

S.3 Mathematics

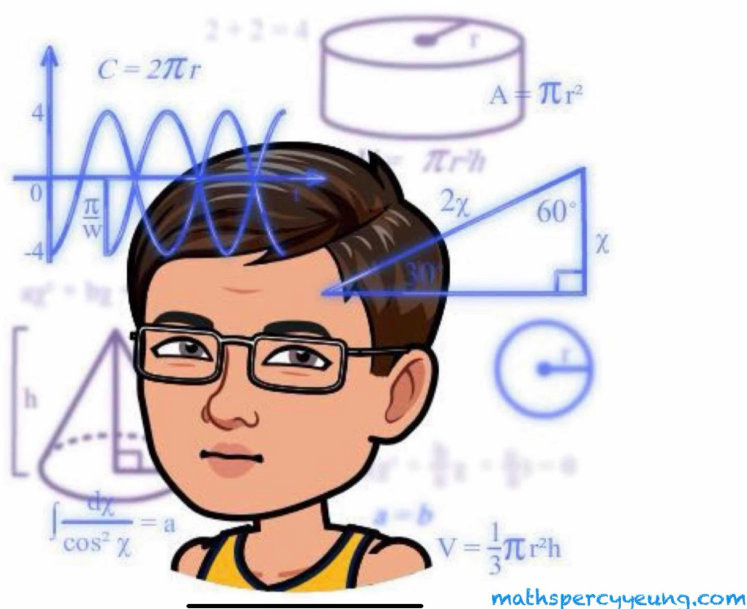
School-Based Exercise

(S.B.E)

Chapter 2

Laws of Integral Indices

Name: _____



Laws of Integral Indices

For non-zero numbers a and b , and integers m and n ,

- | | | |
|--------------------------------|--|------------------------|
| (a) $a^m \times a^n = a^{m+n}$ | (b) $\frac{a^m}{a^n} = a^{m-n}$ | (c) $(a^m)^n = a^{mn}$ |
| (d) $(ab)^n = a^n b^n$ | (e) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ | (f) $a^0 = 1$ |
| (g) $a^{-n} = \frac{1}{a^n}$ | | |

Exercise 1A + 1B

Simplify each of the following and express the answer with positive indices. (1 – 15)

- | | | |
|-------------------------------------|--|--|
| 1. $(a^2)^{-3}$ | 2. $b^{-4} \times b^3$ | 3. $c^3 \div c^{-7}$ |
| 4. $a^2 \times a^{-4} \div a^6$ | 5. $\frac{b^9 b^{-4}}{b^2}$ | 6. $\frac{c^5}{c^0 c^{-3}}$ |
| 7. $(2x^4)^{-5}$ | 8. $(x^7 x^{-3})^{-2}$ | 9. $a^5(a^{-3})^4$ |
| 10. $\frac{y^3}{(y^2)^{-1}}$ | 11. $\frac{(2b^3)^{-4}}{b}$ | 12. $\frac{x^{-1} y^{-4}}{y^2}$ |
| 13. $\frac{m^{-6} n^2}{m^2 n^{-4}}$ | 14. $\frac{a^0 b^{-5}}{(a^4 b^{-2})^{-3}}$ | 15. $x^2 \left(\frac{x^3 y}{y^{-4}}\right)^{-1}$ |

Evaluate each of the following without using a calculator and give the answer as an integer or a simplest fraction. (16 – 21)

- | | | |
|--|---|--|
| 16. $\frac{7^{-5}}{7^0 \times 14^{-3}}$ | 17. $2^{-15} \times 4^{-5} \div (-2)^{-26}$ | 18. $\frac{5^9 \times 5^{-17}}{25^{-3}}$ |
| 19. $\frac{(3^8 \times 9^{-3})^{-4}}{27^{-3}}$ | 20. $\frac{2^{-11}}{(8^{-6} \times 4^8)^3}$ | 21. $\frac{3^9 \times 6^{-7}}{2^{-5}}$ |

Simplify each of the following and express the answer with positive indices. (22 – 27)

- | | | |
|--|--|---|
| 22. $\frac{(p^3 q)^2}{4p^{-4} q^3}$ | 23. $\frac{(x^{-5} y^2)^3}{(x^4 y^{-1})^{-5}}$ | 24. $\frac{-(r^{-3} s^7)^{-1}}{(2r^{-3} s)^2}$ |
| 25. $\frac{a^{-2} (b^{-5})^0}{(a^{-3} b^7)^2}$ | 26. $(x^2 y^{-1})^4 \left(\frac{y^3}{x}\right)^{-3}$ | 27. $\left(\frac{4x^3}{y}\right)^{-1} \left(\frac{3x^2}{y^{-3}}\right)^2$ |

28. Let n be a positive integer. Simplify each of the following expressions and express the answer with positive indices.

(a) $(7^{3n+1})^2$

(b) $2^n \times 8^{2n} \times 4^{-5n}$

(c) $\frac{9^{3n} \times 81^{-n}}{(27^3)^n}$

(d) $\left(\frac{5^{n-1}}{5^{2n+1}}\right)^{-3}$

29. Arrange the following numbers in ascending order.

$$8^{-9}, 4^{-12}, 32^{-5}$$

30. Arrange the following numbers in descending order.

$$2^{-18}, 3^{-12}, 5^{-6}$$

31. It is given that $x^3 = 5$, $y^2 = 10$ and $z^5 = 4$, find the value of $\frac{z(2x^2y^{-1})^{-3}}{2yz^{-4}}$ without using a calculator.

Scientific Notation

A positive number is said to be expressed in scientific notation if it is in the form of

$$p \times 10^n,$$

where $1 \leq p < 10$ and n is an integer.

Exercise 1C

- Express each of the following numbers in scientific notation.
(a) 1 300 000 (b) 239 100 000
(c) 0.095 7 (d) 0.000 000 34
- Express each of the following numbers as an integer or a decimal.
(a) 7.25×10^6 (b) -6.86×10^4
(c) 1.17×10^{-2} (d) -7.34×10^{-7}
- Express each of the following numbers in scientific notation correct to 3 significant figures.
(a) 54 667 000 (b) -974 630
(c) 0.000 137 162 (d) -0.000 007 478 73
- Evaluate each of the following expressions without using a calculator and express the answer in scientific notation.
(a) $800\,000\,000 + 3\,000\,000$
(b) $(7 \times 10^5) \times (2 \times 10^{-9})$

Express each of the following numbers in scientific notation. (5 – 10)

- | | | |
|-------------|-----------------|---------------|
| 5. 20 500 | 6. -108 000 000 | 7. 8 283 000 |
| 8. 0.036 88 | 9. 0.000 002 49 | 10. -0.007 28 |

Convert each of the following numbers to an integer or a decimal. (11 – 16)

- | | | |
|---------------------------|----------------------------|-----------------------------|
| 11. 3.59×10^4 | 12. -5.601×10^2 | 13. 2.12×10^8 |
| 14. 2.36×10^{-3} | 15. -9.27×10^{-5} | 16. -1.059×10^{-6} |

Round off each of the following numbers correct to 3 significant figures and express the answer in scientific notation. (17 – 19)

- | | | |
|----------------|------------------|----------------|
| 17. 45 793 200 | 18. -0.001 433 1 | 19. 0. 029 039 |
|----------------|------------------|----------------|

Evaluate each of the following expressions without using a calculator and express the answer in scientific notation. (20 – 25)

20. $2.7 \times 10^3 + 3.5 \times 10^3$ 21. $6.5 \times 10^{-6} - 1.7 \times 10^{-6}$ 22. $(2 \times 10^5) \times (3.6 \times 10^3)$
 23. $(2.4 \times 10^7) \times (4 \times 10^{-4})$ 24. $(4.8 \times 10^{15}) \div (1.2 \times 10^6)$ 25. $(9.6 \times 10^{-8}) \div (6 \times 10^7)$

26. The equatorial radius of the Moon is approximately 1 740 000 m. Express the radius in scientific notation in each of the units cm, m and km.
 27. The length of a house dust mite is about 0.000 58 m. Express the length in scientific notation in each of the units cm, m and km.
 28. Earth orbits the Sun at an average speed of 107 000 km/h. Express the speed in scientific notation in each of the units km/h and m/s. (Give the answers correct to 3 significant figures if necessary.)

Express each of the following numbers in scientific notation correct to 3 significant figures. (29 – 34)

29. 7324×10^7 30. -53.67×10^4 31. $0.000\ 459\ 8 \times 10^8$
 32. $-21\ 170 \times 10^{-5}$ 33. $0.043\ 87 \times 10^{-2}$ 34. -353.849×10^{-4}

Evaluate each of the following expressions without using a calculator and express the answer in scientific notation. (35 – 40)

35. $9.4 \times 10^6 + 5.7 \times 10^6$ 36. $1.3 \times 10^5 - 4.7 \times 10^4$ 37. $4.1 \times 10^{-3} - 8.3 \times 10^{-2}$
 38. $(5 \times 10^{-7}) \times (2.5 \times 10^4)$ 39. $(2.56 \times 10^2) \div (3.2 \times 10^{-3})$ 40. $200\ 000\ 000 \times 80\ 000$

Use a calculator to evaluate each of the following expressions. Give the answer in scientific notation correct to 3 significant figures. (41 – 46)

41. $3.14^5 \times 10^{-6}$ 42. $\left(\frac{1}{3} + \frac{1}{7}\right)^2 \times 10^5$ 43. $-\frac{2.69}{3.42 \times 10^{-7}}$
 44. $\frac{7.2 \times 10^{12} + 9.6 \times 10^{14}}{5.5 \times 10^6}$ 45. $\frac{1}{3.8 \times 10^{-7} - 4.7 \times 10^{-8}}$ 46. $\sqrt{2015 \times 10^{-16}}$

47. It is given that the volume of Earth is approximately 1.08×10^{12} km³ and the volume of Mars is 0.151 times that of Earth. Estimate the volume of Mars in km³.
 (Give the answer in scientific notation correct to 3 significant figures.)

48. The central processing unit (CPU) of a notebook computer takes 2.91×10^{-10} seconds to execute one instruction. If it takes 4.5 minutes to solve a complicated algebraic problem, find the number of instructions involved.
(Give the answer in scientific notation correct to 3 significant figures.)
49. (a) How many minutes are there in 1 year?
(b) If the average heart beats of David is 68 times per minute, estimate the number of times his heart beats in a decade.
(Take 1 year = 365.25 days.)
(Give the answers in scientific notation correct to 3 significant figures.)
50. In 2013, the number of visitors to Hong Kong was about 54.3 million.
(a) If 47% of visitors stayed overnight, estimate the number of overnight visitors in 2013.
(b) If the average spending per overnight visitor is approximately \$8100, estimate the total spending of overnight visitors in Hong Kong in 2013.
(Give the answers in scientific notation correct to 3 significant figures.)
51. The total capacity of impounding reservoirs in Hong Kong is approximately 586 050 000 m³. Express the total capacity in scientific notation in cm³.

Numeral Systems

Exercise 1D + 1E

1. Consider the denary number 618.302, write down the digit with place value

- (a) 10^2 , (b) 10^0 ,
(c) 10^{-2} , (d) 10^{-3} .

2. In each of the following numbers, write down the place value of the digit '0'.

- (a) 40 378 (b) 1011111_2

Express each of the following numbers in the expanded form. (10 – 15)

3. 2637 4. 62.742

5. 10110.1_2 6. 1001.01_2

7. (a) Express $4 \times 10^3 + 1 \times 10^2 + 9 \times 10^1 + 0 \times 10^0 + 8 \times 10^{-1}$ as a denary number.

- (b) Express $6 \times 100 + 0 \times 10 + 5 \times 1 + 8 \times \frac{1}{10} + 1 \times \frac{1}{100}$ as a denary number.

- (c) Express $5 \times 10 + 8 \times 1 + 4 \times \frac{1}{1000}$ as a denary number.

8. (a) Express $1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1}$ as a binary number.

- (b) Express $1 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1 + 0 \times \frac{1}{2} + 1 \times \frac{1}{4}$ as a binary number.

- (c) Express $1 \times 16 + 1 \times 4 + 1 \times 2 + 1 \times \frac{1}{2} + 1 \times \frac{1}{8}$ as a binary number.

9. Which of the following numbers is the smallest?

1001.1_2 , 1010.01_2 , 110.01_2

10. Arrange the following numbers in descending order.

10101.1_2 , 10110.1_2 , 1111.11_2

Convert each of the following binary numbers to a denary number. (11 – 14)

11. (a) 10_2 (b) 100_2 (c) 1000_2

12. (a) 1001_2 (b) 1111_2 (c) 1011_2

13. (a) 11101_2 (b) 11010_2 (c) 10111_2

14. (a) 101001_2 (b) 100110_2 (c) 110101_2

Convert each of the following denary numbers to a binary number. (15 – 18)

- | | | |
|-------------|---------|---------|
| 15. (a) 7 | (b) 19 | (c) 31 |
| 16. (a) 42 | (b) 53 | (c) 84 |
| 17. (a) 106 | (b) 153 | (c) 186 |
| 18. (a) 216 | (b) 239 | (c) 254 |

MC

1. $7(8^0 \times 7)^{-2} =$

- A. 7^{-3} .
- B. 7^{-2} .
- C. 7^{-1} .
- D. 7 .

2. $\frac{a^3}{a^{-3}b^{-5}} =$

- A. b^{-5} .
- B. a^3b^{-5} .
- C. a^6b^5 .
- D. a^9b^5 .

3. $-(p^3q)^{-2} =$

- A. $-\frac{p^3}{q^2}$.
- B. $\frac{p^3}{q^2}$.
- C. $-\frac{1}{p^6q^2}$.
- D. $\frac{1}{p^6q^2}$.

4. $3^0 \times 9^2 \times (-3)^{-3} =$

- A. 9 .
- B. -3 .
- C. $\frac{1}{3}$.
- D. $-\frac{1}{9}$.

5. $\frac{a^4}{(2a^2b^{-1})^{-1}} =$

- A. $\frac{2a^6}{b}$.
- B. $\frac{a^2b}{2}$.
- C. $\frac{a^2}{2b}$.
- D. $2a^2b$.

6. $\frac{(2^3 \times 10^{-3})^2}{5^{-5}} =$

- A. $\frac{1}{10}$.
- B. $\frac{1}{5}$.
- C. $\frac{4}{5}$.
- D. $\frac{5}{2}$.

7. If x is a non-zero number, then

$\left(\frac{1}{x}\right)^{2015} (-x)^{2014} =$

- A. $-\frac{1}{x}$.
- B. $\frac{1}{x}$.
- C. $-x$.
- D. x .

8. Which of the following numbers is / are expressed in scientific notation?

I. -0.9×10^5

II. 1×10^{-7}

III. 23×10^8

A. I only

B. II only

C. I and III only

D. II and III only

9. $1.64 \times 10^{-10} =$

A. 0.000 000 000 164.

B. 0.000 000 016 4.

C. 164 000 000.

D. 16 400 000 000.

10. $(3.8 \times 10^{-8}) \times (-6 \times 10^3) =$

A. -2.28×10^{-6} .

B. -2.28×10^{-5} .

C. -2.28×10^{-4} .

D. 2.28×10^{-5} .

11. $1.2 \times 10^6 - 8.5 \times 10^5 =$

A. -1.02×10^{11} .

B. -7.3×10^6 .

C. 3.5×10^6 .

D. 3.5×10^5 .

12. $\frac{4 \times 10^6}{5 \times 10^{-3}} =$

A. 8×10^9 .

B. 8×10^8 .

C. 8×10^7 .

D. 8×10^6 .

13. The distance between Hong Kong and Beijing is approximately 1963 km. Express the distance in mm in scientific notation.

A. 1.963×10^3 mm

B. 1.963×10^6 mm

C. 1.963×10^8 mm

D. 1.963×10^9 mm

14. Express $1 \times 2^5 + 1 \times 2^2 + 1$ as a binary number.

A. 100011₂

B. 100110₂

C. 100101₂

D. 1000110₂

15. 101000011₂ =

A. $1 \times 2^8 + 1 \times 2^6 + 1 \times 2^2 + 1 \times 2^1$.

B. $1 \times 2^8 + 1 \times 2^6 + 1 \times 2^1 + 1 \times 2^0$.

C. $1 \times 2^9 + 1 \times 2^7 + 1 \times 2^2 + 1 \times 2^1$.

D. $1 \times 2^9 + 1 \times 2^7 + 1 \times 2^1 + 1 \times 2^0$.

16. Which of the following numbers is the largest?

A. 10101₂

B. 10011₂

C. 11010₂

D. 11001₂

17. 1000000000₂ =

A. 128.

B. 256.

C. 512.

D. 1024.