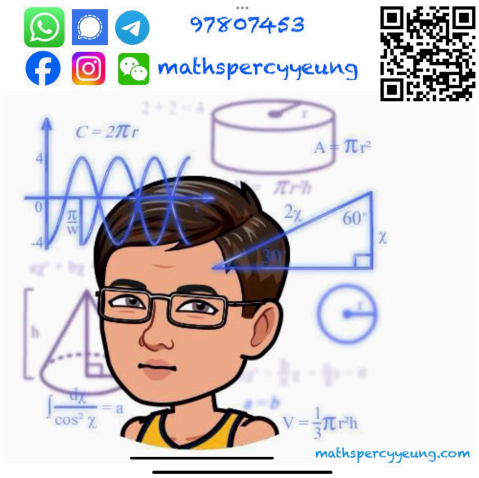
2019-20 F.4
MATH EP
M2



Form 4 2019 - 2020 1st Term Examination

MATHEMATICS Extended Part
Module 2 (Algebra and Calculus)

Question–Answer Book

14th January, 2020. (Tuesday)

8:15 am – 9:30 pm (1 hour 15 minutes)

This paper must be answered in English.

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your name, class and class number in the spaces provided on this cover.
2. This paper consists of Section A and Section B.
3. Answer ALL questions. Write your answers in the spaces provided in this Question-Answer Book.
4. Graph paper and supplementary answer sheets will be supplied on request. Write your name, class, class number and mark the question number box on each sheet.
5. Unless otherwise specified, all working must be clearly shown.
6. Unless otherwise specified, numerical answers must be exact.

	Marks
Section A	/ 34
Section B	/ 16
Grand Total	/ 50

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

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Section A (34 marks)

1. **(a)** Prove by mathematical induction that

$$1 - \frac{3}{1\times 4} - \frac{3}{4\times 7} - \frac{3}{7\times 10} - \ldots - \frac{3}{(3n-2)(3n+1)} = \frac{1}{3n+1}$$

for all positive integers n.

(b) Hence, evaluate $\dfrac{2}{1\times 4} + \dfrac{2}{4\times 7} + \dfrac{2}{7\times 10} + \ldots + \dfrac{2}{28\times 31}$.

(8 marks)

Answers written in the margins will not be marked.

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2. The coefficients of x and x^2 in the expansion of $(1+ax)^n$ are -3 and 4 respectively, where n is a positive integer and a is a constant. Find the values of a and n. (5 marks)

Answers written in the margins will not be marked.

3. Find the coefficient of x^2 in the expansion of $\left(3x-\frac{2}{x^2}\right)^8$. (4 marks)

Answers written in the margins will not be marked.

4. Find the constant term in the expansion of $(2-3x)^7\left(\frac{1}{x}+1\right)^3$ (4 marks)

Answers written in the margins will not be marked.

5. Prove that

$\sin(B+C-A)+\sin(C+A-B)+\sin(A+B-C)-\sin(A+B+C)=4\sin A\sin B\sin C$.

(4 marks)

Answers written in the margins will not be marked.

6. Prove that $\dfrac{\cos 3x}{\cos 3x - \sin 3x} - \dfrac{\cos 3x}{\cos 3x + \sin 3x} = \tan 6x$. (4 marks)

Answers written in the margins will not be marked.

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7. **(a)** Prove that $\csc^2 2x \equiv \frac{1}{4}\left(\sec^2 x + \csc^2 x\right)$.

(b) Hence, find the least value of $\tan^2 x + \cot^2 x$. (5 marks)

Answers written in the margins will not be marked.

Section B (16 marks)

8. **(a)** Show that $\cos^2 A - \cos^2 B = \sin(A + B)\sin(B - A)$. (3 marks)

(b) ABC is a triangle.

(i) Using **(a)**, show that

$$\cos^2 A - \cos^2 B + \sin^2 C = 2\cos A \sin B \sin C.$$

(ii) If $\cos^2 A - \cos^2 B - \cos^2 C = -1$, show that ABC is a right-angled triangle.

(6 marks)

(c) Using **(a)**, or otherwise, show that

$$\cos^2 x - \sin^2 y = \cos(x + y)\cos(x - y).$$

Hence solve the equation

$$\cos^2 2\theta - \sin^2 \theta + \cos \theta \sin 3\theta = 0$$

for $0° \le \theta \le 90°$.

(7 marks)

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