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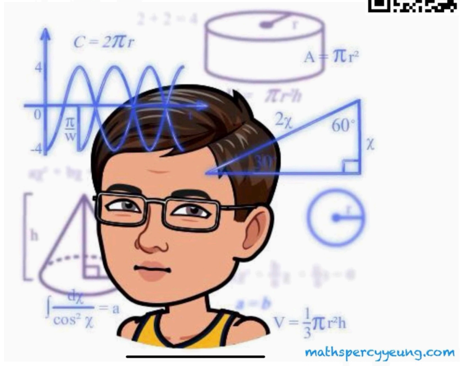
mathspercyyeung



2021-2022 S5
1st TERM UT
MATH EP
M2

2021 – 2022
S5 First Term Uniform Test

MATHEMATICS Extended Part

Module 2 (Algebra and Calculus)

Question–Answer Book

4th November, 2021
10:15 am – 11:15 am (1 hour)
This paper must be answered in English

INSTRUCTIONS

1. Write your name, class and class number in the spaces provided on this cover.

2. This paper consists of TWO sections, A and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question – Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers must be exact.

6. The diagrams in this paper are not necessarily drawn to scale.

Sections	Marks
A Total	**/33**
B Total	**/12**
TOTAL	**/45**

FORMULAS FOR REFERENCE

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$	$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$	$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$
$2\sin A \cos B = \sin(A+B) + \sin(A-B)$	$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$
$2\cos A \cos B = \cos(A+B) + \cos(A-B)$	
$2\sin A \sin B = \cos(A-B) - \cos(A+B)$	

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Section A (33 marks)

1. Let $f(x) = \dfrac{1}{3x^2+4}$. Find $f'(x)$ from first principles. (4 marks)

2. Prove that $\sin^2(x-y)-2\sin x\cos y\sin(x-y)=\cos^2 x-\cos^2 y$. (3 marks)

Answers written in the margins will not be marked

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3. (a) Using mathematical induction, prove that $10+32+66+\cdots+2n(3n+2)=n(n+1)(2n+3)$ for all positive integers n.

(b) Using (a), evaluate $\sum_{k=5}^{50} k(3k+2)$.

(7 marks)

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4. A particle moves with a velocity of 3 m/s along a straight line initially and the acceleration after t seconds is $\dfrac{1}{(t+1)^2}$ m/s^2. Find the distance travelled by the particle in the first t seconds in terms of t. (5 marks)

Answers written in the margins will not be marked

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5. Find $\int \tan^4 \frac{x}{2} \sec^2 \frac{x}{2}\, dx$. (2 marks)

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6. Find $\int \frac{1}{x^2\sqrt{1-9x^2}}\,dx$. (5 marks)

Answers written in the margins will not be marked

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7. (a) Using integration by parts, find $\int x^2(1-e^x)\,dx$.

(b) At any point (x, y) on a curve, the slope of the tangent to the curve is $\dfrac{x^2(1-e^x)}{2}$. The y-intercept of the curve is 1. Find the equation of the curve.

(7 marks)

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Section B (12 marks)

8.

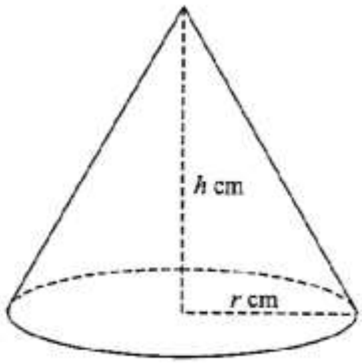

Figure 1

In a Winter Carnival, a display item is in the shape of a right circular cone. It is made of ice and a stabilizer so that the display remains in the shape of a right circular cone with the volume remaining constant. Within the duration of the Carnival, the height of the cone decreases at a constant rate of 2 cm per day. At time t days after the beginning of the Carnival, the base radius and height of the cone are r cm and h cm respectively (see Figure 1).

(a) Show that $\dfrac{dr}{dt} = \dfrac{r}{h}$. (4 marks)

(b) Let S cm^2 be the curved surface area of the cone.

(i) Show that $\dfrac{d}{dt}\left(S^2\right) = \dfrac{2\pi^2 r^2}{h}\left(2r^2 - h^2\right)$.

(ii) At the beginning of the Carnival, the height of the cone is 1.2 times the base radius. The gatekeeper of the Carnival claims that the curved surface area of the display increases during the whole period of the Carnival.

Do you agree with the gatekeeper? Explain you answer.

(8 marks)

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