are 20° and 35° respectively. Express $\overrightarrow{AC}$ in terms of **u** and **v**. (Give your answer correct to 1 decimal place.)



Level 3

49. In ΔOAB, $\left|\overrightarrow{OA}\right| = \left|\overrightarrow{OB}\right| = 3$.

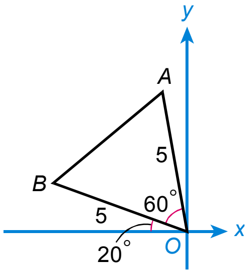


(a) Express $\overrightarrow{OA}$ and $\overrightarrow{OB}$ in terms of **i** and **j**. (Give your answers correct to 1 decimal place.)

(b) Express $\overrightarrow{AB}$ in terms of **i** and **j**. (Give your answer correct to 1 decimal place.)

(c) Hence prove that $\left|\overrightarrow{AB}\right| = 3$.

50. In ΔOAB, $\left|\overrightarrow{OA}\right| = \left|\overrightarrow{OB}\right| = 5$.



(a) Express $\overrightarrow{AB}$ in terms of **i** and **j**. (Give your answer correct to 1 decimal place.)

(b) Hence find $\left|\overrightarrow{AB}\right|$.

EXERCISE 14C

Level 1

51. Consider two points $A(2, 0, 1)$ and $B(1, -1, 2)$. Find $\overrightarrow{AB}$ and $\left|\overrightarrow{AB}\right|$.

52. Consider two points $A(-2, 3, 6)$ and $B(5, 2, -1)$. Find $\overrightarrow{AB}$ and $\left|\overrightarrow{AB}\right|$.

53. Consider two points $A(1, -3, 2)$ and $B(2, -1, 4)$.

(a) Find $\overrightarrow{AB}$ and $\left|\overrightarrow{AB}\right|$.

(b) Find the unit vector which has the same direction as $\overrightarrow{AB}$.

54. Consider two points $A(2, -1, -2)$ and $B(1, 1, 3)$.

(a) Find $\overrightarrow{AB}$ and $\left|\overrightarrow{AB}\right|$.

(b) Find the unit vector which has the same direction as $\overrightarrow{AB}$.

55. Consider two points $A(3, 1, -4)$ and $B(9, -2, 2)$. Find the unit vector which has the same direction as $\overrightarrow{AB}$.

56. Consider two points $A(-2, 1, -3)$ and $B(1, 2, 4)$. Find the unit vector which has the same direction as $\overrightarrow{AB}$.

57. It is given that the coordinates of A and B are $(2, \lambda, 5)$ and $(-\lambda, 4, 1)$ respectively. If $\left|\overrightarrow{AB}\right| = \sqrt{42}$, find the values of λ.

58. It is given that the coordinates of A and B are $(1, -4, -\lambda)$ and $(\lambda + 1, -1, -8)$ respectively. If $\left|\overrightarrow{AB}\right| = 7$, find the values of λ.

59. It is given that $A(1, -2, -3)$, $B(2, 0, 3)$ and C are collinear, where $\overrightarrow{BC} = 2\overrightarrow{AB}$.

(a) Find $\overrightarrow{BC}$.

(b) Find $\overrightarrow{AC}$.

60. It is given that $A(-2, -2, 4)$, $B(1, 2, 1)$ and C are collinear, where $\overrightarrow{BC} = 3\overrightarrow{AB}$.

(a) Find $\overrightarrow{BC}$.

(b) Find $\overrightarrow{AC}$.

61. It is given that the coordinates of A and C are $(2, -5, 3)$ and $(4, 1, -1)$ respectively. If $\overrightarrow{AB} = \frac{1}{2}\overrightarrow{AC}$, express $\overrightarrow{BC}$ in terms of **i**, **j** and **k**.

62. It is given that the coordinates of A and C are $(-1, 4, -2)$ and $(2, 1, 7)$ respectively. If $3\overrightarrow{AB} - \overrightarrow{AC} = \mathbf{0}$, express $\overrightarrow{BC}$ in terms of **i**, **j** and **k**.

Level 2

63. It is given that $\overrightarrow{AB} = \mathbf{i} - 7\mathbf{j} + 4\mathbf{k}$ and $A(2, -1, 3)$. Find the coordinates of B.

64. It is given that $\overrightarrow{AB} = -2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$ and $A(1, -1, 3)$. Find the coordinates of B.

65. It is given that $\overrightarrow{AB} = 2\mathbf{i} - 7\mathbf{j} - 8\mathbf{k}$ and $B(-1, -2, -4)$. Find the coordinates of A.

66. It is given that $\overrightarrow{AB} = -3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$ and $B(2, -2, 1)$. Find the coordinates of A.

67. It is given that the coordinates of A, B, C and D are (1, 2, 1), (2, 0, −1), (3, λ, 5) and (5, −2, 1) respectively. If $\overrightarrow{AB} // \overrightarrow{CD}$, find the value of λ.

68. It is given that the coordinates of A, B, C and D are (2, 1, −2), (0, 2, −6), (1, $2\lambda - 1$, $-\mu - 2$) and (7, $\lambda - 1$, μ) respectively. If $\overrightarrow{AB} // \overrightarrow{CD}$, find the values of λ and μ.

69. $ABCD$ is a parallelogram, where the coordinates of A, B and C are (2, 1, −2), (1, 2, −2) and (3, −1, 1) respectively. Find the coordinates of D.

70. $ABCD$ is a parallelogram, where the coordinates of A, B and C are (−1, k, 2), (3, $k + 1$, 1) and (2, −2, 1) respectively, and k is a constant. Find the coordinates of D.

71. Given four points $A(1, 3, 1)$, $B(4, 1, 2)$, $C(6, 4, 1)$ and $D(3, 6, 0)$, prove that $ABCD$ is a rhombus.

72. Given three points $A(-1, 2, 4)$, $B(2, 1, 2)$ and $C(4s, r, -s)$,

(a) express $\overrightarrow{AB}$ in terms of **i**, **j** and **k**.

(b) express $\overrightarrow{BC}$ in terms of **i**, **j**, **k**, r and s.

(c) If A, B and C are collinear, find the values of r and s.

73. Given three points $A(1, 1, 2)$, $B(0, 2, 5)$ and $C(r, s, -r - 1)$,

(a) express $\overrightarrow{AB}$ in terms of **i**, **j** and **k**.

(b) express $\overrightarrow{BC}$ in terms of **i**, **j**, **k**, r and s.

(c) If A, B and C are collinear, find the coordinates of C.

Level 3

74. Let **a** and **b** be the unit vectors of **u** and **v** respectively, where $\mathbf{u} = 4\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$ and $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$. It is given that $\alpha\mathbf{a} + \beta\mathbf{b} = 5\mathbf{i} - \gamma\mathbf{j} + 5\mathbf{k}$, find the values of α, β and γ.

75. Let **a** and **b** be the unit vectors of **u** and **v** respectively, where $\mathbf{u} = \mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$ and $\mathbf{v} = -6\mathbf{i} + 6\mathbf{j} - 7\mathbf{k}$. It is given that $\alpha\mathbf{a} - \beta\mathbf{b} = (2\gamma + 1)\mathbf{i} - (3\gamma - 1)\mathbf{j} + (\gamma + 2)\mathbf{k}$, find the values of α, β and γ.

Exercise 14D

Level 1

76. For each of the following, find $\overrightarrow{AB}$ and $\left|\overrightarrow{AB}\right|$.

(a) $\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j}$, $\overrightarrow{OB} = 5\mathbf{i} + 3\mathbf{j}$

(b) $\overrightarrow{OA} = \mathbf{i} - \mathbf{j} + \mathbf{k}$, $\overrightarrow{OB} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$

77. For each of the following, find $\overrightarrow{AB}$ and $\left|\overrightarrow{AB}\right|$.

(a) $\overrightarrow{OA} = -\mathbf{i} - \mathbf{j}$, $\overrightarrow{OB} = 4\mathbf{i} - 2\mathbf{j}$

(b) $\overrightarrow{OA} = -\mathbf{i} + \mathbf{j} + 2\mathbf{k}$, $\overrightarrow{OB} = \mathbf{i} - \mathbf{j} - \mathbf{k}$

78. Given two points $A(1, -2)$ and $B(7, 6)$, find

(a) the position vectors of A and B.

(b) the unit vector which has the same direction as $\overrightarrow{AB}$.

79. Given two points $A(-1, 1)$ and $B(2, 3)$, find

(a) the position vectors of A and B.

(b) the unit vector which has the same direction as $\overrightarrow{AB}$.

80. Given two points $A(2, 1, -1)$ and $B(1, 3, 1)$, find

(a) the position vectors of A and B.

(b) the unit vector which has the same direction as $\overrightarrow{AB}$.

81. Given two points $A(-1, 1, 2)$ and $B(2, 2, 1)$, find

(a) the position vectors of A and B.

(b) the unit vector which has the same direction as $\overrightarrow{AB}$.

82. It is given that $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ are the position vectors of P, Q and R with respect to O respectively. If $\overrightarrow{PQ} = 3\overrightarrow{QR} + 2\overrightarrow{RP}$, prove that $\mathbf{p} = \dfrac{4\mathbf{q} - \mathbf{r}}{3}$.

83. Find the coordinates of the mid-point of $A(7, 2, -3)$ and $B(2, -1, 5)$.

84. P is a point on AB such that $\overrightarrow{AP} = 2\overrightarrow{PB}$. If the coordinates of A and B are (1, 3, 2) and (1, 0, 2) respectively, find the coordinates of P.

85. P is a point on AB such that $3\overrightarrow{AP} = \overrightarrow{PB}$. If the coordinates of A and B are (2, −1, −2) and (2, 7, 14) respectively, find the coordinates of P.

86. P is a point on AB such that $3\overrightarrow{AP} = 2\overrightarrow{PB}$. If the coordinates of A and B are (−2, 1, 2) and (1, −2, 3) respectively, find the coordinates of P.

87. $A(2, -1, -7)$ and $P(2, 2, 2)$ are given. If P is a point on AB such that $AP : PB = 3 : 1$, find the coordinates of B.

88. $A(1, 2, 3)$ and $P(-4, 5, -6)$ are given. If P is a point on AB such that $AP : PB = 4 : 3$, find the coordinates of B.

89. The coordinates of A and B are (2, −2, 6) and (2, −6, −10) respectively. For each of the following cases, find the coordinates of C.

(a) C is the mid-point of AB.

(b) C is a point on AB such that $AC : CB = 1 : 3$.

90. The position vectors of the vertices of ΔABC are $\mathbf{a} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = -\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $\mathbf{c} = \mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$. If D is the mid-point of BC, find $\overrightarrow{AD}$.

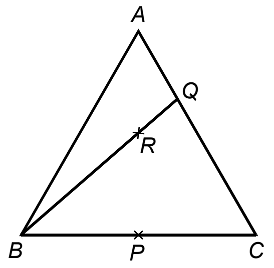
91. The position vectors of the vertices of ΔABC are $\mathbf{a} = 7\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$, $\mathbf{b} = -2\mathbf{i} - 2\mathbf{j} - 6\mathbf{k}$ and $\mathbf{c} = 5\mathbf{i} + \mathbf{j} - 4\mathbf{k}$. If D is the mid-point of BC, find $\overrightarrow{AD}$.

92. The coordinates of A, B and M are $(2y, -z, -2x)$, $(2z, -5z, -y + 1)$ and $(-2x, -y, z)$ respectively. If M is the mid-point of AB, find the coordinates of M.

Level 2

93. It is given that the position vectors of A, P and B are $4\mathbf{i}+\mathbf{j}$, $\frac{13}{2}\mathbf{i}+6\mathbf{j}$ and $8\mathbf{i}+9\mathbf{j}$ respectively. Prove that A, P and B are collinear, and find $AP:PB$.

94. In ΔABC, P is the mid-point of BC, Q is a point on AC such that $AQ:QC=1:2$, R is a point on BQ such that $BR:RQ=3:1$. Let $\overrightarrow{AQ}=\mathbf{u}$ and $\overrightarrow{AB}=\mathbf{v}$.

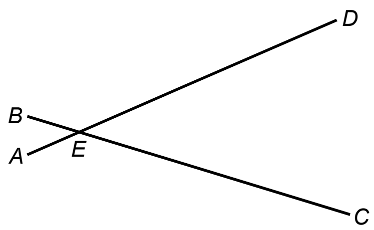


(a) Express $\overrightarrow{AR}$ in terms of $\mathbf{u}$ and $\mathbf{v}$.

(b) Express $\overrightarrow{AP}$ in terms of $\mathbf{u}$ and $\mathbf{v}$.

(c) Hence prove that A, R and P are collinear.

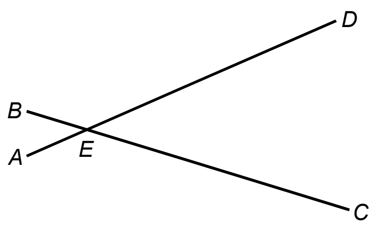
95. In the figure, AD and BC intersect at E. $AE:ED=BE:EC=1:5$. Let $\overrightarrow{AB}=\mathbf{a}$, $\overrightarrow{AC}=\mathbf{b}$.



(a) Express $\overrightarrow{AE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(b) Express $\overrightarrow{AD}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. Hence prove that $AB // CD$.

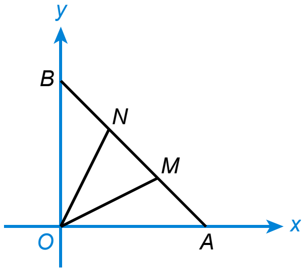
96. In the figure, AD and BC intersect at E. $AE:ED=BE:EC=2:9$. Let $\overrightarrow{AB}=\mathbf{a}$, $\overrightarrow{AC}=\mathbf{b}$.



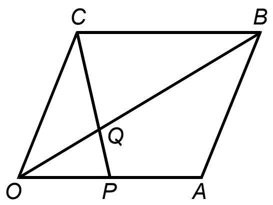
(a) Express $\overrightarrow{AE}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(b) Express $\overrightarrow{AD}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. Hence prove that $AB // CD$.

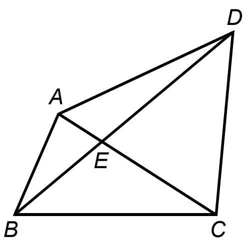
97. In the figure, A and B lie on the x-axis and the y-axis respectively. M and N are points on AB such that $AM = MN = NB$. Prove that $AB^2 = \frac{9}{5}(OM^2 + ON^2)$.



98. The figure shows a parallelogram $OABC$. P is the mid-point of OA. PC and OB intersect at Q. Find $OQ : QB$.



99. The figure shows a quadrilateral $ABCD$. Diagonals AC and BD intersect at E. $\overrightarrow{AC} = 3\overrightarrow{AB} + 2\overrightarrow{AD}$. Let $\overrightarrow{AB} = \mathbf{u}$ and $\overrightarrow{AD} = \mathbf{v}$.



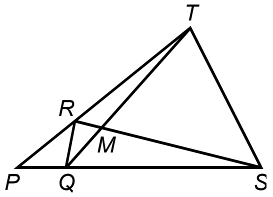
(a) Let $\overrightarrow{AE} = k\overrightarrow{AC}$. Express $\overrightarrow{AE}$ in terms of $\mathbf{u}$, $\mathbf{v}$ and k.

(b) Let $ED : BE = 1 : r$. Express $\overrightarrow{AE}$ in terms of $\mathbf{u}$, $\mathbf{v}$ and r.

(c) Hence find the values of k and r.

(d) Express $\overrightarrow{BE}$ in terms of $\mathbf{u}$ and $\mathbf{v}$.

100. The figure shows ΔPST. Q and R are points on PS and PT respectively such that $\overrightarrow{PS} = 5\overrightarrow{PQ}$ and $\overrightarrow{PT} = 3\overrightarrow{PR}$. QT and SR intersect at M. Let $\overrightarrow{PQ} = \mathbf{a}$ and $\overrightarrow{PR} = \mathbf{b}$.



(a) Express $\overrightarrow{QR}$, $\overrightarrow{QS}$ and $\overrightarrow{QT}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(b) Let $RM : MS = 1 : s$. Express $\overrightarrow{QM}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and s.

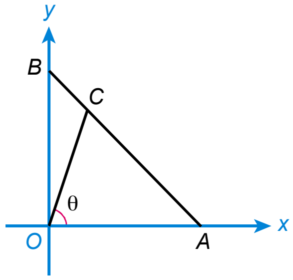
(c) Let $\overrightarrow{QM} = t\overrightarrow{QT}$. Express $\overrightarrow{QM}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and t.

(d) Hence find the values of s and t.

(e) Express $\overrightarrow{QM}$ and $\overrightarrow{MS}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Level 3

101. Let $\angle AOC = \theta$, $\overrightarrow{OA} = \mathbf{i}$ and $\overrightarrow{OB} = \mathbf{j}$. If C is a point on AB such that $AC : CB = 3 : 1$, find θ. (Give your answer correct to 1 decimal place.)



102. In ΔABC, D, E and F are the mid-points of BC, CA and AB respectively. G is the centroid of ΔABC. O is the origin. Prove the followings.

(a) $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \mathbf{0}$

(b) $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} = \overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF}$

(c) $\overrightarrow{OG} = \frac{1}{3}(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC})$

(d) $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC} = \mathbf{0}$

(e) G and the centroid of ΔDEF are the same point.

[Hint: The centroid divides each median in the ratio of 2 : 1.]

103. It is given that the coordinates of A and B are $(1, -2, -8)$ and $(1, -2, 4)$ respectively. AB cuts the xy-plane at P.

(a) Find the coordinates of P.

(b) Find $AP : PB$.

104. The coordinates of A, B and P are $(x-1, 3z+2, -2z)$, $(z-2, -2x, -4y)$ and $(-y, y+2z, -x)$ respectively. If A, B and P are collinear such that $AP : PB = 2 : 3$, find the coordinates of A, B and P.