

Number Systems

Number Systems

Conventional Questions

[19-20]

1. [S.4 19-20 Standardized test 1, #4]

- (a) Simplify $\frac{26}{2+3i}$ and express the answer in the form of $a + bi$, where a and b are real numbers. **(2 marks)**
- (b) If $\frac{26}{2+3i} + (c + ci)(-5 + ci)$ is a real number, where c is a real constant, find the value(s) of c . **(2 marks)**

2. [S.4 19-20 Mid-year, #6]

It is given that $z = 2i(7 + ki) + 4$, where k is a real number. If z is purely imaginary, find the value of k . **(2 marks)**

3. [S.4 19-20 Mid-year, #8]

The sum of the squares of 3 consecutive positive integers is 302. Find the largest integer. **(3 marks)**

4. [S.4 19-20 Mid-year, #12]

- (a) Express $\frac{1}{2-5i}$ in the form of $a + bi$. **(2 marks)**
- (b) If $\frac{29}{2-5i}$ is a root of the quadratic equation $mx^2 + px + q = 0$, where p and q are real numbers, find the values of the p and q . **(2 marks)**

[21-22]

5. [S.4 21-22 Mid-year, #12]

Let $z = \frac{1}{4-3i}$.

- (a) Express z in the form of $a + bi$, where a and b are real numbers. **(2 marks)**
- (b) If $kz - 2 + hi = 0$, where h and k are real numbers, find the value of h . **(3 marks)**

[22-23]

6. [S.4 22-23 Mid-Year, #10]

Express $\frac{2+i}{3-5i}$ in the form $a + bi$, where a and b are real numbers. **(2 marks)**

Number Systems

[23-24]

7. [S.4 23-24 Mid-Year,#11]

The real part of $\frac{k}{7-i}$ is 14, where k is real number. Find the value of k .

(3 marks)

8. [S.4 23-24 Mid-Year,#12]

It is given that $(a+6i)(3-2i)=27+2bi$, where a and b are real numbers. Find the values of a and b .

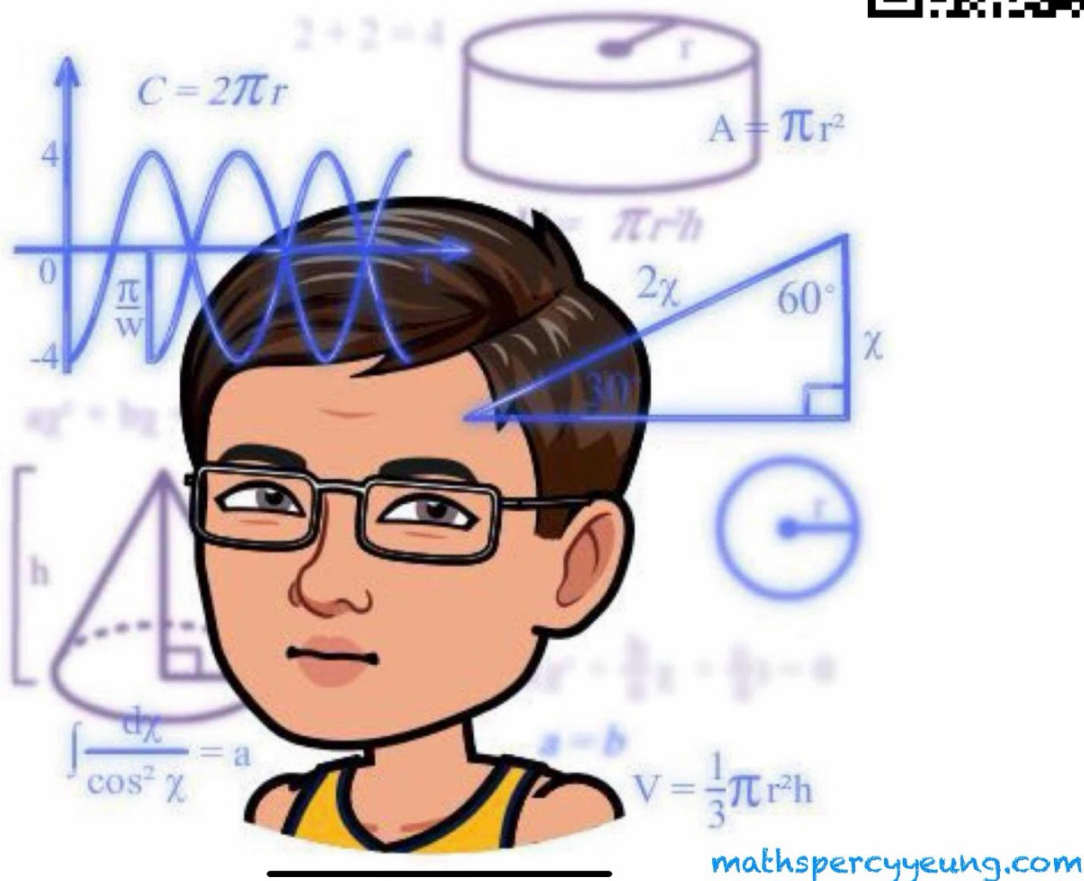
(4 marks)

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Ch.1 Number Systems

Multiple Choice Question

[19-20]

1. [19-20 Standardized test 1, #4]

Simplify $(\sqrt{3} + i)(\sqrt{27} - 3i)$.

- A. 6
- B. 12
- C. $6 + 6\sqrt{3}i$
- D. $12 + 6\sqrt{3}i$

2. [19-20 Standardized test 1, #6]

$i^{2019} - i^{11} - i^5 =$

- A. $-i$.
- B. i .
- C. -1 .
- D. $-3i$.

3. [19-20 Mid-year, #4]

$-(i^7)^5 =$

- | | |
|----------|-----------|
| A. i . | B. $-i$. |
| C. 1. | D. -1 . |

4. [19-20 Mid-year, #5]

It is given that $xi(5 + 2i) = -y + 15i$, where x and y are real numbers. Find the values of x and y .

- A. $x = 3, y = 6$
- B. $x = 3, y = -6$
- C. $x = -3, y = 6$
- D. $x = -3, y = -6$

5. [19-20 Mid-year, #10]

Which of the following are true?

- I. The sum of two irrational numbers may be a rational number.
- II. The product of a rational number and an irrational number may be a rational number.
- III. $2.0\dot{1}\dot{9}$ is a rational number.

- A. I and II only
- B. I and III only
- C. II and III only
- D. I, II and III

[20-21]

6. [20-21 Mid-year, #11]

Find the real part of $\frac{p+3i}{p-3i}$ if p is a real number.

- A. $\frac{p^2+9}{p^2-9}$
- B. $\frac{p^2-9}{p^2+9}$
- C. $\frac{6p}{p^2-9}$
- D. $\frac{6p}{p^2+9}$

7. [20-21 Final Exam, #20]

Let $z = (2+3i)(k+i)$, where k is a real number. If z is purely imaginary, then $k =$

- A. -1 .
- B. $-\frac{2}{3}$.
- C. 1 .
- D. $\frac{3}{2}$.

8. [20-21 Final Exam, #21]

The imaginary part of $3i + 6i^2 + 9i^3 + 12i^4$ is

- A. -9 .
- B. -6 .
- C. 3 .
- D. 12 .

9. [20-21 S.5 Final Exam, #33]

If k is a real number, then the real part of $\frac{2-i^6}{k-i}$ is

- A. $\frac{-k}{k^2+1}$.
- B. $\frac{1}{k^2+1}$.
- C. $\frac{k}{k^2+1}$.
- D. $\frac{3k}{k^2+1}$.

[21-22]

10. [21-22 Mid-year, #19]

If p is a real number, then the imaginary part of $\frac{5pi+i^{22}}{3p-i^{21}}$ is

- A. $\frac{5}{3}$.
- B. $\frac{-8p}{9p^2+1}$.
- C. $\frac{15p^2-1}{9p^2-1}$.
- D. $\frac{15p^2-1}{9p^2+1}$.

11. [21-22 Mid-year, #20]

Let β be a real number. Define $u = w + \frac{2}{w}$ and $v = w - \frac{2}{w}$, where $w = 1 - \beta i$. Which of the following must be true?

- I. The real part of $u + v$ is 2.
 - II. The real part of u and the real part of v are the same.
 - III. uv is a rational number.
- A. I only
 - B. II only
 - C. I and III only
 - D. II and III only

12. [21-22 Final exam, #15]

If k and $\frac{10}{3-i} + ki - i^2$ are real numbers, then $k =$

- A. -1.
- B. 0.
- C. 1.
- D. 4.

13. [21-22 S.5 Mid-year, #26]

Define $z_1 = \frac{3-ki}{1+i}$ and $z_2 = \frac{5(k+7i)}{2-i}$, where k is a real number. If z_1 is a purely imaginary number, then the real part of z_2 is

- A. -13.
- B. -1.
- C. 11.
- D. 17.

14. [21-22 S.5 Final Exam, #21]

If a is a real number, then the real part of $\frac{6i^{13} - 5ai^{10}}{2-i}$ is

- A. $\frac{10a+6}{3}$.
- B. $\frac{10a-6}{5}$.
- C. $\frac{5a+12}{3}$.
- D. $\frac{5a+12}{5}$.

15. [21-22 S.6 Mock, #35]

If k and $\frac{i^{456}}{1-i^{2021}} + ki$ are real numbers, then $k =$

- A. -1 .
- B. $-\frac{1}{2}$.
- C. $\frac{1}{2}$.
- D. 1 .

[22-23]

16. [S.4 22-23 Mid-Year,#7]

Let a be a real number. Which of the following are real numbers?

- I. $-\sqrt{a^2}$
- II. $\sqrt{-a^2}$
- III. $-\sqrt{(-a)^2}$

- A. I and II only
- B. I and III only
- C. II and III only
- D. I, II and III

17. [S.4 22-23 Mid-Year,#15]

If a and $(2+3i)(-1-ai)+6i$ are real numbers, then $a =$

- A. $\frac{3}{2}$.
- B. $\frac{2}{3}$.
- C. $-\frac{2}{3}$.
- D. $-\frac{3}{2}$.

18. [S.4 22-23 Mid-Year,#16]

$$i^{23}(7 - ki) =$$

- A. $k + 7i$.
- B. $k - 7i$.
- C. $-k + 7i$.
- D. $-k - 7i$.

19. [S.5 22-23 mid-year,#25]

If a is a real number, then the real part of $\frac{6+i^7}{a-i} + i^2$ is

- A. $\frac{6a - a^2}{a^2 + 1}$.
- B. $\frac{6 - a}{a^2 + 1}$.
- C. $\frac{6a - a^2}{a^2 - 1}$.
- D. $\frac{6 - a}{a^2 - 1}$.

20. [S.5 22-23 Final,#33]

Let $z = 9i + 8i^2 + 7i^3 + 6i^4 + ki^5$, where k is a real number. If the real part and the imaginary part of z are equal, then the imaginary part of z is

- A. -4 .
- B. -2 .
- C. 0 .
- D. 2 .

21. [S.6 22-23 Timed Practice 2,#34]

If u is a real number, then the imaginary part of $\frac{3+i^7}{u-4i^5} + i^4$ is

- A. $\frac{u^2 + 3u + 20}{u^2 + 16}$.
- B. $\frac{u^2 + 3u + 20}{u^2 - 16}$.
- C. $\frac{12 - u}{u^2 + 16}$.
- D. $\frac{12 - u}{u^2 - 16}$.

22. [S.6 22-23 Timed Practice 4,#34]

If k is a real number, then the imaginary part of $-\frac{2+8ki}{2i}-i(3k+i)$ is

- A. $-3k$. B. $3k-1$.
C. $1-4k$. D. $1-3k$.

23. [S.6 22-23 Timed Practice 5,#38]

If k is a real number, then the imaginary part of $\frac{(3+i)(1-2i)}{1+ki}$ is

- A. $-\frac{5+5k}{1+k^2}$.
B. $\frac{5+5k}{1+k^2}$.
C. $\frac{5-5k}{1+k^2}$.
D. $\frac{5k-5}{1+k^2}$.

24. [S.6 22-23 Timed Practice 6,#34]

If k is a real number, then the real part of $\frac{6-i^5}{2i^4+ki^3}$ is

- A. $\frac{-2+6k}{4-k^2}$.
B. $\frac{12+k}{4-k^2}$.
C. $\frac{-2+6k}{4+k^2}$.
D. $\frac{12+k}{4+k^2}$.

25. [22-23 S6 Mock,#35]

If k is a real number, then the imaginary part of $\frac{i^{28}+i^{23}}{k-i}+i^{17}$ is

- A. $\frac{k^2-k+2}{k^2+1}$.
B. $\frac{k+1}{k^2+1}$.
C. $\frac{-k(k+1)}{k^2+1}$.
D. $\frac{k+1}{k-1}$.

[23-24]

26. [S.4 23-24 Mid-Year,#14]

The imaginary part of $3i^{13} + 4i^{16} + 5i^{19} - 6i^{22}$ is

- A. -9 .
- B. -2 .
- C. 8 .
- D. 10 .

27. [S.4 23-24 Mid-Year,#20]

If $g = \frac{4}{1-ki}$ and $h = \frac{4}{1+ki}$, where k is a real number, which of the followings must be true?

- I. gh is a real number.
- II. The real part of g is equal to the real part of h .
- III. $\frac{1}{g} - \frac{1}{h}$ is a purely imaginary number.

- A. I and II only
- B. I and III only
- C. II and III only
- D. I, II and III

28. [S.4 23-24 Final,#20]

$$\frac{i^{2023} + 2i^{2024}}{3+i} =$$

- A. $\frac{1}{2} - \frac{1}{2}i$.
- B. $-\frac{1}{2} + \frac{1}{2}i$.
- C. $-\frac{7}{10} - \frac{1}{10}i$.
- D. $\frac{7}{10} + \frac{1}{10}i$.

28. [S.5 23-24 Mid-year,#25]

If k is a real number, then the real part of $\frac{k^2}{k+i^3} + \frac{1}{k-i^3}$ is

- A. $\frac{k-1}{k+1}$.
- B. $k^2 - 1$.
- C. $k^2 + 1$.
- D. k .

29. [S.5 23-24 Final,#34]

Let $z = 6 - ai^{11} + 4ai^{14} + 32i^{16} + 8i^{17}$, where a is a real number. If the real part and the imaginary part are equal, then the imaginary part of z is

- A. 8.
- B. 10.
- C. 14.
- D. 18.

30. [S.6 23-24 Timed Practice 2,#28]

Let $u = \frac{5}{x-2i}$ and $v = \frac{5}{x+2i}$, where x is a real number. Which of the following must be true?

- I. $u + v$ is a rational number.
- II. Real part of u is equal to real part of v .
- III. $\frac{1}{u} + \frac{1}{v}$ is a real number.

- A. I only
- B. II only
- C. I and III only
- D. II and III only

31. [S.6 23-24 Timed Practice 4,#27]

Let x be a real number. $\frac{3x-i}{i} - (2-xi) =$

- A. $-3 - 2xi$.
- B. $-1 - 4xi$.
- C. $(3x-3) - xi$.
- D. $(3x-3) + xi$.

32. [S.6 23-24 Timed Practice 6,#35]

The real part of $1 + 2i + (2i)^2 + (2i)^3 + \dots + (2i)^{67}$ is

- A. $\frac{1+2^{68}}{5}$.
- B. $\frac{1-2^{68}}{5}$.
- C. $1+2^{68}$.
- D. $1-2^{68}$.

~End~