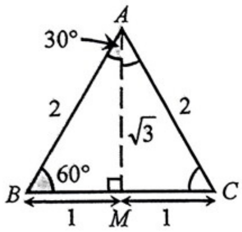
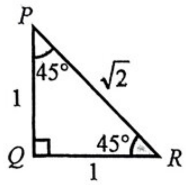
S.3 Mathematics
Chapter 09 –Trigonometric Ratio – Note 01

Name:____________ Class:________( ) Date:_____________

Trigonometric Ratios of Special Angles





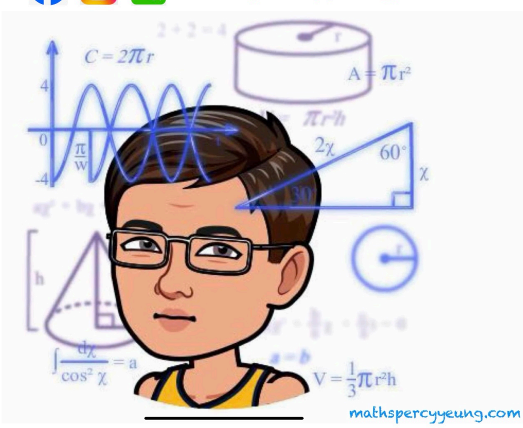
θ	$30°$	$45°$	$60°$
$\sin\theta$			
$\cos\theta$			
$\tan\theta$			

1. Evaluate each of the following without using a calculator.

(a) $\cos 60° - \sin 30°$ **(b)** $\tan 60° \tan 30°$ **(c)** $\cos^2 45° + \sin^2 45°$

2. In each of the following, find the acute angle θ.

(a) $\tan\theta = 1$ **(b)** $2\sin\theta = \sqrt{2}$ **(c)** $\cos\theta \tan 30° = \dfrac{1}{2}$



3. In each of the following, find the acute angle θ.

(a) $3 \tan \theta = \sqrt{3}$

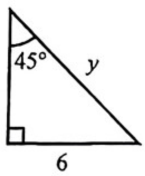
(b) $2 \cos \theta - 1 = 0$

(c) $2 \sin \theta - 3 = -2$

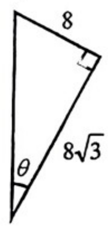
(d) $2 \cos (\theta + 15°) = \dfrac{\tan 60°}{\tan 45°}$

4. Find the unknown in each of the following figures.

(a)



(b)



5. Find the acute angle θ such that $\dfrac{2 \sin \theta}{\sqrt{3}} - 1 = 0$.

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Trigonometric Identities

For any acute angle θ, we have the following relationships:

$$\sin^2\theta + \cos^2\theta = 1 \qquad \text{and} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

Note $\sin^2\theta + \cos^2\theta = 1$ can also be written as **(i)** $\sin^2\theta = 1 - \cos^2\theta$
(ii) $\cos^2\theta = 1 - \sin^2\theta$

1. Simplify each of the following.

(a) $\tan\theta\cos\theta + \sin\theta$

(b) $\sin^2\theta - \sin^2\theta\cos^2\theta$

(c) $\tan\theta + \dfrac{1}{\tan\theta}$

2. Simplify each of the following.

(a) $\tan^2\theta\cos^2\theta$

(b) $\sin^3\theta + \sin\theta\cos^2\theta$

(c) $\dfrac{\cos^2\theta - 1}{\sin^2\theta - 1}$

(d) $\dfrac{\sin^2\theta}{\tan^2\theta} + \sin^2\theta$

3. Given that $\sin \theta = 0.43$, find the value of $\cos \theta$ by using the trigonometric identities. Give the answer correct to 3 significant figures.

4. Given that $\cos \theta = 0.375$, find the value of $\sin \theta$ by using the trigonometric identities. Give the answer correct to 3 significant figures.

5. Simplify $\dfrac{\sin\theta\cos\theta}{\tan\theta} - 1$.

6. Given that $\tan \theta = 3$, find the value of $\sin \theta$ by using the trigonometric identities. Leave your answer in surd form.

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Trigonometric Identities

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$$\sin^2\theta + \cos^2\theta = 1 \quad \text{and} \quad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

Note $\sin^2\theta + \cos^2\theta = 1$ can also be written as **(i)** $\sin^2\theta = 1 - \cos^2\theta$

(ii) $\cos^2\theta = 1 - \sin^2\theta$

1. Given that $\tan\theta = 3$, find the value of $\dfrac{2\sin\theta + \cos\theta}{2\cos\theta - \sin\theta}$.

2. Given that $\cos\theta = \dfrac{1}{4}$, find the value of $\dfrac{2\tan\theta}{\sin\theta}$.

3. Given that $\cos\theta = \dfrac{1}{4}$, find the value of $7\sin^2\theta + 5\cos^2\theta$.

4. Simplify $2\cos\theta\sin\theta + (\sin\theta - \cos\theta)^2$.

5. Simplify $(\cos\theta - \sin\theta)^2 + \dfrac{2\sin^2\theta}{\tan\theta}$.

6. Express each of the following in terms of $\cos\theta$.

(a) $\tan\theta\sin\theta$

(b) $(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)$

7. Express each of the following in terms of $\sin\theta$.

(a) $\cos^2\theta - \sin^2\theta$

(b) $(\sin\theta)(\sin\theta - 2\cos\theta) - (\sin\theta - \cos\theta)^2$

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Trigonometric Identities of Complementary Angles

For any acute angle θ,

$\sin\theta = \cos(90° - \theta)$ $\cos\theta = \sin(90° - \theta)$ $\tan\theta = \dfrac{1}{\tan(90° - \theta)}$

1. In each of the following, find the value of θ.

(a) $\cos\theta = \sin 28°$

(b) $\tan\theta \tan 72° = 1$

(c) $\sin(90° - \theta) = \cos 34°$

(d) $\cos(\theta - 10°) = \sin 19°$

2. Evaluate each of the following without using a calculator.

(a) $\sin^2 27° - \cos^2 63°$

(b) $5\cos^2 40° + 5\cos^2 50°$

(c) $\dfrac{\sin 18°}{3\cos 72°} - 4$

(d) $\tan 25° \tan 35° \tan 55° \tan 65°$

3. In each of the following, find the value of θ.

(a) $\sin\theta = \cos 15°$

(b) $\cos\theta = \sin 24°$

(c) $\tan\theta = \dfrac{1}{\tan 57°}$

(d) $\sin(\theta + 10°) = \cos 38°$

4. Evaluate each of the following without using a calculator.

(a) $\cos 80° - \sin 10°$

(b) $\tan 22° \tan 68° - 1$

(c) $\sin^2 20° + \sin^2 70°$

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Trigonometric Identities of Complementary Angles

For any acute angle θ,

$\sin\theta = \cos(90° - \theta)$ $\cos\theta = \sin(90° - \theta)$ $\tan\theta = \dfrac{1}{\tan(90° - \theta)}$

1. Simplify each of the following.

(a) $1 - \cos^2(90° - \theta)$ **(b)** $\dfrac{2\sin(90° - \theta)}{\tan(90° - \theta)}$

(c) $\tan(90° - \theta)\sin\theta$ **(d)** $\dfrac{\tan\theta\cos\theta}{\cos(90° - \theta)}$

2. Simplify each of the following.

(a) $2\sin(90° - \theta)\tan\theta$ **(b)** $-\sin\theta + 3\cos(90° - \theta)$

3. Simplify each of the following.

(a) $\dfrac{\cos(90^\circ-\theta)}{\sin(90^\circ-\theta)}-\dfrac{2}{\tan(90^\circ-\theta)}$

(b) $\dfrac{1-\sin\theta\cos(90^\circ-\theta)}{\cos\theta\sin(90^\circ-\theta)}$

(c) $\tan^2(90^\circ-\theta)\cos^2(90^\circ-\theta)-1$

(d) $\dfrac{\sin^2(90^\circ-\theta)}{\sin\theta\cos(90^\circ-\theta)}$

4. Simplify each of the following.

(a) $\dfrac{\cos^2(90^\circ-\theta)}{\tan^2\theta}+\cos^2(90^\circ-\theta)$

(b) $\dfrac{\sin^4\theta+\sin^2\theta\sin^2(90^\circ-\theta)}{\cos^2(90^\circ-\theta)}$

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Trigonometric Identities

For any acute angle θ, we have the following relationships:

$$\sin^2\theta + \cos^2\theta = 1 \quad \text{and} \quad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

Note $\sin^2\theta + \cos^2\theta = 1$ can also be written as **(i)** $\sin^2\theta = 1 - \cos^2\theta$

(ii) $\cos^2\theta = 1 - \sin^2\theta$

$$\sin\theta = \cos(90° - \theta) \qquad \cos\theta = \sin(90° - \theta) \qquad \tan\theta = \frac{1}{\tan(90° - \theta)}$$

1. Given that $\tan\theta = \frac{4}{3}$, find the value of $\tan(90° - \theta) - \tan\theta$.

2. Given that $\cos(90° - \theta) = \frac{5}{6}$, find the value of $\frac{\cos^2\theta - \sin^2\theta}{\cos(90° - \theta)}$.

3. Simplify each of the following.

(a) $2\sin^2(90° - \theta) + \cos^2(90° - \theta)$

(b) $\dfrac{\tan(90° - \theta)\cos(90° - \theta)}{\sin^2(90° - \theta)}$

(c) $6\tan\theta - \dfrac{\cos(90° - \theta)}{2\sin(90° - \theta)}$

(d) $\dfrac{3\tan(90° - \theta)}{\cos\theta} - \dfrac{4}{\cos(90° - \theta)}$

4. If $\dfrac{\cos\theta\cos(90° - \theta)}{\tan(90° - \theta)} = \dfrac{1}{4}$, find the value of θ.

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Prove Simple Trigonometric Identities

For any acute angle θ, we have the following relationships:

$$\sin^2\theta + \cos^2\theta = 1 \qquad \text{and} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

Note $\sin^2\theta + \cos^2\theta = 1$ can also be written as **(i)** $\sin^2\theta = 1 - \cos^2\theta$

(ii) $\cos^2\theta = 1 - \sin^2\theta$

$\sin\theta = \cos(90° - \theta)$ $\qquad$ $\cos\theta = \sin(90° - \theta)$ $\qquad$ $\tan\theta = \dfrac{1}{\tan(90° - \theta)}$

1. Prove the following identities.

(a) $\sin^2 x + 1 = 2 - \cos^2 x$

(b) $\cos^2 x - 1 = -\cos^2 x \tan^2 x$

(c) $1 - \tan^2\theta = 2 - \dfrac{1}{\cos^2\theta}$

2. Prove the following identities.

(a) $\sin\theta\cos(90° - \theta) = 1 - \cos\theta\sin(90° - \theta)$

(b) $\dfrac{\cos\theta\cos(90° - \theta)}{\tan(90° - \theta)} + \sin^2(90° - \theta) = 1$

(c) $\sin^2 x + 2\sin^2(90° - x) = \dfrac{\cos^2(90° - x)}{\tan^2 x} + 1$