TMCSS F4MATH_E1_2223

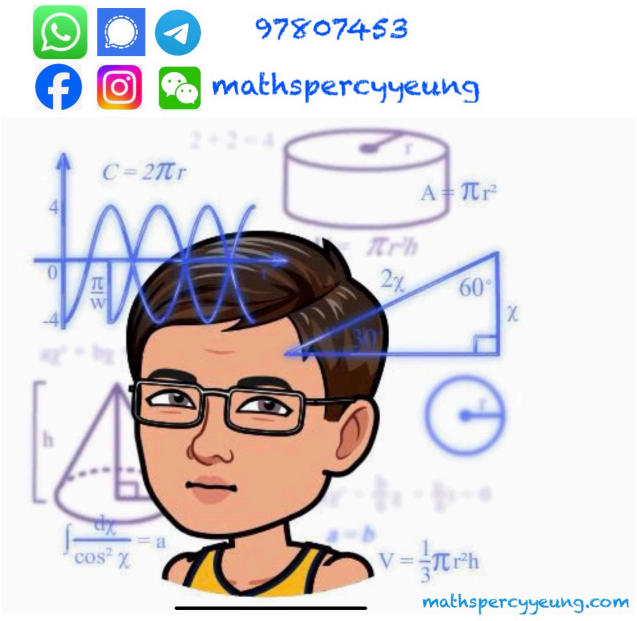
First Term Examination (2022 – 2023)

F.4 Mathematics I

Maximum marks	: 100	Name:	_____
Date	: 12 – 1 – 2023	Class:	_____
Time	: 8:15 – 9:45	Class No.:	_____
Time allowed	: 1.5 hours		

Instructions:

1. This paper consists of THREE sections, A(1), A(2) and B.
2. Attempt ALL questions in this paper. Write your answers in the spaces provided in this set of Question-Answer Sheets. Do not write in the margins. Answers written in the margins will not be marked.
3. Supplementary answer sheets will be supplied on request.
4. Unless otherwise specified, all working steps must be clearly shown.
5. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
6. The diagrams in this paper are not necessarily drawn to scale.



SECTION A(1)	/ 30
SECTION A(2)	/ 37
SECTION B	/ 33
Total	/ 100

SECTION A(1) (30 marks)

1. Simplify $\dfrac{2\alpha^2\beta^{-1}}{(2\alpha^{-3}\beta)^2}$ and express your answer with positive indices. (3 marks)

2. Factorize
   (a) $m^2 - 3mn + 2n^2$,
   (b) $m^2 - 3mn + 2n^2 - 4m + 4n$.

(3 marks)

3. Simplify $\dfrac{x^2-4x+4}{2x-2} \div \dfrac{x^2-4}{x^2-x}$. (3 marks)

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4. The number of cards owned by Marco is 2 times that owned by Joey. If Marco gives 3 cards to Joey, the ratio of the number of cards owned by Marco to that owned by Joey is 7 : 5. Find the total number of cards owned by Marco and Joey. (4 marks)

5. The coordinates of two points P and Q are $(-6, -2)$ and $(4, 8)$ respectively. P' is the reflection image of P with respect to the x-axis. Q is translated upwards by 4 units to Q'.

   (a) Write down the coordinates of P' and Q'.

   (b) Is PQ parallel to $P'Q'$? Explain your answer.

(4 marks)

6. Consider the graph of $y = 8 - 2(x + 3)^2$. (3 marks)

   (a) The direction of opening is ___upwards / downwards___*.

   (* Circle the correct answer.)

   (b) The vertex is _____.

   (c) The y-intercept is _____.

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7. Given that the equation of a straight line L_1 is $2x - 3y + 6 = 0$. (4 marks)

(a) The x-intercept of L_1 is _____.

(b) If a straight line L_2 whose slope is zero intersects L_1 at the y-axis, then the equation of L_2 is _____.

(c) If a straight line L_3 is parallel to L_1 and the x-intercept of L_3 is -5, find the equation of L_3.

8. Given that $f(x) = \dfrac{x^2+1}{x-1}$. Find $3f(-3)$. (2 marks)

9. When a polynomial $g(x)$ is divided by $2x - 1$, the quotient and the remainder are $x - 2$ and 3 respectively. Express $g(x)$ in the expanded form. (2 marks)

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10. Find the real roots of $(3x - 1)(x + 2) = 2(2x - 7)$. (Working steps are required.) (2 marks)

SECTION A(2) (37 marks)

11. Simplify $\dfrac{(32x^6y^0)^{\frac{1}{5}}}{\left(x^{-3}\cdot\sqrt[4]{y}\right)^2}$ and express your answer with positive indices. (3 marks)

12. The graph of $y = (m + 2)x^2 + 2mx + m - 3$ cuts the x-axis at 2 distinct points, where m is a constant and $m \neq -2$.
    (a) Find the range of the values of m. (3 marks)
    (b) Find the smallest value of m if m is an integer. (1 mark)

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13. In Figure 1, two straight lines L_1: $2x - (a + 2)y + 5 = 0$ and L_2: $3ax + 2y + (b - 3) = 0$ are perpendicular to each other, and L_2 passes through $A(1, -2)$. Find the values of a and b.

(4 marks)

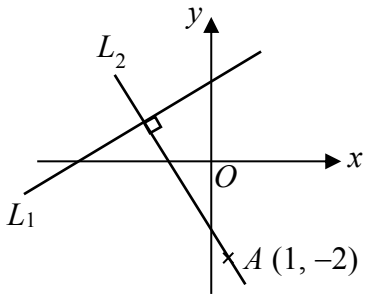

Figure 1

14. Let $f(x) = 6x^3 - 23x^2 + 12x + 20$ and $g(x) = -12x^2 + 22x + 20$.

(a) (i) Show that $3x + 2$ is a factor of $f(x)$.

(ii) Using long division, factorize $f(x)$.

(4 marks)

(b) (i) Find the G.C.D and L.C.M of $f(x)$ and $g(x)$.

(ii) Find the roots of the equation $f(x) = g(x)$.

(5 marks)

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15. Figure 2 shows the graph of $y = f(x) = 3x^2 + 9x - 12$. V is the vertex of the graph.

(a) (i) Write down the coordinates of A and B.

(ii) Find the coordinates of V.

(3 marks)

(b) Given that C is a point on the graph and its coordinates are $(a, 14a)$ with $a > 0$, find the value of a and the coordinates of C. (3 marks)

(c) Someone claims that the area of the quadrilateral $AVBC$ is larger than 150 sq. units. Do you agree? Explain your answer. (4 marks)

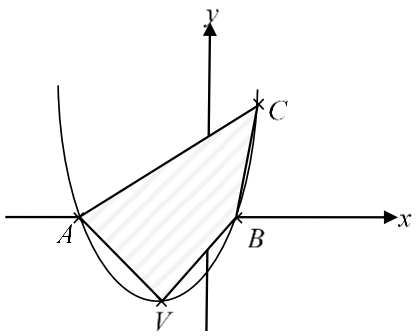

Figure 2

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16. $f(x)$ is a cubic polynomial. It is given that $f(x)$ is divisible by $1 - x$ and when $f(x)$ is divided by $x - 2$, the remainder is 4. When $f(x)$ is divided by $x^2 - 3x + 2$, the quotient is $3x + 7$.

(a) Find $f(x)$. (4 marks)

(b) Someone claims that all the roots of the equation $f(x) = 0$ are rational numbers. Do you agree? Explain your answer. (3 marks)

Section B (33 marks)

17. Form a quadratic equation in x with integral coefficients whose roots are $\frac{1}{a}$ and $2a$, where a is a positive integer. (3 marks)

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18. (a) Express each of the following in the form of $a + bi$. (Working steps are required.)

(i) $(1 + i)(2 - i)$

(ii) $\dfrac{10}{(1+i)(2-i)}$

(3 marks)

(b) Suppose that $\dfrac{10}{(1+i)(2-i)}$ is a root of the equation $x^2 + px = q$, where p and q are real constants. Find p and q. (3 marks)

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19. In Figure 3, $PQRS$ is a square. A and B are two points lying on PS and RS respectively. X, M and N are moving points on PS, PQ and QR respectively such that XN intersects AB at Y and $YNQM$ is a rectangle. Given that $QR = 4$, $AS = 2$ and $AX = 2XY$. Let $YN = a$, where $3 \leq a \leq 4$.

(a) Show that the area of $YNQM = a(10 - 2a)$. (3 marks)

(b) Alfred claims that the maximum area of rectangle $YNQM$ is 12.5 square units by using the method of completing square. Do you agree? Explain your answer. (4 marks)

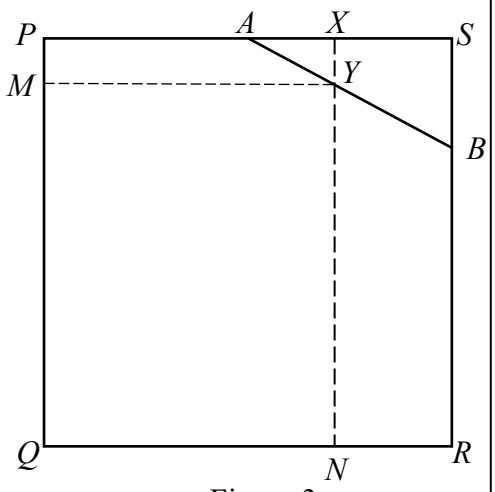

Figure 3

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20. In Figure 4, two straight lines L_1 and L_2 cut the x-axis at A and B respectively. C is the point of intersection of L_1 and L_2. The equations of L_1 and L_2 are $4x - 3y - 16 = 0$ and $y = -4x + 48$ respectively.

(a) Find the coordinates of C. (3 marks)

(b) L_3 is a straight line passing through C such that it divides ΔABC into two halves of equal area. Find the equation of L_3. (3 marks)

(c) G is a point on AC such that area of ΔABG : area of $\Delta BGC = 2 : 3$. Find the coordinates of G. (3 marks)

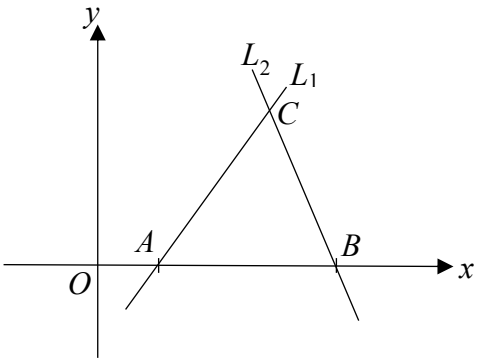

Figure 4

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21. Figure 5 shows the graph of $y = -x^2 + (k + 3)x + (k + 10)$, where $k > 0$. It cuts the x-axis at points $A(\alpha, 0)$ and $B(\beta, 0)$, and cuts the y-axis at point C. M is the mid-point of AB.

(a) Write down the coordinates of C in terms of k. (1 mark)

(b) Find in terms of k,

(i) $\alpha + \beta$ and $\alpha\beta$,

(ii) $(\alpha - \beta)^2$.

(3 marks)

(c) Write down the coordinates of M in terms of k. (1 mark)

(d) Jonathon claims that there are two possible values of k if the distance of MC is $\sqrt{490}$ units. Do you agree? Explain your answer. (3 marks)

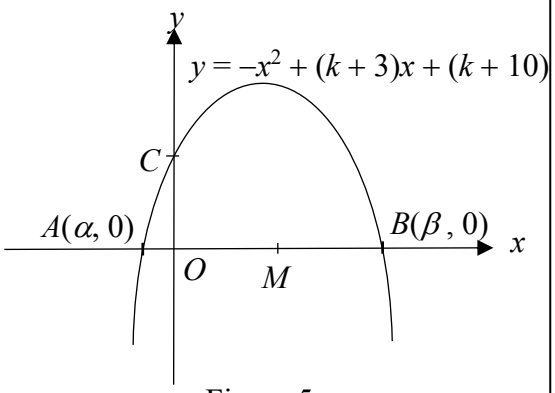

Figure 5

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