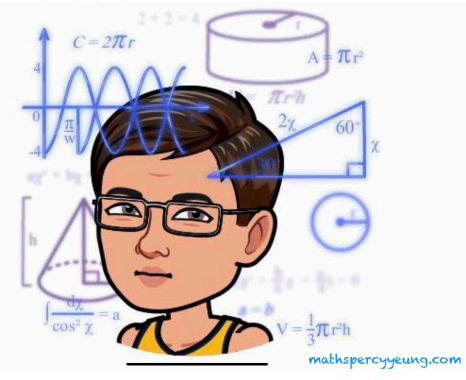
S.3 Mathematics

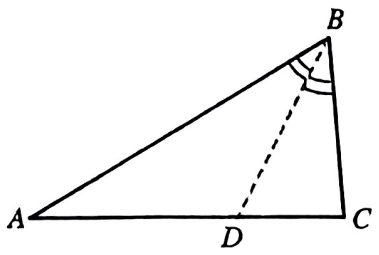
School-Based Exercise (S.B.E)

Chapter 6
Special Lines and Centres in a Triangle

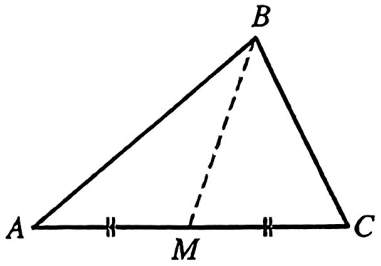
Name: _____ ( )



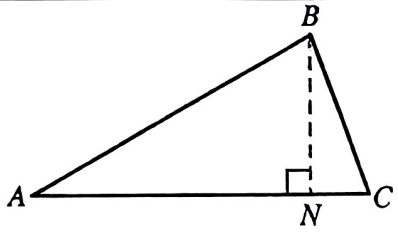
6.1 and 6.3 Angle Bisectors, Perpendicular Bisectors, Median and Altitude



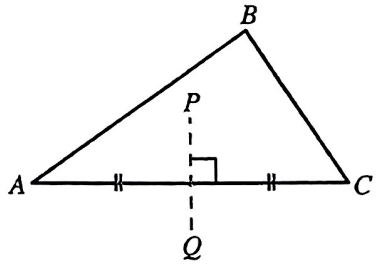
BD is a angle bisector of $\angle ABC$.



BM is a median of ΔABC.



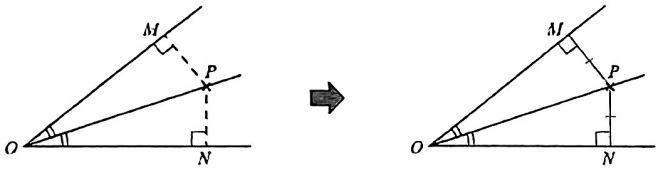
BN is an altitude of ΔABC.



PQ is a perpendicular bisector of AC.

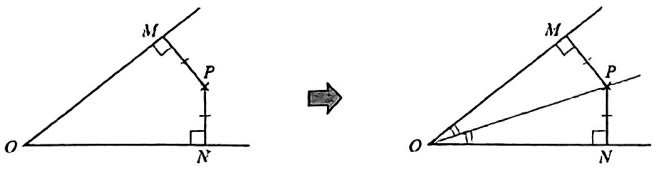
(Perpendicular: $PQ \perp AC$
Bisector: $AQ = AC$).

A straight line that bisects an angle is called an **angle bisector**.



If OP is the angle bisector of $\angle MON$, $PM \perp OM$ and $PN \perp ON$, then $PM = PN$.

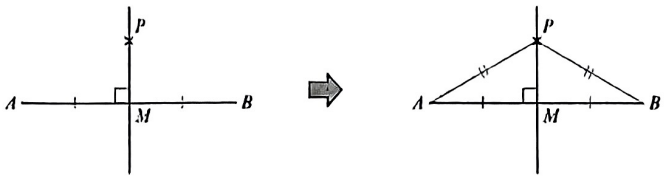
[Abbreviation: $\angle$ *bisector property*]



If $PM = PN$, $PM \perp OM$ and $PN \perp ON$, then OP is the angle bisector of $\angle MON$.

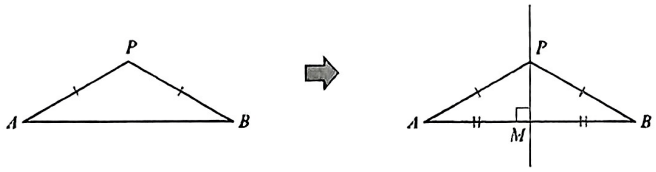
[Abbreviation: *converse of* $\angle$ *bisector property*]

A perpendicular line that bisects a line segment is called a **perpendicular bisector**.



If PM is the perpendicular bisector of AB, then $AP = BP$.

[Abbreviation: $\perp$ *bisector property*]



If $AP = BP$, then P lies on the perpendicular bisector of AB.

[Abbreviation: *converse of* $\perp$ *bisector property*]

Note: PM is the perpendicular bisector of AB in each of the following cases.

Case 1: $AP = BP$ and $AM = BM$

Case 2: $AP = BP$ and $PM \perp AB$

Level 1

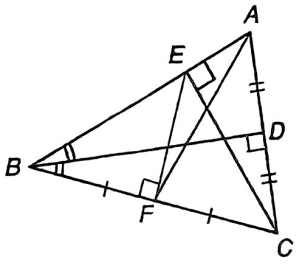
1. Refer to the figure.

(a) Name the angle bisector(s) of $\triangle ABC$.

(b) Name the median(s) of $\triangle ABC$.

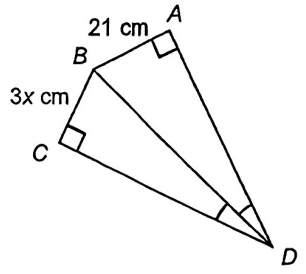
(c) Name the altitude(s) of $\triangle ABC$.

(d) Name the perpendicular bisector(s) of $\triangle ABC$.



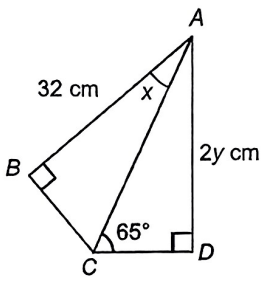
Find the unknown(s) in each of the following figures. **(2 – 5)**

2.



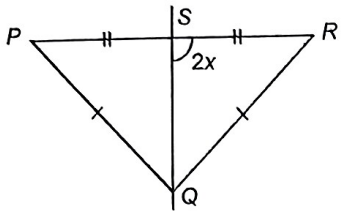
BD is the angle bisector of $\angle ADC$.

3.



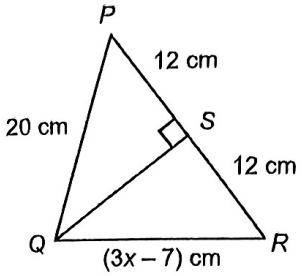
AC is the angle bisector of $\angle BCD$.

4.



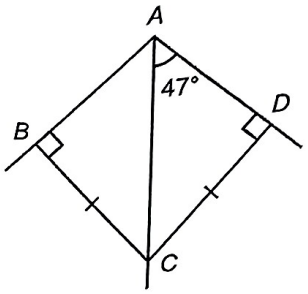
PSR is a straight line.

5.



PSR is a straight line.

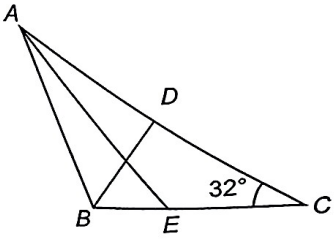
6. In the figure, $AB \perp BC$ and $AD \perp CD$. If $\angle CAD = 47°$, find $\angle ACB$.



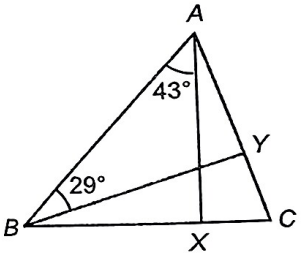
7. In the figure, BD is the perpendicular bisector of AC and AE is the angle bisector of $\angle BAC$. If $\angle ACB = 32°$, find

(a) $\angle CAE$,

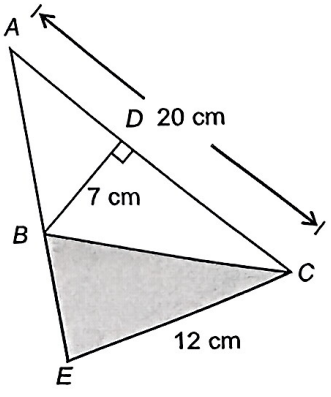
(b) $\angle AEB$.



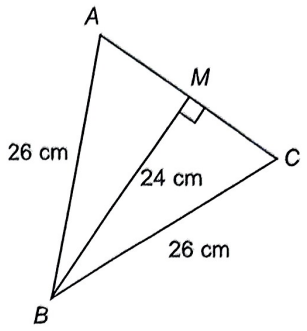
8. In the figure, X and Y are points on BC and AC respectively such that AX and BY are two altitudes of $\triangle ABC$. $\angle ABY = 29°$ and $\angle BAX = 43°$. Find $\angle ACB$.



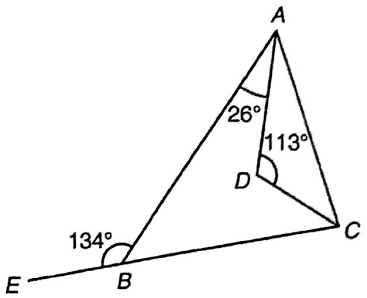
9. In the figure, B is a point on AE such that BC is the angle bisector of $\angle ACE$. D is a point on AC such that $BD \perp AC$. $AC = 20$ cm, $BD = 7$ cm and $CE = 12$ cm. Find the area of $\triangle BCE$.



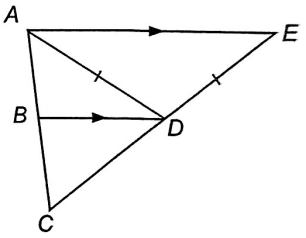
10. In the figure, M is a point on AC such that $BM \perp AC$. $AB = BC = 26$ cm and $BM = 24$ cm. Find the length of AC.



11. In the figure, EBC is a straight line. AD is the angle bisector of $\angle BAC$. $\angle ABE = 134°$, $\angle BAD = 26°$ and $\angle ADC = 113°$. Is CD the angle bisector of $\angle ACB$? Explain your answer.

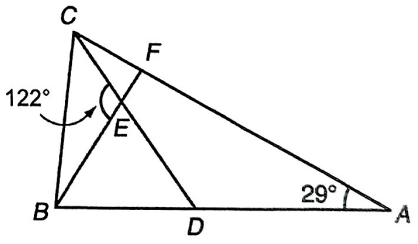


12. In the figure, ABC and CDE are straight lines, $AE \parallel BD$ and $AD = DE$. Is BD the angle bisector of $\angle ADC$? Explain your answer.

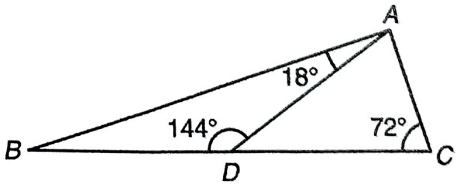


Level 2

13. In the figure, AFC and ADB are straight lines. BF and CD intersect at E such that $BE = DE$. $\angle BAC = 29°$ and $\angle BEC = 122°$. Prove that BF is an altitude of $\triangle ABC$.



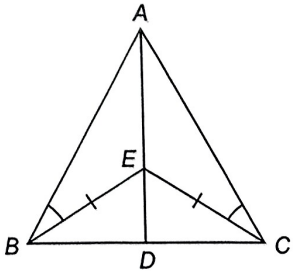
14. In the figure, BDC is a straight line. $\angle BAD = 18°$, $\angle ADB = 144°$ and $\angle ACD = 72°$. Prove that AD is a median of $\triangle ABC$.



15. In the figure, BDC is a straight line. E is a point on AD such that $BE = CE$ and $\angle ABE = \angle ACE$.

(a) Prove that $\angle ABC = \angle ACB$.

(b) Prove that AD is the perpendicular bisector of BC.

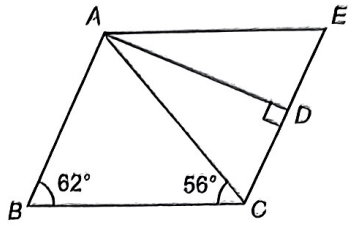


Cross-topic

16. In the figure, $ABCE$ is a parallelogram. D is a point on CE such that $AD \perp CE$. $\angle ABC = 62°$ and $\angle ACB = 56°$.

(a) Prove that $AC = BC$.

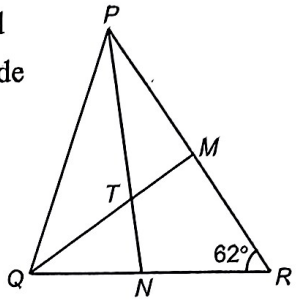
(b) Is AD the perpendicular bisector of CE? Explain your answer.



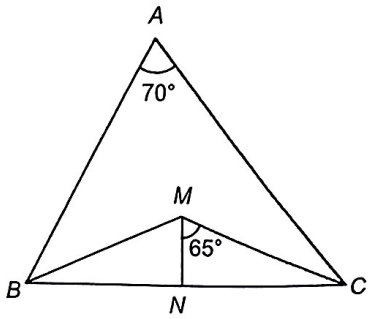
17. In the figure, M and N are points on PR and QR respectively such that PN and QM are the angle bisectors of $\angle QPR$ and $\angle PQR$ respectively. QM is an altitude of $\triangle PQR$. PN and QM intersect at T. If $\angle PRQ = 62°$, find

(a) $\angle PQM$,

(b) $\angle MTN$.



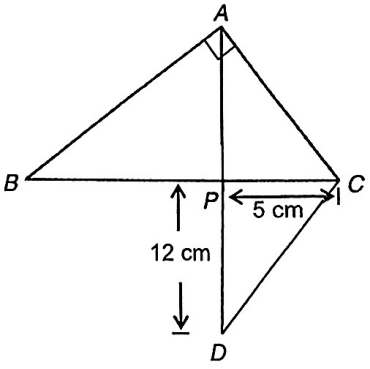
18. In the figure, MC is the angle bisector of $\angle ACB$. N is a point on BC such that MN is the perpendicular bisector of BC. $\angle BAC = 70°$ and $\angle CMN = 65°$. Find $\angle ABM$.



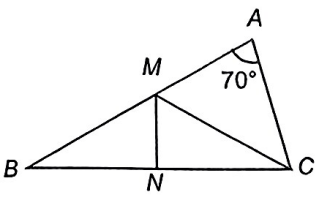
19. In the figure, APD and BPC are straight lines. BC is the perpendicular bisector of AD. $\angle BAC = 90°$, $CP = 5$ cm and $DP = 12$ cm.

(a) Prove that $\triangle ABP \sim \triangle CDP$.

(b) Hence, find the length of AB.



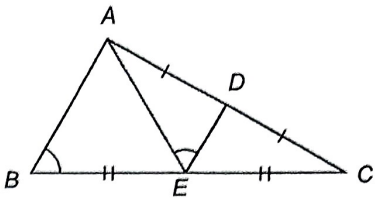
20. In the figure, M and N are points on AB and BC such that MN is the perpendicular bisector of BC. If $\angle BAC = 70°$ and $AB = BC$, find $\angle ACM$ and $\angle AMC$.



21. In the figure, D and E are the mid-points of AC and BC respectively. $\angle AED = \angle ABE$.

(a) Prove that $AE = BE$.

(b) Prove that DE is the perpendicular bisector of AC.

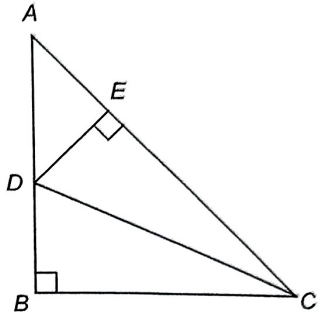


22. In the figure, $\triangle ABC$ is a right-angled isosceles triangle, where $\angle ABC = 90°$ and $AB = BC$. D is a point on AB such that CD is the angle bisector of $\angle ACB$. E is a point on AC such that $DE \perp AC$.

(a) Prove that $AE = DE$.

(b) Prove that $AB = CE$.

(c) If $AC = 102$ cm, find the perimeter of $\triangle ADE$.



Multiple Choice Questions

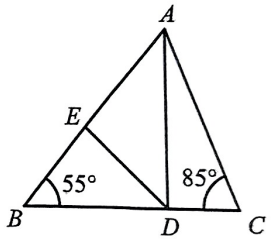
23. In the figure, D is a point on BC such that AD is an altitude of $\triangle ABC$. E is a point on AB such that DE is the angle bisector of $\angle ADB$. $\angle ABD = 55°$ and $\angle ACB = 85°$. Find $\angle AED$.

A. 80°

B. 95°

C. 100°

D. 110°



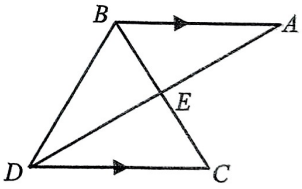
24. In the figure, AD and BC intersect at E. AD is the angle bisector of $\angle BDC$ and $AB \,//\, CD$. Which of the following must be true?

A. BC is the angle bisector of $\angle ABD$.

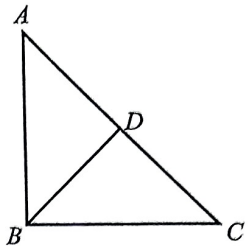
B. DE is a median of $\triangle BCD$.

C. BE is the perpendicular bisector of AD.

D. $\triangle ABD$ is an isosceles triangle.



25. In the figure, D is a point on AC such that BD is an altitude of $\triangle ABC$ and the angle bisector of $\angle ABC$. Which of the following must be true?



I. D is the mid-point of AC.

II. $\triangle ABC$ is a right-angled triangle.

III. $\triangle ABC$ is an isosceles triangle.

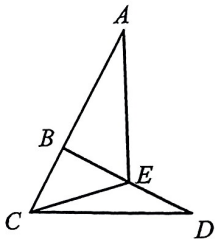
A. I and II only

B. I and III only

C. II and III only

D. I, II and III

26. In the figure, ABC and BED are straight lines. $\angle CAE = \angle BDC$, $\angle AEB = \angle ACD$ and $AB = BD$. Which of the following must be true?



I. $\triangle BCE$ is an isosceles triangle.

II. CB is an altitude of $\triangle CDE$.

III. CE is a median of $\triangle BCD$.

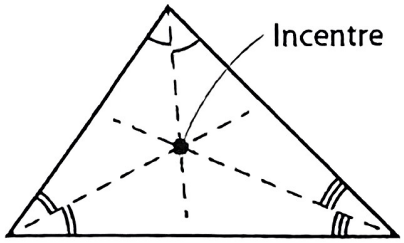
A. I and II only

B. I and III only

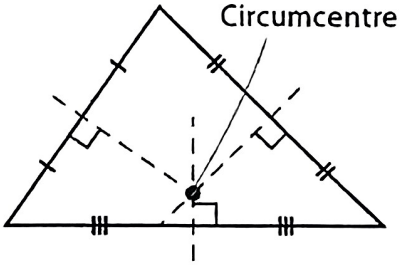
C. II and III only

D. I, II and III

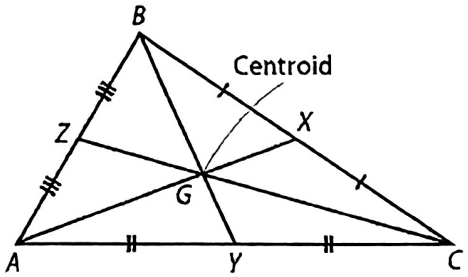
6.2 and 6.3 Centres of a Triangle



Incentre is the interception point of three **angle bisectors** .

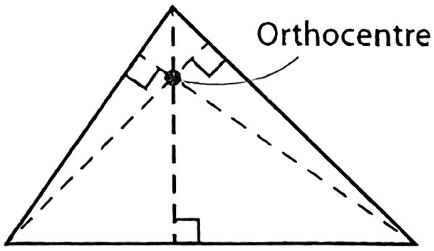


Circumcentre is the interception point of three **perpendicular bisectors.**



Centroid is the interception point of three **medians.**

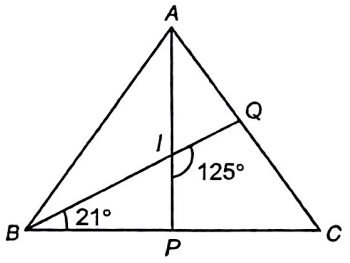
[Note: $AG:GX=BG:GY=CG:GZ=2:1$]



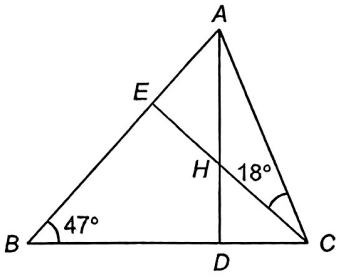
Orthocentre is the interception point of three **altitudes.**

Level 1

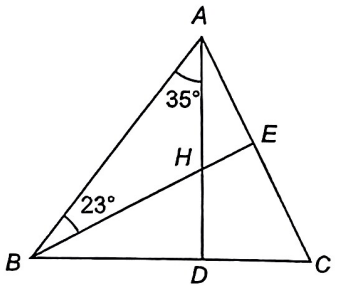
1. In the figure, I is the incentre of $\triangle ABC$. AI and BI are produced to meet BC and AC at P and Q respectively. $\angle PBI = 21°$ and $\angle PIQ = 125°$. Find $\angle ACB$.



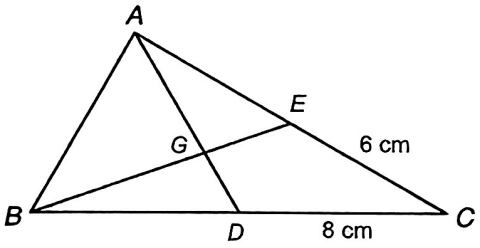
2. In the figure, H is the orthocentre of $\triangle ABC$. AH and CH are produced to meet BC and AB at D and E respectively. $\angle ABC = 47°$ and $\angle ACE = 18°$. Find $\angle CAD$.



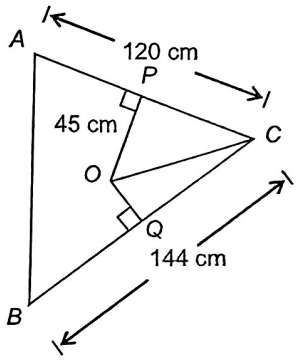
3. In the figure, H is the orthocentre of $\triangle ABC$. AH and BH are produced to meet BC and AC at D and E respectively. $\angle ABE = 23°$ and $\angle BAD = 35°$. Find $\angle ACB$.



4. In the figure, G is the centroid of $\triangle ABC$. AG and BG are produced to meet BC and AC at D and E respectively. $CD = 8$ cm, $CE = 6$ cm and the perimeter of $\triangle ABC$ is 35 cm. Find the length of AB.

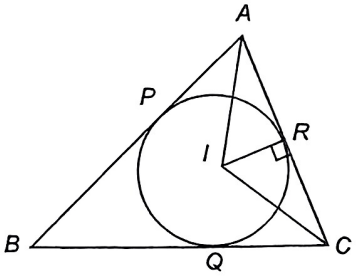


5. In the figure, O is the circumcentre of $\triangle ABC$. P and Q are points on AC and BC respectively such that $OP \perp AC$ and $OQ \perp BC$. $AC = 120$ cm, $BC = 144$ cm and $OP = 45$ cm. Find the length of OQ.



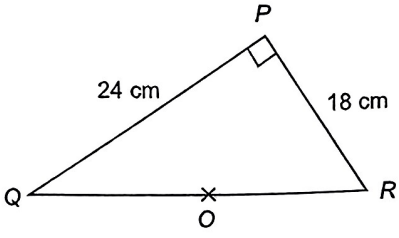
Cross-topic

6. In the figure, I is the incentre of $\triangle ABC$ and PQR is the circle that fits into $\triangle ABC$. $IR \perp AC$, $IA = IC = 17$ cm and $AC = 30$ cm. Find the circumference of the circle in terms of π.



Cross-topic

7. In the figure, O is the circumcentre of $\triangle PQR$ and lies on QR. $PQ = 24$ cm, $PR = 18$ cm and $\angle QPR = 90°$. Find the area of the circle that passes through the three vertices of $\triangle PQR$ in terms of π.

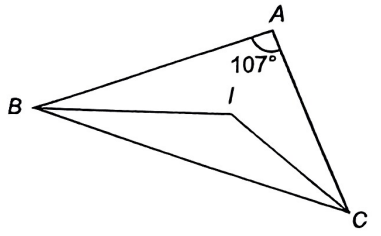


Level 2

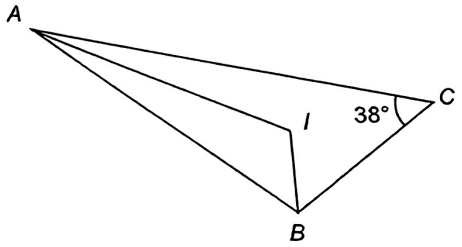
8. In the figure, I is the incentre of $\triangle ABC$. It is given that $\angle BAC = 107°$.

(a) Find $\angle IBC + \angle ICB$.

(b) Hence, find reflex $\angle BIC$.

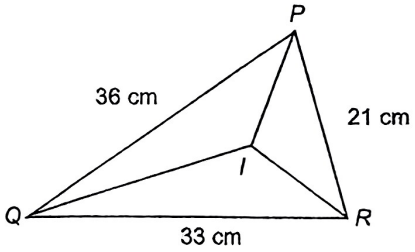


9. In the figure, I is the incentre of $\triangle ABC$.
If $\angle ACB = 38°$, find $\angle AIB$.



Cross-topic

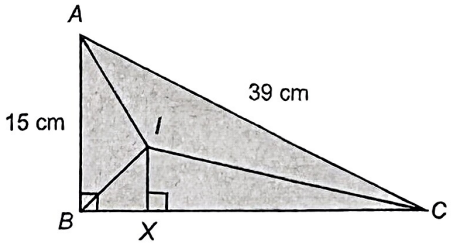
10. In the figure, I is the incentre of $\triangle PQR$. If $PQ = 36$ cm, $QR = 33$ cm and $RP = 21$ cm, find the ratio
area of $\triangle IPQ$: area of $\triangle IQR$: area of $\triangle IRP$.



11. In the figure, I is the incentre of right-angled triangle ABC. X is a point on BC such that $IX \perp BC$. $AB = 15$ cm and $AC = 39$ cm.

(a) Find the area of $\triangle ABC$.

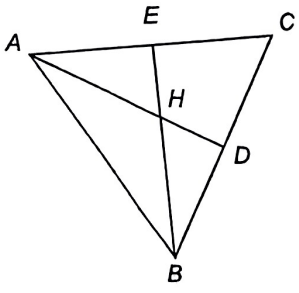
(b) Find the length of IX.



12. In the figure, H is the orthocentre of $\triangle ABC$. AH and BH are produced to meet BC and AC at D and E respectively and $DH = EH$.

(a) Prove that $\triangle AEH \cong \triangle BDH$.

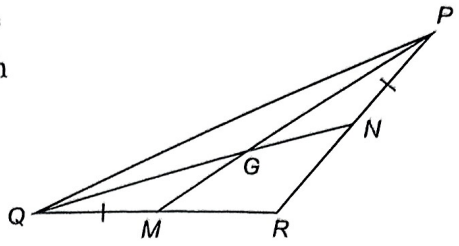
(b) If $\angle ACB = 60°$, find $\angle BAH$.



13. In the figure, G is the centroid of $\triangle PQR$. PG and QG are produced to meet QR and PR at M and N respectively such that $PN = QM$.

(a) Prove that $\angle MPR = \angle NQR$.

(b) Is $GP = GQ$? Explain your answer.

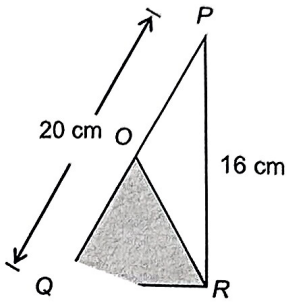


Cross-topic

14. In the figure, O is the circumcentre of $\triangle PQR$ and lies on PQ. $PQ = 20$ cm, $QR = 12$ cm and $PR = 16$ cm.

(a) Prove that $\triangle PQR$ is a right-angled triangle.

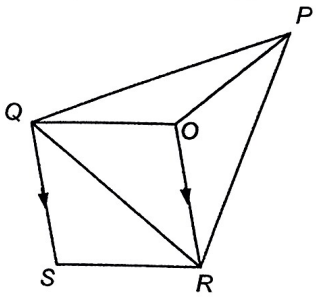
(b) Find the area of $\triangle OQR$.



15. In the figure, O is the circumcentre of $\triangle PQR$ and $QS \parallel OR$.

(a) Prove that QR is the angle bisector of $\angle OQS$.

(b) If $\angle PQR = 63°$ and $\angle RQS = 38°$, find $\angle POR$.



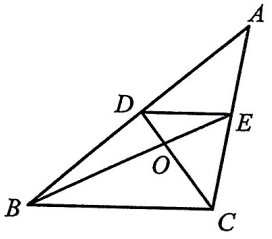
Level 3

16. In the figure, D and E are two points on AB and AC respectively such that CD and BE are medians of $\triangle ABC$ and they intersect at O.

(a) Prove that $\triangle ABC \sim \triangle ADE$.

(b) Prove that $\triangle BCO \sim \triangle EDO$.

(c) Using the results of **(a)** and **(b)**, prove that $CO : DO = 2 : 1$.

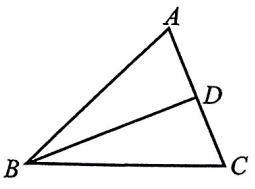


17. In the figure, $BA = BC$ and D is a point on AC such that $BD \perp AC$.

(a) Prove that $\triangle ABD \cong \triangle CBD$.

(b) Are the in-centre, the orthocentre, the centroid and the circumcentre of $\triangle ABC$ collinear? Explain your answer.

[Hint: Points are collinear if they lie on the same straight line.]



Multiple Choice Questions

18. Which of the following must be the centre of the inscribed circle of a triangle?

A. The orthocentre of the triangle

B. The centroid of the triangle

C. The in-centre of the triangle

D. The circumcentre of the triangle

19. Which of the following points must lie inside a triangle?

I. The in-centre of the triangle

II. The orthocentre of the triangle

III. The centroid of the triangle

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

20. If $\triangle ABC$ is a right-angled triangle, which of the following points must lie inside $\triangle ABC$?

I. The centroid of $\triangle ABC$

II. The orthocentre of $\triangle ABC$

III. The circumcentre $\triangle ABC$

A. I only

B. II only

C. III only

D. I, II and III